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PRECISE LEVELLING DATA PROCESSING NEAR TERRACED LANDFORMS

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Abstract. Precise levelling results are affected by the Earth's gravity field, especially in areas of abrupt changes of landscape, such as terraced landforms. To eliminate the effect of the gravity field gradient, corrections need to be used in precise levelling data processing. To estimate the expected range of the correction due to the gravity field gradient (here called the gravimetric correction) within a region of terraced landforms, an experiment was proceeded in Estonia. Gravity data together with GNSS coordinates were acquired in 2011 in an area where a levelling section crosses the North Estonian Klint (height difference of 30 m within the levelling section). The gravimetric correction for the given 300 m long section proved to be 1.2 mm. Practically the same correction value can be obtained using interpolation of existing gravity data. However, in the case study area the gravity database had an extremely good quality which may not be the case elsewhere in which case gravimetric information needs to be collected alongside levelling. In height network calculations it is important to note that in such challenging areas all points should obtain their height values from an adjustment or from a point on the same side of a terrace, otherwise errors in heights may be as large as the gravimetric correction across the terrace.

Keywords: levelling, gravity field, gravimetry, cliff, Estonia.

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Introduction

The Earth's gravity field affects precise levelling. Thus, a correction that accounts for the changes in the gravity field in a survey area needs to be added to the levelling results. The magnitude of the correction value appears to be the most significant in areas of abrupt changes of landscape. The objective of current research is to determine and learn to predict the magnitude of these corrections to levelling profiles that are nearby or cross terraced landforms.

Terraced landforms are areas of abrupt change in height within a short horizontal distance. The highest coastal cliffs, reaching 1370 m vertically, are in Canada, Baffin Island. Inland terraces can reach up to 1300 or 4600 meters, depending on the strictness of the definition. (Wikipedia 2012) In Europe, some of the better known coastal terraces are in the United Kingdom and Ireland (see an example on Fig. 1), also in France and Italy. In Estonia, terraced landforms reach up to 56 m in coastal regions.

Fig. 1. Cliffs of Moher in Ireland reach up to 214 m
Source: from www.cliffsofmoher.ie

Estimation of magnitude of the correction due to changes in the gravity field, from here on called the gravimetric correction, is based on gravity data acquired in a case study. Geopotential numbers are used for determining the differences in surveyed geometric heights and "gravimetrically corrected" normal heights.

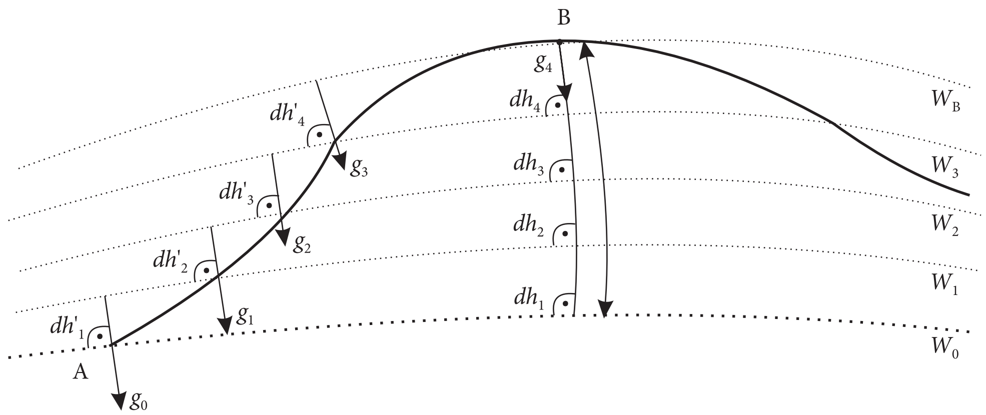

Fig. 2. Equipotential surfaces, levelled heights and orthometric heights [modified from Heiskanen and Moritz (1967)]

The submission is structured as follows. At first, the theoretical background of the gravity field's relation to levelling is described. Following that, a case study of collecting and processing specific gravity data for investigating the gravimetric corrections is described. This leads to a discussion on the influence of gravity field gradients on height network planning and levelling. A brief summary concludes the paper.

1. Theoretical background

As is well known, the Earth's gravity varies according to the locations of observation points. The gravity field is stronger on the poles and weaker on the equator due to the centrifugal effect on the rotating Earth. The variations in the gravity field are also due to the heterogenous nature of the Earth's interior and crust. Accordingly, the equipotential surfaces of the Earth are curved (Fig. 2). Related difficulties are explained in e.g. Heiskanen and Moritz (1967) and shortly reviewed below.

Where the gravity field is stronger, the distance between the equipotential surfaces is shorter (for example in point A compared to point B on Fig. 2). Let us look at levelling across such a heterogenous gravity field. The starting point A of a route is located at the reference surface of levelling (for example on the geoid), which is an equipotential surface W_0. To find the height of the endpoint B that is located on the equipotential surface W_B, geometric levelling is conducted. The measured height difference $\Delta H'$ between points A and B is obtained by summing of measured height increments dh'. The actual orthometric height difference ΔH of point B is the length of the plumb line (passing through the point B) in between the reference surface W_0 and the surface W_B. Thus, it is the sum of plumb-line increments dh between the two surfaces.

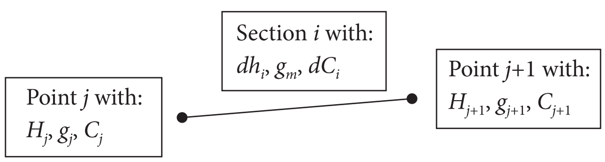

Fig. 3. Two levelling stations and a section between them with corresponding values of height, gravity and geopotential, the used symbols are explained in text

Since the equipotential surfaces are not parallel i.e. $dh' \neq dh$, the measured height difference is not equal to the orthometric height difference, i.e. $\Delta H' \neq \Delta H_{AB}$, creating a situation where the surveyed height depends on the trajectory of the levelling route. This means that in a closed levelling loop the sum of height increments is not necessarily equal to zero:

$$\oint dh' \neq 0. \tag{1}$$

To avoid such vagueness, the height increment dh_i' within a section i (Fig. 3) is calculated into geopotential value increment dC_i by multiplying it with the average gravity value g_m on the section:

$$dC_i = g_m \cdot dh_i', \tag{2}$$

where g_m is usually calculated as the average between gravity values of sections' endpoints:

$$g_m = \frac{g_j + g_{j+1}}{2}. \tag{3}$$

The geopotential value of point B is then calculated from the geopotential value of point A and the sum of geopotential increments dC_i:

$$C_B = C_A + \sum dC_i. \tag{4}$$

In a closed levelling loop the theoretical sum of geopotential increments equals to zero and the remaining deviation from zero reflects only inaccuracies of levelling that can be adjusted by the usual methods (e.g. proportionally to section lengths or by a least squares adjustment).

After finding the geopotential value at point B, it is converted to a conventional height value H_B by:

$$H_B = \frac{C_B}{\overline{g_B}}, \tag{5}$$

where the value of $\overline{g_B}$ depends on the desired height system. For instance, for the normal height system, the value of $\overline{g}$ corresponds to $\overline{\gamma}$, the average value of normal gravity along the ellipsoidal normal. Thus, normal heights are calculated as:

$$H_B^n = \frac{C_B}{\overline{\gamma}_B}. \tag{6}$$

The value of $\overline{\gamma}$ can be rigorously calculated from:

$$\overline{\gamma} = \gamma_0 \left[1 - \left(1 + f + m - 2f \sin^2 \varphi\right) \frac{H}{a} + \frac{H^2}{a^2} \right], \tag{7}$$

where a is the major semi-axis of the reference ellipsoid; $f = (a-b)/a$ is the flattening of the ellipsoid; b is the minor semi-axis of the ellipsoid; φ is the geodetic latitude; $m = \omega^2 a^2 b / GM$; ω is the angular velocity of the Earth's rotation and GM is the gravitational mass constant. Also, the geopotential value of the initial point A used in Eq. (4) is calculated from its height H_A and normal gravity $\overline{\gamma}_A$ by multiplying the two.

The difference between the measured geometric height H' and the normal height H^n is the so-called gravimetric correction.

An alternative algorithm for calculating the gravimetric correction often used in practical computations (e.g. Planserk Ltd. 2010) is the following:

$$H_B^n - H_A^n = dh_{AB} + f_{AB}, \tag{8}$$

where H_B^n and H_A^n are normal heights of points A and B; dh_{AB} is the measured height difference between A and B; f_{AB} is the so called normal correction (a gravity correction for obtaining normal heights) and is calculated from:

$$f_{AB} = -\frac{1}{\gamma_m}\left(\gamma_{0B} - \gamma_{0A}\right) H_m + \frac{1}{\gamma_m}\left(g - \gamma\right)_m dh_{AB}, \tag{9}$$

where γ_m is taken to be 980 000 mGal; γ_{0A} and γ_{0B} are normal gravity values on points A and B; H_m is the average height of A and B; $(g-\gamma)_m$ is the average gravity anomaly on points A and B. Normal gravity values on the reference ellipsoid are obtained from the Helmert 1901 equation:

$$\gamma_0 = \gamma_e (1 + \beta \sin^2 \varphi - \beta_1 \sin^2 2\varphi), \tag{10}$$

where values of $\gamma_e = 978030$, $\beta = 0.005302$ and $\beta_1 = 0.000007$ need to be adopted in case of the Baltic Height System 1977 (BH'77).

The aforementioned gravimetric/normal corrections can be calculated either for the full levelling profile or by each individual levelling station.

Next, the magnitude and behaviour of the gravimetric correction is investigated using information collected for a case study at a terraced landform.

2. Case study

2.1. Data collection

Gravity surveys were conducted in Tabasalu, North Estonia. A profile of gravity values was surveyed on a road that crosses a terrace, the North-Estonian Klint (on the background of Fig. 4. The road, being cut into the terrace, is steep, but levelling along it is possible. In fact, a section of a high-precision height network has been levelled along the same road. The misclosure of this section was said to be particularly high which arose the question of the effect of gravity field change on levelling in Tabasalu.

Gravity data were acquired using a LaCoste& Romberg model G (LCR G-65) relative spring gravimeter (Fig. 5). On each survey point with an interval

Fig. 4. Gravity surveys on the road that cuts into the North-Estonian Klint (on the background)

Fig. 5. LaCoste&Romberg model G gravimeter used for the collection of gravity data

of about 50 m (corresponding to an average distance between levelling stations on slopes), at least three readings were taken. The reading reflecting the gravity value was always reached by turning the metering screw clockwise. To avoid temperature changes within the instrument, it was kept hidden from direct sunrays (by an umbrella seen on Fig. 5) and during transport the instrument was covered by a white cover. The instrument was handled with extreme care as not to jolt or shock it. On one instance the instrument did receive a small shock, the consequences of which were visible in the results and were treated as a jump in the drift function. During surveys unnecessary movements around the instrument were avoided, readings were not taken when large trucks passed. To remove the effect of a number of inaccuracies caused by instrumental and environmental factors, surveying was repeated on several points, allowing for the gravimeter's drift calculation.

The coordinates of the gravity points were determined in the EUREF-EST geodetic system (longitude λ, latitude φ and height h). Later the coordinates were re-computed into the Estonian rectangular L-EST'97 coordinates x, y and BH'77 heights H. Positioning was proceeded using RTK GPS (Real Time Kinematic Global Positioning System), the Trimble VRS Now virtual stations' service and a Trimble 5800 GPS receiver (also visible on Fig. 5). The expected uncertainty of planar coordinates is ±1...2 cm, of heights ±3 cm. However, an earlier data analysis has revealed that the uncertainty for such RTK GPS heights can reach up to ±10 cm (Türk *et al.* 2011).

2.2. Data processing

First, the gravimeter's readings were reduced to the height of the survey point and the effect of lunar tides was removed by GRAVS2 software developed by the Estonian Land Board (2012). Next, the gravimeter's drift was modelled by a linear function and removed from the data. Remaining deviations on repeated points were not systematic.

The relative gravity surveys conducted in Tabasalu were connected to the Suurupi absolute gravimetric point (Oja *et al.* 2009, situated some 10 km West from the study area) by a digital quartz spring Scintrex CG5 gravimeter. The uncertainty of surveys with this instrument does not exceed the uncertainty of LCR gravimeters. A least squares adjustment was proceeded to calculate gravity values on the survey points.

From the covariance matrix of the adjustment, standard deviations (STDEV) for the gravity values were obtained, the largest deviation value reached 0.06 mGal. Since calibration parameters determined for the G-65 gravimeter (Oja *et al.* 2010) were not considered, the possible loss in accuracy was calculated from the calibration parameters. The largest change in gravity values measured on the profile was 6.6 mGal which corresponds to the calibration parameters' polynomial component (F_{pol}) of 0.011 mGal and periodic component (F_{per}) of 0.010 mGal. The resulting uncertainty $\sigma(g)$ of gravity measurements was thus found to be

$$\sigma(g) = \sqrt{STDEV^2 + F_{pol}^2 + F_{per}^2} =$$

$$\sqrt{0.06^2 + 0.011^2 + 0.010^2} = 0.062 \approx 0.06(\mathrm{mGal}). \quad (11)$$

The free-air gravity correction is about 0.31 mGal/m, meaning the gravity field weakens by 0.31 mGal with every meter that the observation point moves higher from the initial surface. As the uncertainty of GPS height determination could reach up to 10 cm (see the end of previous section), the gravity survey point could have an additional error of $0.31 \cdot 0.1 = 0.03$ mGal. Hence, the accuracy of GPS positioning is sufficient for the purposes of this research.

A digital elevation model (DEM) can also be used for obtaining height information. However DEM models lack accuracy in terraced areas, including errors that can reach half of the terrace's height. For example a 3"x3" DEM of Estonia has errors of up to 10 metres on the Tabasalu terrace (Talvik 2012: 87), which is clearly not sufficient for the present goals of obtaining viable height information for gravity survey points.

Most commonly, for the purposes of levelling network data processing existing gravity data are used instead of conducting special gravity surveys. For instance, within the case study area the gravity data coverage is very dense, see Ellmann *et al.* (2009). Using interpolation and initial height values, gravity values on the Tabasalu route points were predicted from the existing gravity anomaly data for comparison with field data. The interpolated values proved to have an accuracy of about ±0.6 mGal near the terrace (Talvik 2012, Ch. 6.2), which is 10 times lower than that of field data.

2.3. Calculations of the gravimetric correction

As described above, to determine the effect of the gravity field gradient along a levelling profile to the levelling results, the height increments measured need to be converted into geopotential numbers, if necessary, adjusted and later converted to conventional normal height values. This was proceeded using the gravimetric data acquired on the Tabasalu profile, assuming the accuracy of survey points' height values (obtained by using RTK GPS, see Sec. 2.1) to be the same as that of precise levelling.

The surveyed geometric height difference between the profile endpoints A and B is:

$$\Delta H' = H_B' - H_A' = 32.490 - 2.576 = 29.914\,(\mathrm{m}),$$

where H_B' and H_A' are the surveyed height values. The normal height difference between the endpoints A and B [calculated using Eqs. (2)–(4) and (6)] amounts to:

$$\Delta H^n = H_B^n - H_A^n = 32.4887 - 2.5759 = 29.9128\,(\mathrm{m}).$$

The discrepancy between the geometric and normal height differences (i.e. the gravimetric correction) between endpoints A and B is therefore:

$$\Delta H' - \Delta H^n = 29.9140 - 29.9128 = 0.0012\,(\mathrm{m}) = 1.2\,(\mathrm{mm}).$$

Hence, the gravity gradient along the approx. 2 km long Tabasalu profile creates a 1.2 mm discrepancy between surveyed geometric and gravimetrically corrected normal heights, a significant error in terms of precise levelling.

The distribution of the gravity gradient's effect on levelling along the profile was also investigated. The value of $H' - H^n$ for every levelling station was plotted against the distance of the gravity point from the terrace (Fig. 6). The distance was calculated as a planar distance from the survey point that is situated on top, closest to the edge of the terrace.

As mentioned, the profile does not follow the terrace itself exactly, but is cut into it. In fact the 30 meter height difference observed at the terrace is covered by 300 meters of levelling profile on the road. This can be seen in observed gravity values that continue to increase below the terrace until the distance of 300 meters from the upper edge (Fig. 6).

From Fig. 6 it becomes obvious that the gravimetric correction is the largest at the immediate neighbourhood of the terrace, where the height and gravity values change rapidly between levelling station points. This means that the 1.2 mm correction is in fact distributed only within 300 meters from the edge, the immediate neighbourhood of the terrace.

The correlation between the gravity correction and height or gravity gradient was further investigated. In the case of Tabasalu a 0.2 mm gravimetric correction is caused by a 1 mGal change in the gravity field (Fig. 7) or a change of height by 5 metres (Fig. 8).

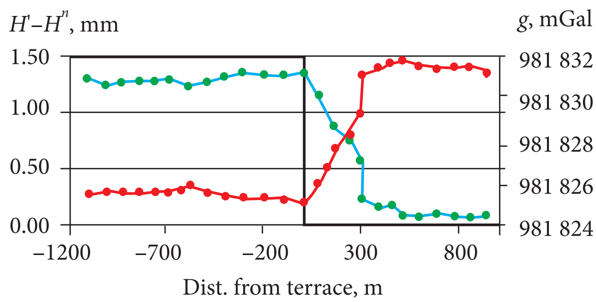

Fig. 6. Value of $H' - H^n$ (blue line) and the observed gravity values g (red line) for the Tabasalu profile, an illustrative shape of the terrace (black line)

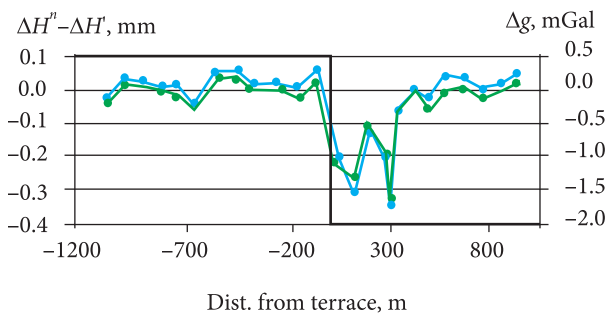

Fig. 7. Value of $\Delta H^n - \Delta H'$ (green line) and the gravity change Δg between two station points (blue line) for the Tabasalu profile, an illustrative shape of the terrace (black line)

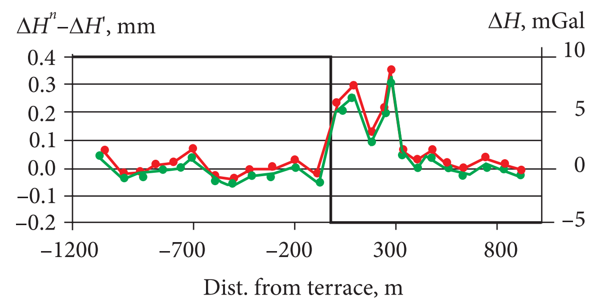

Fig. 8. Value of $\Delta H' - \Delta H^n$ (green line) and the height change ΔH between two station points (brown line) for the Tabasalu profile, an illustrative shape of the terrace (black line)

Thus it is obvious that areas with large height or gravity gradients need more attention in levelling data processing.

For numeric data and further information on the gravity field behaviour in the study area the reader is advised to consult Talvik (2012), a complementary discussion can also be found in Talvik *et al.* (2014).

3. Discussion

The gravimetric corrections calculated empirically for the case study area have proved to be very significant – within 300 meters from the upper edge of the terrace (which corresponds to the actual length of the descent), the gravimetric correction can cumulate to 1.2 mm. This is a reminder that the use of such corrections is vital in precise levelling data processing. Thus the quality of gravimetric data affects levelling accuracy directly. The gravity data need to be available in levelling areas or collected alongside with the levelling fieldwork.

During a recent campaign a precise levelling section crossed the terrace on nearly the same route as the gravity profile described in the case study. Using the same reference ellipsoid and gravimetric system, interpolating gravity values from an existing gravity database and calculating the correction using the algorithm described by formulae (8)–(10) the correction amounts to 1.3 mm which is consistent with the findings of this case study. It is an excellent agreement, considering that the interpolated gravity data was about 10 times less accurate than a special survey. However, it is important to note that the existing gravity data used for interpolation to real levelling stations were extremely dense (about 4 points/km^2). The correction calculation might not be as successful with lower quality databases.

It is important to note that if correct normal height values were necessary at levelling section endpoints A and B only, the distribution of gravimetric corrections on survey points between the endpoints would not be significant. However, attention should be paid to corrections when planning a multi-order height network where higher order points have a gravimetric correction added, but lower order points do not. In the worst case of higher order points A and B lying on two different sides of a terrace and a lower order levelling section starting from A but crossing the terrace towards B, but not closing on B, the resulting error could amount to the level of the whole gravimetric correction between points A and B.

The terraced landform investigated in the current study reached a height of 30 meters. However, the effect of higher terraces would be significantly larger. Using known information on the magnitude of the gravimetric correction, the effect of landforms with a different height can be predicted (see Talvik 2012).

Conclusions

To evaluate and learn to predict the effect of the gravity gradient along a levelling section to the levelling results, a field experiment on a terraced landform was conducted. Gravity data were acquired using a relative spring gravimeter; uncertainty of 0.06 mGal was achieved. Positioning was proceeded using GPS technology; the uncertainty of achieved coordinates was likely not exceeding 0.10 m. Data collected were processed as if they had the accuracy of precise levelling.

Height increments were converted into geopotential number increments and later calculated into conventional normal height values. This process eliminates the effect of the Earth's gravity field from levelling results. By comparing the surveyed geometrical height increments with the gravimetrically corrected normal height increments, the magnitude of the gravimetric effect was found. The findings illustrate the connection of both the height and gravity gradient along a levelling profile with the resulting correction to the levelling data.

Gravity field gradient along a levelling profile has a significant effect on the levelling results. Being able to predict the magnitude and the distribution of the gravity gradient's effect to levelling in more challenging regions allows for a thoughtful levelling network planning.

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