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DETERMINING THE FORM OF ERROR DISTRIBUTION OF GEODETIC MEASURING

Valeriy GLADILIN[1], Vadym BELENOK[2], Nataliia SHUDRA[3]

[1]*Department of Geodesy, Cartography and Land Management, Bila Tserkva National Agrarian University, Soborna pl. 8/1, 09117 Bila Tserkva, Kyivska oblast, Ukraine*
[2]*Department of Aerospace Geodesy and Land Management, National Aviation University, Kosmonavta Komarova av. 1, 03058 Kyiv, Ukraine*
[3]*Department of Engineering Geodesy, Kyiv National University of Construction and Architecture, Povitroflotsky av. 31, 03037 Kyiv, Ukraine*

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Abstract. Industrial equipment is a dynamic system and has deformations not only during installation but also during operation. Under the influence of variable load and displacement of the center of gravity, the soil under the foundation settles unevenly, and accordingly, the equipment deforms unevenly, which is a threat to the equipment, the greater the load corresponds to more subsidence.
Separation of partial deformations from full is important for determining the elements of straightening equipment for its uninterrupted and trouble-free operation. The presence of significant total deformation does not affect the performance of the equipment. The most critical deformations are partial deformations. Absolute vertical deformations are calculated as the difference in sediment between adjacent sediment marks, which are fixed on the equipment in the same measurement cycle.
Comparing the values of deformations with the allowable technical conditions, decide on the need for straightening and adjustment of equipment.
The accuracy of installation is characterized by a tolerance of 0.1÷0.5 mm on the relative position of the equipment, which is conjugate mounted at a distance of several tens or hundreds of meters. For installation of the equipment with such accuracy carry out special geodetic works with use of methods and technical means of measurements specially developed for this purpose in geodesy, metrology and mechanical engineering.

Keywords: deformation (full, general, partial), geodetic measurement, error distribution, installation of precision technological equipment.

Introduction

There are fundamentally different schemes and methods of installing equipment, the choice of which depends on its layout, the nature of production and the accuracy of the relationship of individual elements of equipment that are part of a single technological complex, construction and installation conditions, as well as operating conditions. Many installation schemes involve the use of known methods of geodetic marking: the method of polar coordinates, rectangular coordinates, angular and linear serifs, creative measurements. High accuracy of these methods is achieved by creating a particularly accurate geodetic basis using high-precision tools for measuring angles and distances, which reduce the impact of various systematic and random measurement errors.

All geometrical parameters of installation of precision technological equipment are obtained from measurements of geodetic constructions with their subsequent processing and evaluation. As a result of measurements we receive a number of measured sizes:

$$\begin{aligned} &x_1, x_2, \ldots, x_n, \\ &p_1, p_2, \ldots, p_n, \end{aligned} \tag{1}$$

where $x_1, x_2, \ldots, x_n$ are the measured values; $p_1, p_2, \ldots, p_n$ – weights (probability distribution) of the measured values. The weights of the measured values are quite difficult to determine, but quite often the measured values, which are obtained under the same measurement conditions, are assigned weights equal to one, $p_1 = p_2 = \ldots = p_n = 1$ or equal to the given weights $p_1 = p_2 = \ldots = p_n = 1/n$, which most often distorts the

*Corresponding author. E-mail: *vgladilin.55@gmail.com*

probability distribution of the measured values. To eliminate these shortcomings, an algorithm for determining the probability density of the measured values is proposed. In geodetic constructions we will allocate such stages of processing of results of measurements (which are resulted in Figure 1).

1st Stage. Removal of information from the equipment which registers the measured values (theodolite, level, total station) or from the constant carrier and comparison of object of measurement with a working measure (direct deduction on scales or from a magnetic tape (magnetic disk, flash drive), its translation in numerical computer code and reference to computer memory during machining.

2nd Stage. Primary processing. Contains rationing of measurement data, reduction to a certain frame of reference, statistical processing with an assessment of the degree of confidence, filtering, and rejection. The aim is to obtain the initial results (for example, the graph of the initial curve, the coordinates of the points to be determined, the deformation of structures and equipment) of the experiment.

3rd Stage. Interpretation of the obtained initial results, which is an assessment of the desired characteristics of the model of the physical object under study or process (horizontal and vertical displacements (deformations)). In geodetic measurements are often recorded not interested in the characteristics of the phenomena p, but only their functionally dependent or stochastic manifestations $F = Ap$, the problem of interpretation is reduced to solving the equation $Ap = F$, where F – the measured values (with some error), A – transformation operator (matrix in the case of algebraic equations), p – the required values. To calculate the horizontal and vertical displacements and deformations of industrial equipment determine the coordinates of the points of this equipment by different methods (geodetic). In many cases, this task is incorrectly set (Tihonov & Arsenin, 1974). To calculate the accuracy of geodetic measurements in the design and alignment of geodetic networks and structures, evaluation of measured values, it is necessary to know the laws of probability density distribution and their characteristics. Currently, when calculating and estimating the accuracy of measurements use the law of normal distribution (Viduev & Kondra, 1969), which is the limit to which the sum of other full-scale distributions asymptotically approaches, which does not accurately reflect the structure of measured values, with a small amount of measurements. To correct this situation, the calculation must be performed taking into account the distribution structure. Estimates of measurement results are:

– mathematical expectation $M(X)$;
– dispersion $D(X)$;
– standard deviation $\sigma = \sqrt{D(X)}$;
– asymmetry $S(X)$;
– excess $E(X)$.

Mathematical expectation is always calculated, more often it is replaced by the arithmetic mean. The variance is a measure of the scattering of the measured values around the mathematical expectation (arithmetic mean, weight average, structural average). Asymmetry and excess are calculated less often and although they are quantitative characteristics of the distribution, but have no clarity and under the normal law of distribution the asymmetry is zero. The measured values do not always obey the normal distribution law, especially with a small number of measurements.

1. Methods

The problem of probability theory in geodetic measurements can be described by the following scheme: the known composition of the sample and the law of probability distribution, it is necessary for a given experimental scheme to estimate the probability of obtaining the most plausible value of the experiment. Mathematical statistics solves the inverse problem: the results of measurements determine the properties of the distribution law. The complete characteristic of the distribution law is the density distribution of probability.

Each implementation t of the sample T of limited volume n it is possible to put a correspondingly ordered sequence:

$$t_1 < t_2 < \ldots < t_n \in T, \qquad (2)$$

which is obtained from the sample (1) by sorting the measured values in ascending order In accordance with the definition of the probability density function is a function $p(\tau)$, the integral of which is equal to the distribution function:

$$\int_{en1}^{en2} \theta(t-\tau)\, p(\tau)\, d\tau = F(t). \qquad (3)$$

Thus, the probability density $p(\tau)$ is a solution of the Fredholm integral equation of the first kind, i.e. to estimate the density function by sampling a limited volume is to find an approximate solution of Equation (3). Finding the exact solution can be in the presence of the exact right part – the distribution function $F(t)$. In this case, the exact function $F(t)$ is unknown, but there is only a sample of $t_1, t_2, \ldots, t_n$, of limited volume n. We look for the solution $p(\tau)$ in the class of functions continuous on the segment $[en1; en2]$, we will look for a truncated distribution, setting a sufficiently wide interval $[en1; en2]$, which is given by the researcher.

The deviation of the right parts from each other is estimated in a quadratic metric:

$$m_F = \sqrt{\int_{en1}^{en2} \left[F_1(t) - F_2(t)\right]^2 dt}\,, \qquad (4)$$

and the deviation of decisions – in a uniform metric:

$$m_p(p_1, p_2) = \max_{t \in [en1;\, en2]} \left|p_1(\tau) - p_2(\tau)\right|. \qquad (5)$$

For each implementation t of the sample T, the function $F_n(t)$ is uniquely defined and has all the capabilities of the distribution function: increases from 0 to 1 and is a continuous case, while it is piecewise linear and increases only at sequence points (2), with strict inequality (2) the function $F_n(t)$ is given by the relation:

$$F_n(t)=\begin{cases}0 \; if \; t<t_k, \\ \frac{1}{n} \; if \; t_k<t<t_{k+1}, \\ 1 \; if \; t>t_n,\end{cases} \tag{6}$$

$k=1,2,\dots,n-1$, that is the magnitude of the jumps is equal to $1/n$.

In the general case, in accordance with the Glivenko-Cantelli theorem, the empirical distribution function

$$F_n(t)=\frac{1}{n}\sum_{i=1}^{n}\theta(t-t_i) \tag{7}$$

for a sufficiently large n it is as close to $F(t)$ as it pleases with a probability close to unity.

In Equations (3) and (7) $\theta(t)$ is a single jump function (Heaviside function):

$$\theta(t)=\begin{cases}0 \; if \; t\le 0, \\ 1 \; if \; t>0.\end{cases} \tag{8}$$

But we have a sample of limited volume, so equation (3) is solved approximately. The problem of finding an approximate solution of the equation on the approximate right-hand side refers to incorrectly posed problems, because small changes in the argument can lead to significant changes in the function.

To find an approximate solution to an incorrect problem

$$Ap(\tau)=F(t), \tag{9}$$

where A is the transition operator from the differential distribution function to the integral distribution function in the case when the right-hand side is given at points $\tau_1,\tau_2,\dots,\tau_l$ to the point of a random independent error

$$y_i=F(\tau_i)+\xi_i. \tag{10}$$

By the method of structural risk minimization, we find a minimum of N and λ functional:

$$J(N,\lambda)=\left|\frac{\frac{1}{l}\left[Y-F(\lambda)\right]^T Ry^{-1}\left[Y-F(\lambda)\right]}{1-\sqrt{\frac{(N+1)\left[1+\ln(l)-\ln(N+1)-\ln(\eta)\right]}{l}}}\right|. \tag{11}$$

Measure at the points $\tau_i=i\pi/(2(l+1))$, $i=1,2,\dots,l$, the value of the empirical distribution function $F_n(t)$ will be considered as $F(t)$ plus some error and expression (10) will take the form:

$$y_i=F_n(t_i)=F(\tau_i)+\xi_i. \tag{12}$$

In the functionality of empirical risk (11)

$$Y=\{Y_1,\dots,Y_l\};\; Y_i=y_i-\int_0^{\tau_i}\frac{\varphi_1(t)}{\psi_1}dt;\; F(\lambda)=\{F_i^{\lambda}\};$$

$$F_i^{\lambda}=\int_0^{\tau_i}\left\{\sum_{j=2}^{N}\lambda_j\left[\varphi_j(t)-\frac{\psi_j}{\psi_1}\varphi_1(t)\right]dt\right\};\; \psi_j=\int_0^1\varphi_j(t)dt,$$

the minimum of function (11) is searched for the most suitable confidence probability $\eta=0.9973$, but for this it is necessary to have more than 730 measurements in accordance with Eq. (24).

Inverse covariance matrix Ry^{-1} of the vector y:

$$Ry^{-1}=\begin{bmatrix} r_1 & \rho_1 & 0 & \dots & 0 & 0 & 0 \\ \rho_1 & r_2 & \rho_2 & \dots & 0 & 0 & 0 \\ 0 & \rho_2 & r_3 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \rho_{l-2} & r_{l-1} & \rho_{l-1} \\ 0 & 0 & 0 & \dots & 0 & \rho_{l-1} & r_l \end{bmatrix}, \tag{13}$$

where

$$r_1=\frac{nF(\tau_2)}{F(\tau_1)\left[F(\tau_2)-F(\tau_1)\right]};$$

$$r_l=\frac{n\left[1-F(\tau_{l-1})\right]}{\left[1-F(\tau_l)\right]\left[F(\tau_l)-F(\tau_{l-1})\right]};$$

$$r_i=\frac{n\left[F(\tau_i)-F(\tau_{i-2})\right]}{\left[F(\tau_i)-F(\tau_{i-1})\right]\left[F(\tau_{i-1})-F(\tau_{i-2})\right]};$$

$$\rho_i=-\frac{n}{F(\tau_{i+1})-F(\tau_i)};$$

$$i=1,2,\dots,l-1;\; l=5n/\ln(n).$$

From expression (13) for the inverse covariance matrix it follows that its coefficients depend on the unknown function $F(t)$, the derivative of which is the probability density

$$p^{(N)}(t)=\sum_{j=1}^{N}\lambda_j\varphi_j(t). \tag{14}$$

The degree of complexity of the assessment, i.e. the number of members of the decomposition N is selected depending on the sample size n, using the method of structural minimization of empirical risk in accordance with Eq. (11). The probability density is found in the form of expansion by basic functions $\varphi_j(t)$ by the method of least squares (Gladilin, 1996). We look for the minimum of the functional (11) for each N provided

$$\int_0^1\left[\sum_{j=1}^{N}\lambda_j\varphi_j(t)\right]dt=1, \tag{15}$$

that is, the area bounded by the distribution density curve must be equal to one. In Eq. (15), the basis function has the form:

$$\varphi_j(t)=\sqrt{\frac{4}{\pi}}\cos\left(\left(2j-1\right)\frac{\pi}{2}t\right);\ t\in\left[0;1\right];\ j=1,2,\ldots,N\,. \quad (16)$$

The resulting variation series (2) is reduced to the interval [0; 1] by the Equation

$$T(n)=\frac{t_n-A}{B-A}; \quad (17)$$

$$\left.\begin{aligned}A&=\min_n\{t_n\}-en1\cdot\nu\\ B&=\max_n\{t_n\}+en2\cdot\nu\end{aligned}\right\}; \quad (18)$$

$$\nu=\left[\frac{\max_n\{t_n\}-\min_n\{t_n\}}{n-1}\right], \quad (19)$$

where $en1, en2$ are the left and right coefficients of the distribution range around the mathematical expectation (arithmetic mean, structural mean); the density $p(t)$ is calculated in the interval $t\in\left[A;B\right]$, in the general case $A\neq B$. In fact, the distribution is in the range [–∞, ∞], but the values of the coefficients can be taken in the range $0<en1,\ en2\leq 6$. In order to more accurately find the values of the coefficients $en1, en2$ define the left z_L and right z_R structural coefficients of the distribution intervals:

$$z_L=\frac{A-t_{str.mean}}{\sigma_{str.mean}};\ z_R=\frac{B-t_{str.mean}}{\sigma_{str.mean}};\ z_0=ABS\left(\frac{z_L}{z_R}\right)=1,$$

provided that the ratio $z_0=1$.

Assign $en1=en2=\pm2$, obtain $z_0=0.74609$; assign $en1=en2=\pm3$, obtain $z_0=1.00879$; assign $en1=en2=\pm4$, obtain $z_0=1.08156$; assign $en1=en2=\pm5$, obtain $z_0=1.13704$.

By the selection method we obtain: $en1=-3.62$, $en2=+\ 3.359$, thus $z_L=-2.20398$, $z_R=\ 2.20012$ and $z_0=1.00175$, which is closest to unity. For these values $en1=-3.62$, $en2=+\ 3.359$, we calculated by the Equations (19) and (18): $\nu=\left[7.8-\left(-6.2\right)\right]/19=0.73684$, $A=-6.2-3.62\cdot0.73684=-\ 8.8674$, $B\ =\ 7.8\ +\ 3.359\ \cdot\ 0.73684\ =\ 10.2751$.

The values of A and B and the intermediate values of t of expression (2) are listed in Table 2.

The obtained values of the probability density are checked for the normal distribution law in accordance with the Pearson agreement criterion χ^2.

The process of geodetic measurements (Gladilin, 1996) is the receipt of a message, its transmission in the form of a signal (reading on the scale of the device), signal reception and processing. Measured signals differ from communication signals because when comparing the measured value with the working measure there are two signals – reference and measurable (which is compared with the reference). In the rings of the measuring circuit signal conversion occurs according to the laws of computer theory. In the process of measurements, the geodetic instrument is used to receive and/or process information, which is performed in three stages (Figure 1). The measuring device in the process of measurement is included in the circuit, which includes: the object of measurement, the device (apparatus, instrument) that receives and analyzes the measurement results. The circuit includes the external environment, we consider it from the standpoint of computer science theory as a ring of transmission of the measuring signal. The circuit for transmission and processing of the measuring signal is shown in Figure 1.

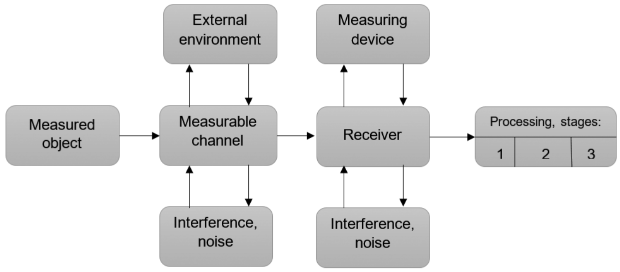

Figure 1. Generalized information transmission chain (measured signal)

The measured value by its size is a random value of the error of its measurement is also random (when eliminating the systematic component of the error; the study of systematic errors is considered, for example, in Tereshchuk et al., 2019). The random state of the object of measurement and its errors leads to some uncertainty, but before the measurements it is known that the value measured is within certain limits, and its value must be obtained with some degree of accuracy, the preliminary information about the size of the object reduces uncertainty and the expected error will show the limits of this uncertainty, i.e. the more accurate the measurement result, the more information about the object being measured.

For the amount of information, the determining measure of uncertainty is entropy. The amount of information obtained about the object reduces uncertainty. The degree of uncertainty does not depend on the specific values of the independent quantity and is related to the law of probability distribution. The measure of the uncertainty of an object with many states (measured values (1)) is the functional that determines the entropy:

$$H(X)=H\left(p_1,p_2,\ldots,p_n\right). \quad (20)$$

If the measurement states are equally probable, then the functional (16) will take the form

$$H(X)=\log_2(n). \quad (21)$$

To determine the shape of the distribution of errors of geodetic measurements, determine the value of the entropy coefficient by the Equation:

$$K_E=\frac{D\cdot n}{2\sigma}10^{-\frac{1}{n}\sum_{i=1}^{L}n_i\lg n_i}, \quad (22)$$

where D is the width of the histogram column; n is sample size; σ is the root mean square error of the measured values; L is the number of columns of the histogram;

n_i is the number of values that fall into the i is column ($i = 1, \ldots, L$). The counter excess is determined by the Equation:

$$\varepsilon = \frac{1}{\sqrt{E(X)}}, \quad (23)$$

which for any known distributions in which $E(X) \geq 1$ is in the range [0; 1]. The asymmetry for all centered measured values is zero, $S(X) = 0$ so it cannot structurally determine the shape of the distribution of measured values.

The confidence probability of determining the shape of the distribution of measured values was determined by a simplified Equation (excluding the left and right distribution intervals)

$$\eta = \frac{n-1}{n+1}. \quad (24)$$

Estimation of the distribution density is performed by the Equation:

$$p^{(N)}(t) = M \sum_{j=1}^{N} \lambda_j \varphi_j \left[M(t-A) \right], \ t \in [A; B]. \quad (25)$$

According to the described algorithm and Equations (1)–(25) a program for calculating the probability density for any exponential *DensProb* distributions in the *FORTRAN* programming language is created, the input values are: n, $en1$, $en2$, $t \in T$ output: probability densities, root mean square deviation, asymmetry, excess, entropy, entropy coefficient, counter excess, mean probability.

In Figure 2 shows a diagram of the equipment in the coordinate system $(0xy)$, which indicates the numbering of equipment points, their rectangular coordinates (x, y), small arrows indicate the directions of displacement (deformation) of these points relative to point 11, which is taken as the center of equipment. The scheme is taken from the work (Gladilin et al., 2019).

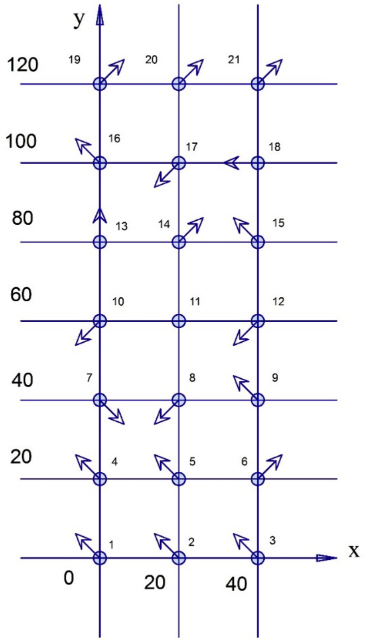

Figure 2. The scheme of placement of points on the equipment and directions of their deviation after deformation

Corrections to the position of the equipment points relative to point 11 are shown in Table 1.

The value for calculating the probability density (corrections to geodetic measurements), which are taken from (Gladilin et al., 2019), are given in Table 1.

Table 1. Corrections (t) in measurements between point 11 other points of equipment

Point numbers	Corrections, mm	Point numbers	Corrections, mm
11–10	–1.6	11–12	1.4
11–13	2.5	11–09	4.4
11–16	–3.0	11–06	–3.8
11–19	7.8	11–03	3.8
11–14	–6.2	11–08	–5.9
11–17	6.6	11–05	3.1
11–20	1.4	11–02	–1.8
11–21	–3.4	11–01	–5.0
11–18	4.5	11–04	4.7
11–15	–3.2	11–07	3.8

When processing the data from Table 1 according to the described algorithm according to Equations (3)–(25) the following results were obtained: arithmetic mean $t_{mean} = 0.505$, standard deviation $\sigma = 4.3465$, structural mean $t_{str.mean} = 0.712$, which is calculated by the Equation:

$$t_{str.mean} = \frac{\sum_{i=1}^{n} t_i p_i}{\sum_{i=1}^{n} p_i}. \quad (26)$$

Structural standard deviation is: $\sigma_{str.mean} = 4.4029$.

Asymmetry $S(X) = -0.0488$, excess $E(X) = 1.605$, counter excess $\varepsilon = 0.789$ (by Eq. (23)), confidence probability $\eta = 0.9048$ (by Eq. (24)); entropy coefficient $K_E = 1.4384$ (by Eq. (22)), the entropy $H(X) = 4.2344\ bit$ indicates that the loss of information in the distribution does not occur; Pearson's criterion $\chi^2 = 700400$ and, accordingly, the distribution of t does not obey the normal, the minimum functional (11) is found at $N = 5$, there are no systematic errors in a number. The calculation of the probability density was performed in the range from $en1 = -3.62$ to $en2 = 3.359$, the scale factor $M = 0.052240$ is calculated by the Equation:

$$M = \frac{1}{B-A}. \quad (27)$$

Decomposition coefficients: $\lambda_1 = 1.312137$; $\lambda_2 = -0.724554$; $\lambda_3 = 0.2849023$; $\lambda_4 = -0.2944348$; $\lambda_5 =$

Table 2. Calculated probability densities using the program *DensProb*

t	−8.8674	−7.9558	−7.0443	−6.1327	−5.2212	−4.3096
$p^{(N)}(t)$	0.0001	0.0009	0.023	0.055	0.080	0.090
t	−3.3981	−2.4866	−1.5750	−0.6635	0.2481	1.1596
$p^{(N)}(t)$	0.078	0.055	0.029	0.013	0.014	0.033
t	2.0712	2.9827	3.8942	4.8058	5.7173	6.6289
$p^{(N)}(t)$	0.062	0.090	0.105	0.103	0.085	0.056
t	7.5404	8.4520	9.3635	10.2751	–	–
$p^{(N)}(t)$	0.028	0.007	0.0007	0.0002	$\sum p^{(N)}(t) =$	1.0079

−0.7371129. Estimation of the distribution density is performed by Equoation (25):

$$p^{(5)}(t) = M\sum_{j=1}^{5}\lambda_j\varphi_j\left[M(t-A)\right],\ t\in\left[A;B\right].$$

In accordance with the calculated values of excess, entropy coefficient and counter-excess, the distribution of the two modal (Novickij & Zograf, 1991), the calculated probability densities for the measured values (Table 1) in the interval $\left[A;B\right] = \left[-8.8674;\ 10.2751\right]$ are given in Table 2, according to which the graph of the probability density distribution, which is shown in Figure 3.

We will perform an interval estimate of the found (arithmetic and structural) mean values at a confidence level of $\eta = 0.9048$ using the Equations:

$$t_{mean} - z_L \cdot \sigma_{mean} \le t_{mean} \le t_{mean} + z_R \cdot \sigma_{mean}, \tag{28}$$

$$t_{str.mean} - z_L \cdot \sigma_{str.mean} \le t_{str.mean} \le t_{str.mean} + z_R \cdot \sigma_{str.mean}. \tag{29}$$

Substituting all values into Equations (28) and (29),

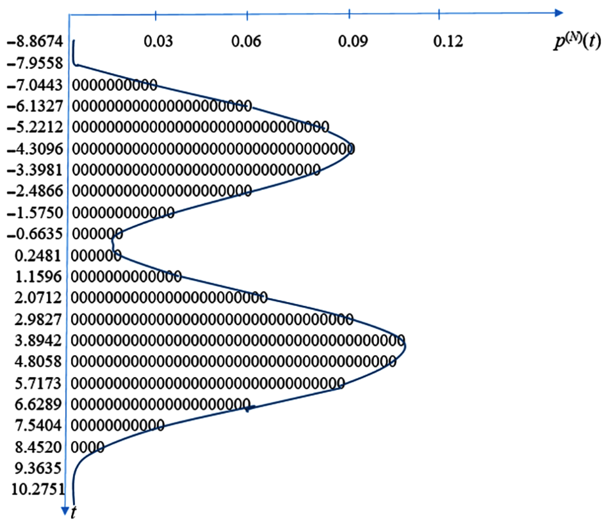

Figure 3. Type of the found distribution of density of probability of corrections. The graph is rotated by 90°

we obtain:

$$-9.07460 \le t_{mean} \le 10.06782;$$

$$-8.87886 \le t_{str.mean} \le 10.28646,$$

in Table 1: $t_{min} = -6.2$, $t_{max} = 7.8$.

The measured values given in Table 1 do not exceed the limits defined by (28) and (29), which means that the measurements were performed without gross errors and they are subject to further processing to identify dominant factors.

Conclusions

These calculations show that not all the results of geodetic measurements obey the normal distribution law, this is clearly seen from the graph in Figure 3, and therefore it is necessary to establish the distribution law for each geodetic measurement in accordance with the algorithm described above, and then calculate the measured values from taking into account the obtained distribution law.

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