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DYNAMIC PROPERTIES OF AIRCRAFT COMPONENTS AT REAL CONSTRAINTS

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Abstract. The basic purpose of this article is consideration of the problems connected with the application of a method of concentrated weights in the tasks of mechanical system dynamics with non-classic internal and external constraints. The method of concentrated weights is a convenient method to analyse the dynamic properties of elastic mechanical systems. It has relative simplicity of definition of the parameters of the equivalent discrete system and clearness of computing algorithms and provides comprehensible accuracy of definition of the lowest natural frequencies. A doubtless advantage of the method is its convenience of modelling non-classic constraints of fastening and internal constraints between elements of complex systems. Such problems arise *when making decisions about the* practice tasks of the analysis of the dynamics of real systems. The method is used for the analysis of vibrations of a beam with variable parameters at the presence of elastic supporting of the beam and attached additional concentrated weight.

Keywords: aircraft component, dynamics, non-classic constraint.

1. Introduction

This article is connected with the European 6FP project AISHA. The basic purpose of the project is the development of a continuous monitoring system of a technical condition integrated into a structure. Progressive methods and means of the control over use of ultrasonic technology are developed. In thin-walled structures, it uses properties of elastic Lamb waves. The final stage demonstrates the execution of full-scale fatigue tests on components of real aviation structures to demonstrate the working capacity and efficiency of methods and means of

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non-destructive testing. The object of testing is the tail beam of an MI-8 helicopter (Fig 1). It is a typical aircraft component made of aluminium alloy.

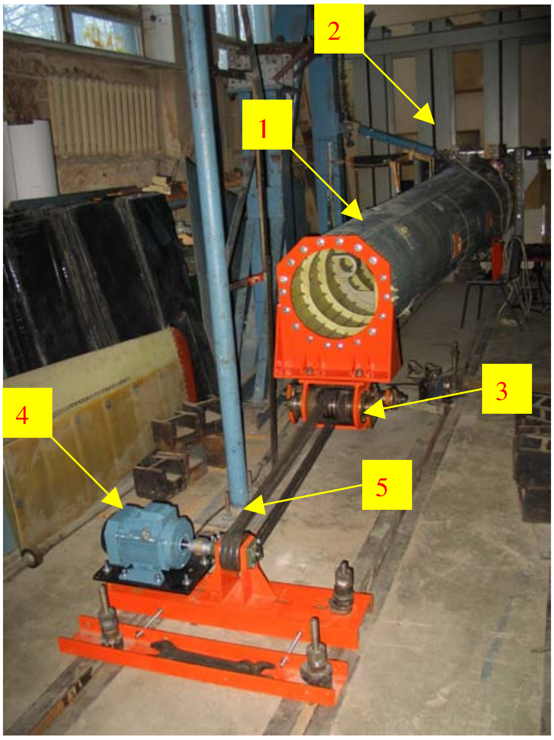

Fig 1. Stand of dynamic testing of an MI-8 helicopter tail beam (1–tail beam, 2–support, 3–mechanical vibrator, 4–motor, 5–V-belt gear)

At the planning stage of an experiment, there are problems with choosing an optimum mode for the tests that would combine the demanded distribution of stresses, duration of tests, and relative simplicity of excitation of the mechanical load. In connection with this, there was the necessity to analyse the dynamic characteristics of the tested object.

There are plenty of methods for the dynamic analysis of a mechanical system (Timoshenko *et al.* 1974, Tse *et al.* 1983, Mei 2006, Romeo 2006). Exact analytical methods have limited application, and their role is mainly the reference solution for the estimation of the accuracy of approximated methods. Among the latter, a prominent place is occupied by Ritz's methods, various versions of energetic methods, and methods of replacing a system having continuous parameters with a system having a limited number of concentrated weights (Zirkelback *et al.* 2006, James *et al.* 1989, Newland 1989, Collar *et al.* 1987). The universal numerical method of finite elements (FEM) is also one of the methods of this group (Huebner 1975).

There are many methods to analyse the dynamic behaviour of complex elastic structures, and special estimation of accuracy by comparison with a precise analytical solution usually shows the rational selection of model parameters and allows obtaining acceptable results of simulation. But modelling of boundary conditions is often not adequate. Real experimental data therefore exhibits an unacceptable difference from the results of simulation in such cases.

In this research, there are two aims. The first is the method of the concentrated application of weights to define the dynamic characteristics of a real aircraft component with continuously distributed weight. The second (and this is the main aim) is the correct simulation of non-classic internal and external constraints at the application of a method of concentrated weights.

2. Method of concentrated weights

In connection with the relatively long length of a beam in comparison with the diameter of its cross-section, the beam schematisation of such structure is admissible at the analysis of its dynamic characteristics. The equation of movement in this case looks like

$$m(z)\frac{\partial^2 v}{\partial t^2}+\frac{\partial^2}{\partial z^2}\left(EJ_x\frac{\partial^2 v}{\partial z^2}\right)=q(z,t)\,, \qquad (1)$$

where $v(z,t)$ is deflection of a beam, $m(z)$ is intensity of weight distribution, EJ_x is bending stiffness of the cross-sections of a beam, and $q(z,t)$ is intensity of the distributed external load. If $q(z,t)\cong 0$, free movement without participation of external forces is realized.

Generally, if the mass and stiffness of the cross-sections is distributed non-uniformly, an analytical solution cannot be received. The approximated method of concentrated weights is therefore used for the purpose of this research. For definition of natural frequencies and forms, the actual beam with continuously distributed weight must be replaced by a weightless beam with the same bending stiffness, but with a finite number of concentrated weights in a finite number of nodes $(k+1)$. For this purpose, the beam has been sheared into finite number of parts k. For each of them weight M_i and its centre coordinate z_{ic} was defined, and then it was distributed between nodes $(i\text{-}1)$ and i, which are finite nods of the part. From a condition of static equivalence, there are the next expressions

$$M_i=\Delta M_{i-1}+\Delta M_i$$

$$M_i\cdot z_{ic}=\Delta M_{i-1}\cdot z_{i-1}+\Delta M_i\cdot z_i=\int_{z_{i-1}}^{z_i} z_i\cdot m(z)dz$$

where ΔM_{i-1} and ΔM_i are two components of weight of the part attached to units $(i\text{-}1)$ and i accordingly.

If the sheared beam has enough many of parts, within one part the distribution of weight can be accepted as linear and then the last two equations can be written down as

$$M_i=\Delta M_i+\Delta M_{i-1}\cong\frac{m_i+m_{i-1}}{2}\cdot\Delta z_i$$

$$M_i z_{ic}=\Delta M_{i-1}z_{i-1}+\Delta M_i z_i\cong\frac{m_i z_i+m_{i-1}z_{i-1}}{2}\cdot\Delta z_i$$

or in the dimensionless form

$$\bar{M}_i = \Delta\bar{M}_i + \Delta\bar{M}_{i-1} \cong \frac{\bar{m}_i + \bar{m}_{i-1}}{2}\cdot\Delta\bar{z}_i,$$

$$\bar{M}_i\cdot\bar{z}_{ic} = \Delta\bar{M}_{i-1}\cdot\bar{z}_{i-1} + \Delta\bar{M}_i\cdot\bar{z}_i \cong \frac{\bar{m}_i\cdot\bar{z}_i + \bar{m}_{i-1}\cdot\bar{z}_{i-1}}{2}\cdot\Delta\bar{z}_i$$

where $\Delta\bar{M}_i = \Delta M_i/(m_0 l)$, $\Delta\bar{z}_i = \Delta z_i/l$.
As a result, relative weights can be expressed in the dimensionless form as

$$\Delta\bar{M}_i = \bar{M}_i\cdot\frac{\bar{z}_i - \bar{z}_{ic}}{\Delta\bar{z}_i},\quad \Delta\bar{M}_{i-1} = \bar{M}_i\cdot\frac{\bar{z}_{ic} - \bar{z}_{i-1}}{\Delta\bar{z}_i}, \qquad (2)$$

where

$$\bar{z}_{ic} = \frac{\bar{m}_i\cdot\bar{z}_i + \bar{m}_{i-1}\cdot\bar{z}_{i-1}}{2M_i}\cdot\Delta\bar{z}_i$$

It is obvious, that the total relative weight in node *i* is $\bar{m}_i = \bar{M}_i + \bar{M}_{i+1}$.

On the basis of the principle of superposition, displacement *Δi* in node *i* (*i=0, 1, … k)* as result of the action of the system of vertical forces P_j concentrated in all nodes, is defined by the following sum:

$$\Delta_i = \sum_{j=1}^{n}\delta_{ij}\cdot P_j = -\sum_{j=1}^{n}\delta_{ij}\cdot m_j\cdot\Delta''_j\ , \qquad (3)$$

where δ_{ij} is the factor of elastic compliance.
In the second part of equation (3), the force P_j is replaced by the inertial force that appears in case of natural vibrations. These vibrations are harmonious with frequency ω. Therefore

$$\sum_{j=1}^{n}\delta_{ij}\cdot m_j\Delta''_j + \Delta_i = 0 \qquad \Delta_i = A_i\cdot Sin\omega t\ ,$$

where A_i is the amplitude of vibration of node *i*.
As a result the next system of *k* the linear homogeneous algebraic equations should be solved

$$\delta_{i1}m_1A_1 + \ldots + \left(-\frac{1}{\omega^2} + \delta_{ii}m_i\right)A_i + \ldots + \delta_{in}m_nA_n = 0\,.$$
$$(i = 1, \ldots, k) \qquad (4)$$

For greater generality, this system of equations is convenient for writing down in a dimensionless view

$$\bar{\delta}_{i1}\bar{m}_1A_1 + \ldots + (-x + \bar{\delta}_{ii}\bar{m}_i)A_i + \ldots + \bar{\delta}_{in}\bar{m}_nA_n = 0\ , \quad (5)$$

where $x = \frac{EJ_{x0}}{m_0 l^2}\cdot\frac{1}{\omega^2}$, $\quad \bar{\delta}_{ij} = \frac{\delta_{ij}}{l^3/(EJ_{x0})}$, $\quad m_j = \frac{m_j}{m_0 l}$

It is known that a non-trivial decision of system (5) exists if the determinant of the coefficients of the equations is equal to zero.

$$Det\{[\bar{\delta}_{ij}\bar{m}_j] - x[I]\} = 0\ , \qquad (6)$$

Roots x_k of equation (6) define natural frequencies of vibrations.

$$\omega_k = \frac{1}{l^2}\cdot\sqrt{\frac{EJ_{x0}}{x_k m_0}} \qquad (7)$$

After their definition, the natural forms can be received too.

To verify the definition of dynamic characteristics by a method of concentrated weights, calculation for a console beam of constant cross-section and uniformly distributed weight was carried out. It was established that if there is *k=8…10*, then at least the first two natural frequencies practically coincide with their exact values.

3. Analysis of dynamic characteristics of a beam under ideal boundary conditions

The method described above was used for the numerical analysis of natural frequencies and forms of a beam of the helicopter at perfect boundary conditions: jamming at one tip. In figure 2, the scheme of the test system is presented. The thin-walled tail beam of the helicopter has elastic attachment 1 to the motionless tip, and in the middle part of the beam, additional weight 2 (about 94 kg) is attached. On the free tip of the beam, mechanical vibrator 3 weighing about 50 kg is installed.

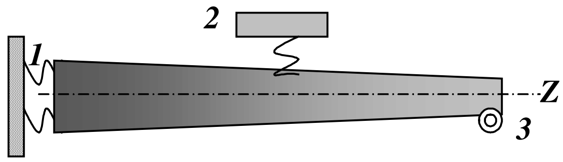

Fig 2. The beam has elastic support 1 and mass 2 also has the same kind of connection with the beam;
3 is the vibrator

In figure 3, relative distribution of bending stiffness is shown. If it is accepted that the beam and additional weight have absolutely rigid constraints, then the relative distribution of weight among the nodes is in table. The weight of the beam itself, the attached weight in the middle zone of the beam, and the weight of the vibrator is common weight of dynamic system.

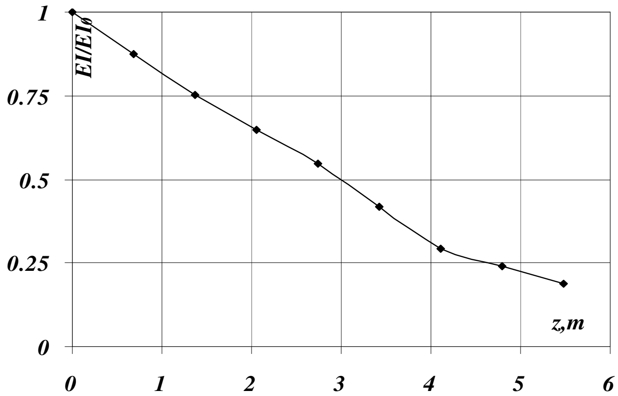

Fig 3. Distribution of the relative stiffness of the tail beam cross-sections

In figure 4, the results of calculating the first two natural frequencies and forms are shown. In connection with the significant bending stiffness of the beam and its light weight, it has high frequencies. The second form of vibrations is obeyed by the requirements of planned tests: the maximal curvature of deformed axes of a beam and maximal stress takes place in the middle part of the beam.

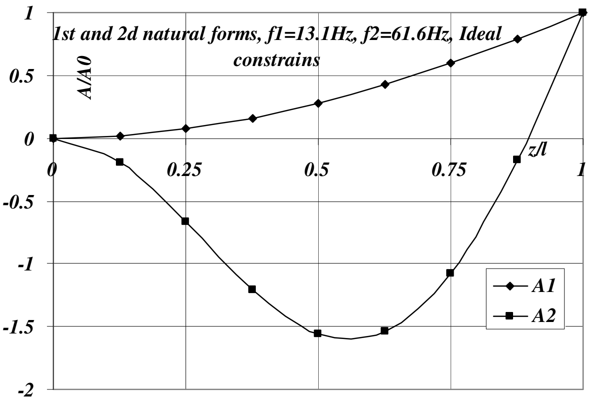

Fig 4. First and second natural forms of vibration

Mass distribution among the nodes of the system

Weight number	0	1	2	3	4	5
m/m_0	1	0.043	0.063	0.061	0.243	0.241

Weight number	6	7	8	9	Total
m/m_0	0.049	0.041	0.039	0.220	1.00

4. Account of influence of non-classic internal and external constraints

Two types of deviations from ideal boundary conditions are examined:
– Elastic fastening;
– An elastic attachment of additional concentrated weight.

Let the characteristic of the elastic hinge look like $M_0 = K\theta$, where K is elastic hinge stiffness, θ is the angle of turn in the hinge, M_0 is the bending moment in the hinge.

If single force acts in a node, then the bending moment $M_0=1z_j$. It causes a rigid turn of the beam around the axis of the hinge $\theta_j = M_0 / K = z_j / K$. As a result, in section j there will be an additional deflection caused by the elastic compliance of the hinge

$$\delta_{0ij} = \theta_j z_i = \frac{z_i z_j}{K}$$

or in the dimensionless form

$$\bar{\delta}_{0ij} = \frac{\delta_{0ij}}{l^3/(EJ_{x0})} = \frac{\bar{z}_i \bar{z}_j}{Kl/(EJ_{x0})} \tag{8}$$

To consider the influence of the elastic compliance hinge fastenings, it is necessary to calculate total compliance $\bar{\delta}_{\Sigma ij} = \bar{\delta}_{ij} + \bar{\delta}_{0ij}$. The subsequent calculation of frequencies and forms remains the same.

The matrix of elastic compliance should be updated for the account of elasticity of constraint. Let additional weight m_a be attached to node k by means of an elastic connection with stiffness C. It is obvious that the connection of additional weight increases the number of degrees of freedom and means that the number of equations of movement will be equal to $n+1$. Thus

$$\delta_{i,n+1} = \begin{cases} \delta_{ik}, \ldots\ldots\ldots\ldots\ldots i \neq k \\ \delta_{kk} + 1/C, \ldots i = k \end{cases} \tag{9}$$

Both kinds of elastic connections are available in the dynamic system examined in above. For the real object prepared for test, an approximated estimation of stiffness of connections has been executed. Their relative values are equal accordingly

$$\bar{K} = \frac{K}{EJ_{x0}/l} = 2.0, \qquad \bar{C} = \frac{C}{1/\delta_{kk}} = 1.0$$

In figure 5, the results of calculating the first two natural frequencies and forms of the beam in this dynamic system are presented. The natural form value for additional concentrated weight is equal to 1.42. This means the displacement of additional concentrated weight is 1.42 times more than the displacement of the free tip of the beam and has the opposite phase of vibrations. It is apparent that the first natural form of beam oscillation is mainly defined by the angular compliance of trailer support. The axis of the beam is practically straight line, which testifies to the small effect of bending. The second natural frequency and the form in this case are more similar to the first frequency and the form of vibrations at rigid jamming. Nevertheless, this form is much more complex.

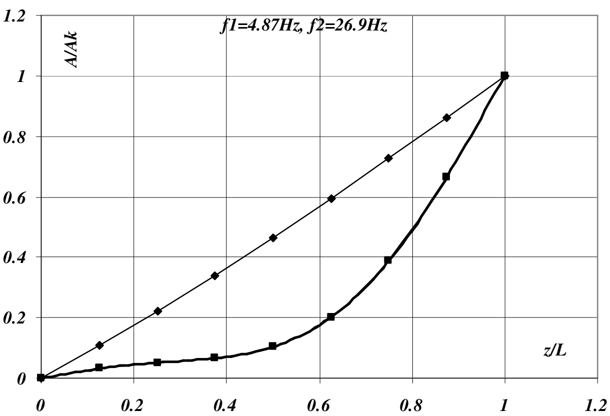

Fig 5. First and second natural frequencies and forms at non-classic constraints

The done analysis shows the exclusively strong influence of the elastic constraints between components of system to dynamic characteristics, in particular, to natural forms and frequencies of vibrations.

5. Conclusion

The method of concentrated weights is a convenient means to analyse the dynamic properties of elastic mechanical systems. It has relative simplicity of definition of the parameters of the equivalent discrete system and clearness of computing algorithms and provides comprehensible accuracy of definition of the lowest natural frequencies. A doubtless advantage of this method is its convenience of modelling non-classic constraints of fastening and internal constraints between elements of complex systems. Such problems arise in practical problems of the analysis of the dynamics of real systems. The calculations presented for the real testing system demonstrate the very important effect of real boundary conditions on the dynamic properties of elastic systems.

Acknowledgement

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ORLAIVIO KOMPONENTŲ DINAMINĖS SAVYBĖS ESANT REALIEMS APRIBOJIMAMS

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S a n t r a u k a

Pagrindinis straipsnio tikslas – analizuoti atskiras problemas, susijusias su koncentruotų masių metodu mechaninių sistemų dinamikos uždaviniuose su neklasikiniais vidiniais ir išoriniais ryšiais. Koncentruotų masių metodu yra patogu analizuoti tamprių mechaninių sistemų dinamines savybes. Šio metodo savybės yra tokios: ekvivalentinės diskrečios sistemos parametrų paprastas nustatymas; aiškus skaičiavimo algoritmas; pakankamai tikslus laisvųjų virpesių žemo dažnio nustatymas. Modeliuojant neklasikinius vidinius ir išorinius elementų ryšius sudėtingoje mechaninėje sistemoje išryškėja neabejotini metodo privalumai. Tokios problemos iškyla analizuojant realias dinamines sistemas.
Šių tyrimų metu metodas buvo panaudotas analizuoti sraigtasparnio sijos virpesius, kai kinta tampraus įtvirtinimo ir tampriai pritvirtintos koncentruotos masės parametrai.

Reikšminiai žodžiai: orlaivio komponentai, dinamika, neklasikinis ryšys.