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APPROXIMATION OF GRAVITATIONAL CALCULATION BASED ON ANALYTICAL MODELS

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Abstract. Gravity meters can be used to measure all effects that make up the Earth's gravity field. Many of these effects are caused by known sources such as the Earth's rotation, distance from the Earth's center, topographic relief, and tidal variation. Physical fields are the main component of many centuries of the paradigm of all of Earth Sciences. The changing gravitational field is very important subject of research in the scientific aspect and practical. This paper applies two analytical models to simplify the gravitation calculation, which are sphere and cone models. Examples and finite element applications for the two models are studied also and discussed. The results of this study reveal that the possibility of using the proposed method with using presented analytical, finite element and numerical models to estimate the better determination of the characteristics of the local gravity of natural and man-made objects of sizes up to several tens of kilometers.

Keywords: gravitational field, finite element, abnormal masses, direct and inverse problems, gravity.

Introduction

Historically, gravity has played a central role in the studies of dynamic processes in geodetic applications and Earth's interior or surface motion and is also important in exploration geophysics (Yin & Sneeuw, 2021; Hilst, 2004; Al-Garni, 2011). Therefore, the correct interpretation of the desired dynamic geodetic observations is needed to build the scientific basis for the study of solid Earth processes. Redistribution of large amounts of rocks and ores is a significant change in the gravity field, and underestimation of the impact of movable weight by leveling which can be the cause of the misconception about the picture and the vertical movements influence on the results of geodetic measurements (Charco et al., 2007).

The modern finite element of earth shapes generally uses dynamical models which consists of gravitational field of larger bodies and often includes self-gravitation between elements. If bodies are gravitating to represent as an infinitely extended in a horizontal plane then the simplified formula are needed. It is possible in the case of the horizontal dimensions of the body twice its depth. This allows the use of analytical model expressed "infinite" flat layer. There are examples of an analytical approach to analysis the influence of gravitating bodies in simple form, for example vertical cylinder. But formula works only for points situated on its axis (Charco et al., 2007). The cone is also limited in terms of space and location gravitating cone (Hilst, 2004; Mazurov & Pankrushin, 2006). The study of a complex nature of the system as the changing of Earth's surface with time self-organizing is a very urgent and important task for geodynamic studies. It needs for a successful solution comprehensive observations of various kinds with the mathematical treatment. A number of analytical and numerical mathematical models, available in the literature, can be used to fit ground deformation and gravity data to infer source location, depth and density as found in (Shandarin & Sathyaprakash, 1996; Battaglia & Hill, 2009).

Understanding the local gravitational field and its accounting is important for monitoring the dynamic earth activity (like, volcanic) and correct interpretation of geodetic observations of various kinds (Stepanova et al., 2021; Mazurov & Pankrushin, 2006; Charco et al., 2007). Determination of the gravity field and its transforms are not trivial task, and often, to achieve the goal of the gravity should be make a combination of analytical descriptions of some elementary spatial bodies with subsequent finite element partition of a complex element of relief. Many bodies of simple shape and a constant density have an effect on the force of gravity, expressed analytically in closed form. For a sphere of radius R with a constant density or

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consists of concentric layers, the known Equation as follows:

$$\Delta g = \frac{4}{3}\pi \times G \times R^3 \times \Delta\rho \times \frac{Z}{(x^2 + y^2 + z^2)^{3/2}}, \qquad (1)$$

where: $\Delta\rho$ is the difference between the density of the perturbing body and the density of its environment; G is the gravitational constant; and x, y, z are the coordinates of the center of the sphere, often these are called point body gravitating masses.

When interpreting the results of observations of Earth's surface movements, it should take into consideration the changes in the gravity field and the approximation gravitating in the study area. To achieve the goal of modeling, the complexity of the model selection should be based on the accuracy of the experimental data. A model that approximates the gravitational influence of a relief to high-precision geodetic and gravimetric measurements at their complex mathematical treatment should be consistent with the chosen model of a gravitating body. In solving the direct problems in the results of geodetic measurements (leveling, angular measurement), the observations must be corrected for deviation of the plumb line. For this purpose, most researchers consider gravitating body set as a collection of elementary bodies such as cubes. The net effect of all gravitating body made up of the sum effects of each elementary cube. However, the main objective of this paper is to present two analytical models to simplify the gravitation calculations which can be used to estimate the Earth's surface motion.

1. Analysis models

The analysis of various analytical models and approximation calculation are mostly used to describe the local gravitating objects. To determine the gravitational characteristics of objects, several analytical models of elementary spatial bodies are applied such as: homogeneous sphere, two-dimensional horizontal layer, spherical polyhedron, cuboids, spherical prisms, cylindrical prisms and others.

Researches (Mazurov et al., 2004a; Mazurov, 2007) have examples of the solution of inverse problems as a result of joint mathematical processing of multiple geodetic and gravimetric observations on the Earth's surface and are evaluating not only the coordinates of points and their displacements, but also the masses gravitating bodies, as well as changes of these masses. The incorrectness of inverse problems is the inability to find the unique solution of the integral equation. These examples are given in relation of the volcanic eruption and the preparation for it. Gravitating bodies were spherical deep chamber of the center of volcano and its cone surface.

Suffice typical elementary, which forms the body to the earth's surface, is a relief cone or generally a truncated cone. In addition to the natural environment of volcanoes, it may be individual components of mountain ranges. In the field of man-made, we have not only the cone-shaped elevation, but also tapered recess (Mazurov, 2007). Open mining space of rocks is generated conical that are, for example, kimberlitic deposits. Development of the diamond deposit Mir (Yakutia) has led to the formation of a cone-shaped quarry depth of 520 meters. Mould boards are often truncated cones.

1.1. Sphere models

Cone gravitating body was approximated by a sphere (one dot weight) (Mazurov et al., 2004b). It will be quite reasonable to clarify approximating cone model by increasing the number of point masses. But it is desirable that the number of estimated parameters remains the minimum required. This will meet the requirements of larger redundancy measurement, which is required during mathematical treatment. For example, suppose that the estimated parameters will be the total mass of the cone, but dispersed in space in a certain way (five points) (Figure 1).

It is necessary to divide a cone into five coextensive parts to find the coordinates of the gravity center. The number of coextensive parts may not be five coextensive parts only, but it can be divided more than five based on the calculations area and the required accuracy.

The steps of this model can be divided as follows:

Step 1. Horizontal plane clipped the top truncated cone volume of one-fifth of the total volume of the cone as shown in Figures 1 and 2;

Step 2. Lower truncated cone was divided into four quadrants by two vertical mutually perpendicular planes as shown in Figure 2b.

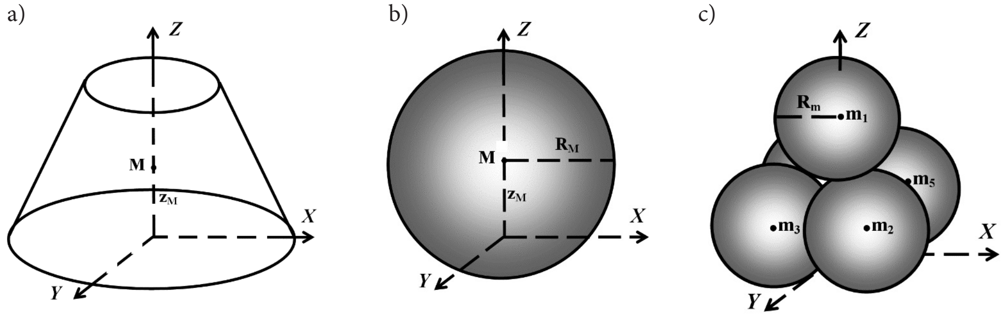

Figure 1. Truncated cone and the approximation model of its gravitational influence:
a – a truncated cone; b – a single point model; c – five point model

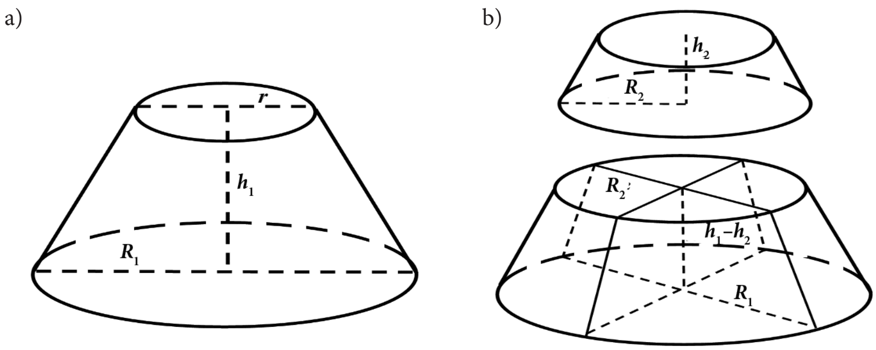

Figure 2. Truncated cone and its volume division into five equal parts

Assume: R_1 is the radius of the lower base, r is radius of the upper base and h_1 is height of the cone.

From Figure 2, it can be calculated the radius R_2 of the horizontal section and elevation of the upper cone, as shown on the following steps. Also, we can be used the Eqs (2) to (3) to calculate the volume of a truncated cone for the source V_1, and upper V_2 , respectively, after cutting a horizontal plane as represented in Eqs (2) and (3):

$$V_1 = \frac{\pi h_1}{3}(R_1^2 + r^2 + R_1\ r); \tag{2}$$

$$V_2 = \frac{\pi h_2}{3}(R_2^2 + r^2 + R_2\ r). \tag{3}$$

In this study, the cone was divided into five bodies of equal volume. Taking into account that $V_2 = (1/5)V_1$. Therefore, If we consider the image of the cone section vertical plane as shown in Figure 3, the sides of the triangle relationship can be calculated based on the following equation:

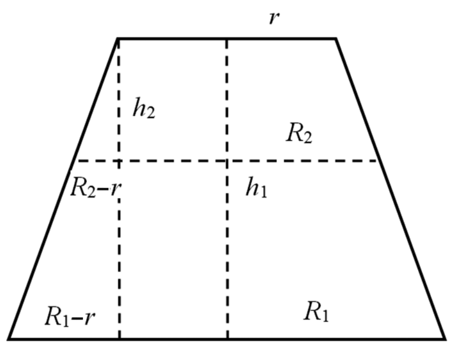

Figure 3. Cross-section of the cone vertically

$$h_2 = h_1 \frac{R_2 - r}{R_1 - r}. \tag{4}$$

Therefore, the volumes of truncated cones can also be submitted via the difference between the radiuses of the cubes. Making the substitution h_2 into the Eq. (3), then the volume V_2 can be calculated as follows:

$$\frac{\pi \times h_1}{3} \times \frac{(R_2 - r)}{(R_1 - r)} \times (R_2^2 + r^2 + R_2 \times r) = \\ \frac{1}{5} \times \frac{\pi \times h_1}{3} \times (R_1^2 + r^2 + R_1 \times r); \tag{5}$$

$$(R_2 - r) \times (R_2^2 + r^2 + R_2 r) = \\ \frac{1}{5} \times (R_1 - r) \times (R_1^2 + r^2 + R_1 r). \tag{6}$$

Then:

$$R_2^3 - r^3 = \frac{1}{5}(R_1^3 - r^3).$$

Hence:

$$R_2 = \sqrt[3]{\frac{1}{5}(R_1^3 - r^3) + r^3}. \tag{7}$$

Now, it is possible to determine analytically the coordinates of the gravity center of the upper truncated cone. For this purpose, we are used the classical approaches of the strength theory of materials to the amount of such calculations through the static moments of elementary geometric shapes (rectangle, triangle, circle sector). The vertical section of the truncated cone is trapezoid, which can be divided into two symmetric rectangular trapeziums. A rectangular trapezoid can be represented by a combination of rectangle and triangle. From the result of dividing, the sum of the static moments of the figures for the amount of space will be coordinates of the center of the gravity. From the trapeze center of gravity, the z direction coordinates can be calculated as follows:

In case of $x = 0$, therefore:

$$Z = \frac{\frac{r h_2^2}{2} + \frac{(R_2 - r) h_2^2}{6}}{r h_2 + \frac{(R_2 - r) h_2}{2}}. \tag{8}$$

While with four lower volume sectors, the process of coordinates calculation will be proceed as follows: for a 90-degree sector of the symmetry, plane will be held at an angle of 45°. The cross-section of the plane will be a rectangular trapezoid; we can present a combination of a rectangle and a triangle to compute the area of figures S_1 – rectangle, S_2 – triangle as follows:

$$S_1 = R_2(h_1 - h_2) \text{ and} \\ S_2 = 0.5 \times (h_1 - h_2) \times (R_1 - R_2). \tag{9}$$

Therefore, the coordinates of the center of gravity of the whole trapeze can be calculated as follows:

$$X' = \frac{S_1 + \frac{R_2}{2} + S_2\left(\frac{R_1 + 2R_2}{3}\right)}{S_1 + S_2}; \quad (10)$$

$$Z' = \frac{S_1\left(\frac{h_1 - h_2}{2}\right) + S_2\left(\frac{h_1 - h_2}{3}\right)}{S_1 + S_2}. \quad (11)$$

The radius of the horizontal section of the cone at a height equal Z' is determined as:

$$R_3 = R_1 - (R_1 - R_2)\frac{z'}{h_1 - h_2}. \quad (12)$$

Let's find coordinates of the centre of gravity from a quarter of this section. It is a quarter of a circle with radius R_3. Taking into account the equation of a circle, the coordinates can be calculated as follows:

$$x_c = y_c = \frac{\int_0^{R_3} dx \int_0^{\sqrt{R_3^2 - x^2}} y dy}{\frac{1}{4}\pi R_3^2} = \frac{4R_3}{3\pi}. \quad (13)$$

Table 1 presents the formula calculating the coordinates of the centers of gravity of all five equals gravitating mass of a truncated cone in the coordinate system with the origin at the center of the lower base.

Table 1. The coordinates of the centers of gravity point masses

	Mass 1	Mass 2	Mass 3	Mass 4	Mass 5
X	0	x_c	$-x_c$	$-x_c$	x_c
Y	0	y_c	y_c	$-y_c$	$-y_c$
Z	$h_1 - h_2 + Z$	z'	z'	z'	z'

Therefore, the gravity can be calculated after determining the coordinates of gravity point masses based on Eq. (1), and the next section represents an example for the gravity calculation based on these formulas.

Observations analysis

Figure 4 illustrates the experimental works for the leveling observations of cone model. On a flat surface (local portion) with the value of the force of gravity $g_0 = 980$ Gal appears anomalous gravitating mass $\boldsymbol{M}$ in the shape of a truncated cone with the lower base of radius R, the radius r of the upper base and a height h, a density $\delta = 2.63$ g/cm^3. This causes a change to the gravity vector g_0 n the surrounding space. At each point, the gravity change is different g_M^A, g_M^B, g_M^1. The consequence of the appearance of abnormal mass $\boldsymbol{M}$ is also a plumb line μ_M^1 at the state leveling, which, in turn, causes the displacement of the bubble level of the device. After bringing it into the center of the sighting axis leveling will show a report a_M on the back staff and b_M front staff. The excess $h_M^{A,B} = a_M - b_M$ will be different from the excess measured $h_0^{A,B} = a_0 - b_0$ before the abnormal mass $\boldsymbol{M}$ by the amount $\delta h_M^{A,B} = h_M^{A,B} - h_0^{A,B}$.

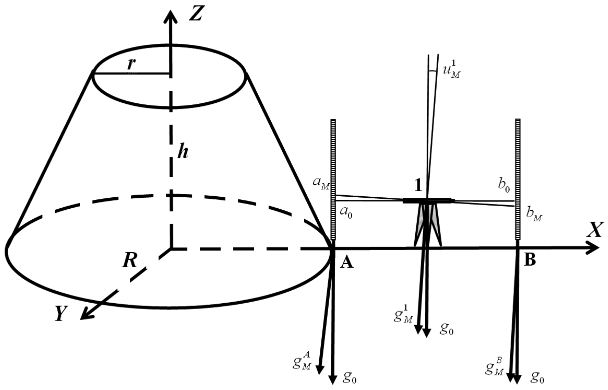

Figure 4. The first station leveling

Procedure and results of the numerical experiment are presented as follows:

For a truncated cone with the geometrical characteristics of R = 500 m, r = 200 m, h = 500 m, its volume (V) will equal 0.204 km^3, with homogeneity of rocks with a density δ = 2.63 g / cm^3 mass (M) = 5.37×10^8 Tones. It was modeled leveling line from the base of the truncated cone, 1 km in length consisting of ten stations in 100 meters from each other. Staff distances 50 meters. Figure 4 shows the first station with differences between staff position at ***A*** and staff position at ***B*** was 100 m. in addition, at the installation site leveling, but not at the height of the device, and in the XY plane. Simulated procedure was assumed to be carried out for determining the absolute values of the force of gravity.

In this example, the random errors are not introduced only methodological errors are evaluated. The results of differences of the gravity from one to five point's approximation models are illustrated in Table 2.

This data for comparison was obtained after a computational experiment using the classic formula of physical geodesy and gravimetric Hofmann & Moritz, 2006).

From Table 2, it can be seen that the difference between the single-point and five-point model in determining the gravity are up to 0.1 μGal. In addition, the difference in close proximity to the cone on the leveling of excess are estimated up to 0.2 mm at 1 km. This difference is insignificant for the horizontal angle. The difference in the plumb line is 0.1″, it is for the cone with the above geometric parameters. For the rare larger cone gravitating bodies, the differences between single-point and five-point models are shown on a large scale. From these results, it can be concluded that the description of the local gravitational field is improved compared with existing methods by about 3–4% for local objects with dimensions of territory about 20–100 km^2 and mountain and foothill terrain.

Table 2. Differences in the plumb line, gravity and leveling excesses, one-point and five-point approximated model of the cone for the leveling

Station number (i)	1	2	3	4	5	6	7	8	9	10
Coordinate X (m)	550	650	750	850	950	1050	1150	1250	1350	1450
Deflection of the vertical (5 point) – (1 point)″	–0.10	–0.08	–0.05	–0.04	–0.02	–0.02	–0.01	–0.01	–0.01	–0.00
g (5 point) – (1 point) μGal	–75	–24	–52	–54	–48	–41	–35	–29	–24	–20
h (1 point) mm	0.98	0.74	0.58	0.46	0.38	0.31	0.26	0.22	0.19	0.17
h (5 point) mm	1.03	0.78	0.60	0.48	0.39	0.32	0.27	0.23	0.20	0.17
h (5 point) – h (1 point) mm	0.05	0.04	0.03	0.02	0.01	0.01	0.01	0.00	0.00	0.00

1.2. Cone sector model

Based on the analysis of the material to determine the Earth's gravitational field, gravitational potential, normal building components plumb lines by reducing the measured gravitational field and gravity anomalies, conclusions are required about the need to consider the characteristics of gravity when performing high-precision geodetic and gravimetric work as their the effect is comparable with the accumulated measurement errors. For the unambiguous interpretation of the observed changes of gravity, heights are needing to be measured, at least in some of the control points to detect changes in gravity at local scales associated with geodynamic and technological processes in the subsurface layers of the lead re-gravimetric observations, with which you can identify the influence of gravity of the masses on the geodetic measurements. The advantage of using the box model is methodically improved the definition of gravitational characteristics of the natural and man-made objects. The basis for this improvement is its use of more precise analytical mathematical patterns for gravity that explicitly take into account the influence of the characteristics of each of the box analytic integral formula, which excludes some methodological errors.

Determination of gravitational characteristics of local natural and man-made objects on the finite element model of the surface topography allows more detailed account in the design of geodetic networks, organization of field work and the subsequent interpretation of the results of geodetic and gravimetric observations. In this paper, we analyzed the influence of the gravitational model of approximation cone-shaped relief, shown developed method of determining the characteristics of gravity and made it to the approbation of the simulated and real objects. Development of methods for determining the characteristics of gravity based on the model of a circular cone with a base radius *R*, height *H* (Figure 5), the density of rocks $\Delta\rho$. This was considered a local area of the earth surface, which has a conical gravitating mass height of 750 m, the radius of the base reserves of 450 m and density of rocks is 2.63 g/cm^3. These values of the gravitational potential, gravity, plumb lines around a cone are using for a truncated cone space. The truncated cone is divided into six layers in height, and the cone – 10 layers. Each of the layers is divided into 60 degree sectors. Then, each sector is divided into a cone 20, 18, 16, 14, 12, 10, 8, 6, 4, 2 zones and a truncated cone 20, 18, 16, 14, 12, 10 zones as

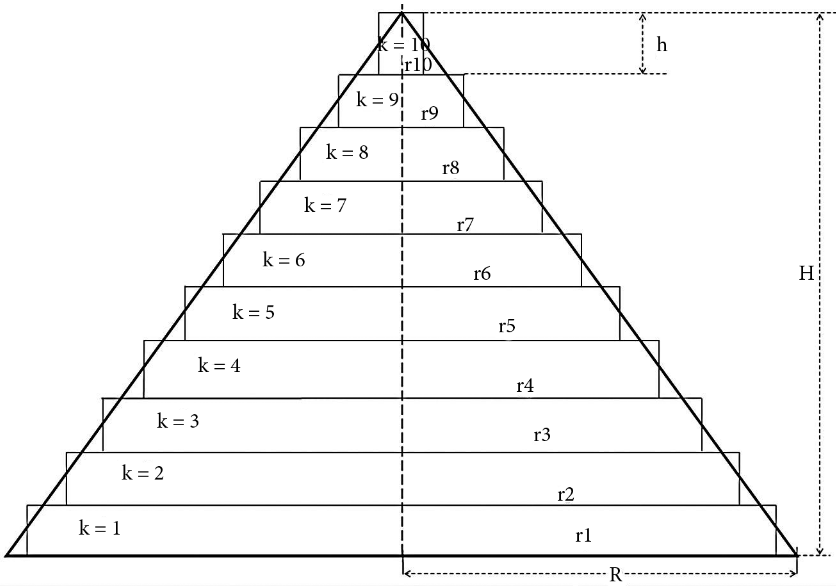

Figure 5. Vertical section of the cone

elevation of each layer other (Figure 6). The computational experiment gravitational potential approximated cone system comprising 6600 material points, and the potential of a truncated cone 5400 system of material points as shown in Figure 5. Then, the value of the gravitational potential T_K^C ()aused by these cones at any point C of the surrounding space will be found.

The following steps are proposed for computing the local gravity based of cone model:

1. Cone horizontal planes are divided by K bodies with base radius, starting from the bottom, r_1, r_2, ..., r_K. The height of each cone is $h = H/K$. Figure 5 shows a vertical section of the cone to $K = 10$.

2. Each k-th layer is tapered with a radius r_i and the lower base of the upper base radius r_{i+1} is replaced by its approximating the k-th layer of the same cylindrical height h under the condition that their volumes:

$$V_k(r_i;r_{i+1}) = V_{cylinder}(r_k) \tag{14}$$

or

$$\frac{1}{3}\pi h(r_i^2 + r_i r_{i+1} + r_{i+1}^2) = \pi h r_k. \tag{15}$$

Therefore, the radius of the base of the approximating cylinder is calculated by the formula:

$$r_k = \frac{1}{3}(r_i^2 + r_i r_{i+1} + r_{i+1}^2). \tag{16}$$

3. Each cylindrical layer is divided into concentric rings J_k at equal distance $dr = r_k/J_k$, where $J_k = 2\times(11 - k)$. Each ring is divided into N circumferential sectors (curvilinear parallelepiped) with a pitch angle $\alpha = 360/N$. Figure 6 shows an example of a finite element partition with $k = 1, 2, ..., 10$; $\alpha = 6°$.

4. To replace each volumetric finite element mass point are the coordinates of its center of mass. For each ring sector k-th layer cylindrical coordinate z (vertical) is calculated by the formula:

$$Z_k = \frac{h}{2} + h(k-1). \tag{17}$$

Coordinates x, y center of mass of each finite element are determined using methods known in the theory of strength of materials of the formulas for the annular sector and its symmetry properties.

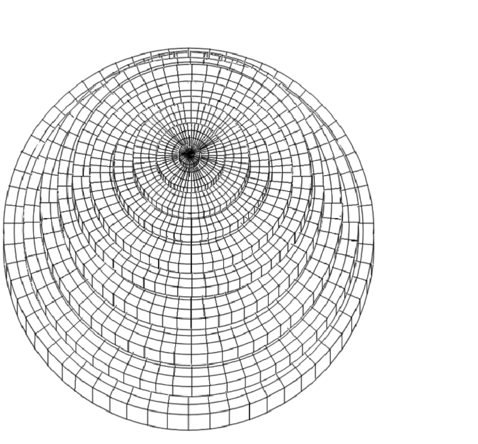

Figure 6. Finite element partition of the cone

For sector angle value α to the inner radius $r_{k,j}$, outer radius $r_{k,j+1}$ is the distance $r_{m_{K,(j,j+1)}}$ rom the center of the ring O forming the center of mass of the ring sector m (Figure 7) by the formula:

$$rm_{k,(j;j+1)} = \frac{4}{3}\frac{\sin\left(\frac{\alpha}{2}\right)}{\alpha}\frac{\left(r_{k,j+1}^3 - r_{k,j}^3\right)}{\left(r_{k,j+1}^2 - r_{k,j}^2\right)}. \tag{18}$$

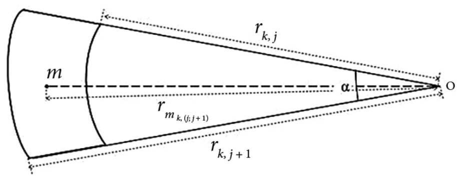

Figure 7. Finding the center of mass of the ring sector

5. Upon receiving the distance value $rm_{k,(j;j+1)}$, the coordinates x, y center of mass of each finite element can be calculated as following:

$$x_{k,(j;j+1),n} = rm_{k,(j;j+1)}\cos\beta_n; \tag{19}$$

$$y_{k,(j;j+1),n} = rm_{k,(j;j+1)}\sin\beta_n, \tag{20}$$

where:

$$\beta_n = \frac{\alpha}{2} + \alpha\times(n-1),\ n = 1,2,...,N.$$

6. The volume of each ring sector is defined as:

$$V_{k,(j;j+1)} = \alpha(r_{k,j+1}^2 - r_{k,j}^2)h. \tag{21}$$

Taking into account the density of breed's Δρ, the weight of sector of j-th ring of k-th layer can be calculated as:

$$m_{k,(j;j+1)} = V_{k,(j;j+1)}\delta. \tag{22}$$

7. The gravitational potential at the point C of the surrounding space, called a point mass quantity $m_{k,(j;j+1)}$ with coordinates, $x_{k,(j;j+1),n}$, $y_{k,(j;j+1),n}$, $z_k = \frac{h}{2} + h(k-1)$ is defined as:

$$T_{k,(j;j+1),n}^C = G\frac{m_{k,(j,j+1),n}}{r_{k,(j;j+1),n}^C}, \tag{23}$$

where: G is gravitational constant; $r_{k,(j;j+1),n}^C$ is a distance from the center of mass of the final element to point C.

The total gravitational potential at the point C of the surrounding space, called a cone, is calculated as the sum of the potentials of the point masses:

$$T_K^C = G\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N}\frac{m_{k,(j,j+1),n}}{r_{k,(j;j+1),n}^C}. \tag{24}$$

8. The final value of the potential W_c given height above the ellipsoid and gravitating cone defined by the sum of the perturbing potential T_c normal potential U, the value of which on the surface level of the ellipsoid U_0 = 62636861.074 m^2c^{-2}.

9. Components of the gravity vector at the point C defined by the formulas:

$$\left.\begin{aligned} g_{cx} &= G\sum_{i=1}^{n} m_i; \\ g_{cy} &= G\sum_{i=1}^{n} m_i; \\ g_{cz} &= G\sum_{i=1}^{n} m_i\left(\frac{z_c - z_i}{r_i^3}\right); \end{aligned}\right\}, \tag{25}$$

where x_i, y_i, z_i are the coordinates of the center of gravity of the elementary volume of gravitating mass m_i, and x_c, y_c, z_c are coordinates of the point C. In the calculation, you must take into account the components g_z gravitating influence of the Earth at a height H_c ellipsoidal:

$$g_C^H = 980\,000 - 0.3086 H_C. \tag{26}$$

The resulting value of the force of gravity at the point C is defined by square root sum of squares of the components of the gravity vector.

10. Getting the value plumb line along the x axis.

11. Calculation height anomaly (the difference geodesic and normal height) by perturbing potential T_c formula Bruns.

A refined technique detail gravitating objects finite element method. When replacing a ball cube (Figure 8), there is methodological errors, the essence of which is as follows: cube replaced sphere of the same volume with a radius:

$$R_\varphi = \sqrt[3]{\frac{3a^2}{4\pi}}. \tag{27}$$

For $a = 1$ (the distance from the center to the edge of the cube perpendicular equal to 0.5), the value of R_φ is 0.62.

In the analysis of local areas with arbitrary relief, it is offered to perform finite element approximation of Parallelepiped of different heights, and then the ultimate gravity is obtained by summing the analytical calculations of gravitational influences of each element. The advantage presented by the technological approach to the calculation of gravitational characteristics of the natural and man-made objects is to improve the quality assessments of their determination through a more precise calculation of the gravitational influence of each of the box's integral formula, excluding some methodological errors. Technologically optimize the process of calculating the gravitational characteristics contribute to a well-developed system using digital elevation model (DEM). Technique and the subsequent creation of technology account the gravitational influence of natural and man-made objects have been used digital models of some really existing local objects. For the subsequent determination of the values of gravity and its principal transforming at points located close to the model under the ground surface, the experimental studies were performed.

2. The experimental studies

The initial data for the determination of gravitational characteristics of objects using real DEM St. Helens volcano site (Mount St. Helens), located in Skamania county Washington State (USA), as well as a real DEM hilly coastal area on the Kamchatka Peninsula and DEM, built on the basis of modeling a local area of the seabed off the coast of the peninsula. Test natural local objects are shown in Figures 8, 9 and 10.

DEM can be created according to various sources surface coordinates. In the first experiment, the DEM is built on the results of a vectoring of topographic map. Figure 10 shows a section of DEM volcano St. Helena coordinate with sections 30×30 m in the x and y respectively. For detailed definitions of gravitational characteristics of the whole area around the cone of the volcano below the total number of finite elements processed were 51 051. Height of the box to the size of the base 30×30 m reach more than 1 km. The total area of the territory around the cone of the volcano to determine its characteristics of gravity is 45.54 km^2. Initial coordinates X, Y, Z points DEM converted to coordinates X', Y', Z' as the beginning of item coordinate system, which calculates the gravitational characteristics. In the experiment, the example of St. Helens volcano, these characteristics were determined for ten different

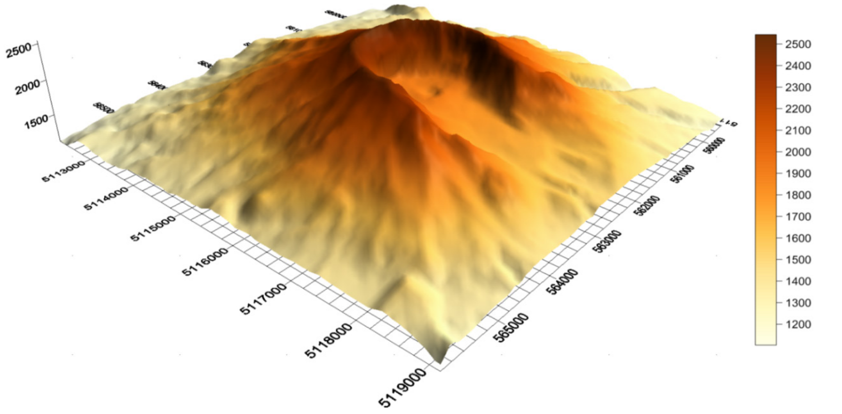

Figure 8. The 3D surface of volcano Saint Helena (Dimension in meter)

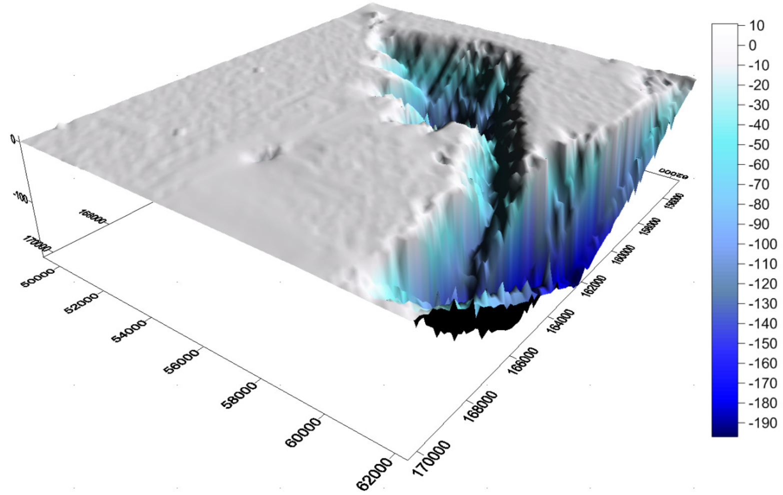

Figure 9. Simulated coastal area of the seabed

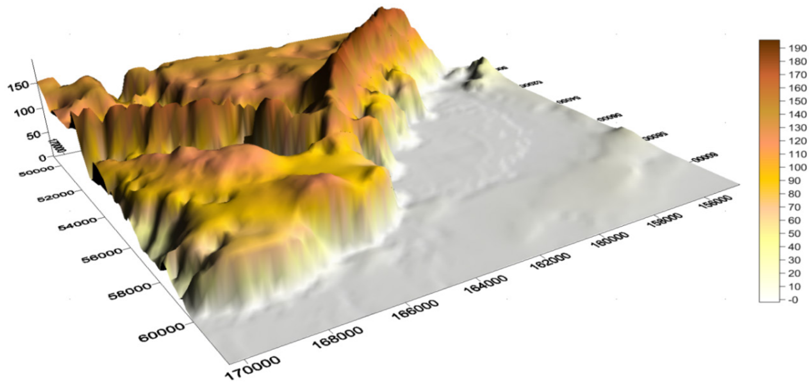

Figure 10. The surface of the coastal territory of the Kamchatka Peninsula

points. The Figures 8, 9, 10 and 11 are produced using Matlab programming and Surfer's extensive modeling tools and programming.

The second investigated the natural local object was the site of hilly coast of the Kamchatka Peninsula. In the experiment, it takes account of different density of coastal areas and adjacent water masses of the sea area. Sushi adopted average density of 2.63 g/cm^3, and the density of sea water 1.1 g/cm^3. GRID Network seafloor contains 8300 elements, the surface of the land. the size of the 8000 elements grounds along the axes x, y 160×160 m. Gravity characteristics were determined for the four points of a natural object. The total area of land hilly coast of the Kamchatka Peninsula to determine its gravitational characteristics of 205.4 km^2, and modeled the coastal area of the seabed is 197.8 km^2. As the finite element used box, during mathematical processing was determined by the value of gravity of the entire object as a set of finite elements; uniform elongated parallelepiped with their subsequent summation. The normal force of gravity to the accepted model of the earth (ellipsoid) can be calculated by the formula:

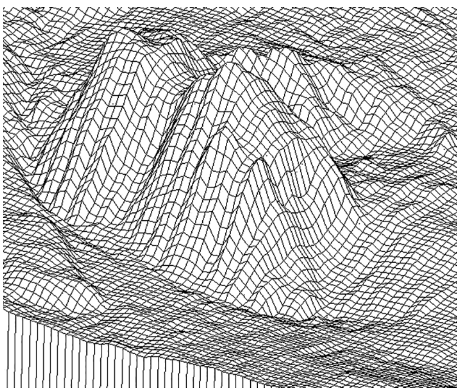

Figure 11. A wire frame view of the volcano area DEM St. Helena

$$\gamma_0 = \gamma_e (1 + \beta \times \sin^2 \phi - \beta_1 \times \sin^2 2\phi). \quad (28)$$

In which the coefficients are taken for option geodetic reference system Moritz. The result is a value γ_0, equal to 9.807290173 m/c^2. Furthermore, to calculate the value of the disturbing potential T across the surface under study found the box for each value of the volume V, mass M and radius characterizing the position of the center of the base of the box. After determining the values T_i for all parallelepiped calculated their total potential U and the height anomaly ζ, the formula Bruns. For components plumb line in the meridian plane ξ (“), and the first vertical plane η (“) use the following formula:

$$\xi = -\frac{1}{\gamma} T_x, \eta = -\frac{1}{\gamma} T_y, \quad (29)$$

where:

$$T_x = G \times \left(\frac{\left(x' + \frac{a}{2} \right) I'}{r^3} \right);$$

$$T_y = G \times \left(\frac{\left(y' + \frac{a}{2} \right) I'}{r^3} \right).$$

At the last stage value determined gravity ten points selected in the experiment, according to the following formula:

$$g = \gamma_{0\ \text{св.в}} + \Delta g, \quad (30)$$

where:

$$\gamma_{0\ \text{св.в}} = \gamma_0 - 0.3086H,$$

H is height gravimetric points whose characteristics were calculated.

An experiment was carried out to determine the gravitational characteristics of a ten-point area of the volcano St. Helens volcano site (Mount St. Helens), located in Skamania county Washington State (USA). The results showed that the minimum value of the gravity = 980.0428 Gal and maximum = 980.3847 Gal values plumb line in the meridian plane and the plane of the first vertical are: ξ_{min} = –3.8 , ξ_{max} = 4.7, η_{min} = –5.0, η_{max} = 4.8, respectively. The calculated height anomaly has the following values ζ_{min} = 6.6 m, ζ_{max} = 28.1 m.

As a result of experiments are calculated gravity values of four points a simulated local coastal area of the sea bed located both in the valley and at higher elevations. These minimum and maximum values of gravity totaled 980.6770 Gal and 980.7365 Gal, respectively. The calculated values of plumb line in the meridian plane and the plane made the first vertical ξ_{min}= –0.24, ξ_{max} = 0.34, η_{min} = –0.91, ηmax= 0.11, equal to the height anomaly ζ_{min} = 0.7 m, ζ_{max} = 7.1 m. The minimum and maximum values of the four points of the force of gravity derived from the digital elevation model of coastal hilly area of the Kamchatka Peninsula are irrelevant 980.68729 Gal and 980.72830 Gal, respectively. The calculated values of plumb line in the meridian plane and the plane of the first vertical up: ξ_{min} = –0.87, ξ_{max} = 1.16, η_{min} = –2.39, η_{max} = 0.26, equal to the height anomaly ζ_{min} = 0.02 m, ζ_{max} = 0.10 m.

Conclusions

This study presents five point models of gravitating truncated cone to calculate the coordinates of the centers of gravity for five coextensive masses which are derived analytically rigorous mathematical formula. Gravimetric observations can be clarified using the derived formulas to approximate the conical body’s terrestrial relief (open mining, waste dumps, volcanoes, etc.). The experiments confirm the possibility of using the proposed methods with the use of analytical, finite element and numerical models to better determine the characteristics of the local gravity of natural and man-made objects of sizes up to several tens of kilometers. In this case, the number of selected finite elements is significantly reduced compared to the cubic approximation.

The easiest option describing the gravitational influence of the local relief elements is using of the point model. For some applications, it may be sufficient. However, there are situations that require more accurate approximation. Our research results can improve the accuracy of the description of the local gravitational field in comparison with existing methods by about 3–4% for local objects from Table 2 with dimensions of territory about 20–100 km^2 with mountain and foothill terrain. The basis for improving the accuracy is a combination of analytical models, finite element method, and digital elevation models.

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