

SUPPLIER SELECTION FOR HOUSING DEVELOPMENT BY AN INTEGRATED METHOD WITH INTERVAL ROUGH BOUNDARIES

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Abstract. Residential whole-decoration is an important initiative for housing industrialization in China. Selecting the most suitable component supplier for housing development is of great significance for both property developers and buyers in the implementation of such a strategy. To address such a problem, this study uses hesitant fuzzy linguistic term sets to express the inaccurate judgments of individuals and then introduces a novel probability aggregation approach based on interval rough boundaries to enable a realistic presentation of the collective evaluations of a group. Then, we propose a hybrid multi-expert multiple criteria decision-making model by integrating the Best Worst Method (BWM) and Combined Compromise Solution (CoCoSo) method based on the interval rough boundaries. A case study about the supplier selection for housing development is carried out, which demonstrates the feasibility and applicability of our proposed hybrid model. A comparison study is also performed to further validate the robustness of the model.

Keywords: multi-criteria decision making, supplier selection for housing development, interval rough boundaries, Combined Compromise Solution method, Best-Worst method, hesitant fuzzy linguistic term set, probabilistic linguistic term set.

Introduction

As environmental protection in China is more and more prominent, energy saving and environment friendly housing development becomes one of the top priorities for the Chinese government (Zhang et al., 2011; Shi et al., 2012). For the conventional sale mode of Chinese housing, most residential houses sold on the market are unfurnished, which means that owners need to fit-out their residences in the way of separate decoration. In this situation, the supply of construction equipment and components fails to form a supply chain system of scale production and serialization, which results in low quality of decoration, poor equipment versatility, and unreasonable performance cost ratio of residential houses. Meanwhile, it also brings problems such as the waste of fit-out materials and the difficulty in handling construction waste (Ministry of Housing and Urban-Rural Development of the People's Republic of China [MOHURD], 2008).

A residential house with whole-decoration is also called a finished residence, which refers that all fixed surfaces of a functional space are completely paved or finished, and the basic equipment, such as the kitchen and bathroom, are all installed before the property developer turns the residence to the owner. In 1999, the concept of whole-decoration was first proposed in China (Ministry of Housing and Urban-Rural Development of the People's Republic of China [MOHURD], 1999). Until now, many provinces in China, such as Shanghai¹, Hainan², and

¹ The promotion of residential whole-decoration strategy in Shanghai, available at http://www.gov.cn/jrzq/2012-10/02/content_2237316.htm

² The promotion of residential whole-decoration strategy in Hainan, available at http://www.gov.cn/xinwen/2017-05/23/content_5196084.htm

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Zhejiang³, have begun to promote whole-decoration residential houses. The main reasons are as follows:

- 1) The advanced management system of real estate enterprises can be used to effectively manage the fit-out process, thus significantly reducing labor, material waste, and realizing resource conservation;
- 2) Property developers can integrate their supplier resources, shorten the circulation of fit-out materials, and reduce the fit-out cost through large-scale purchase, so as to develop high-quality residential products with a relatively low cost;
- 3) The close integration of construction and fit-out process is conducive to standardizing the residential fit-out behavior and reducing damage to the structure of a building, which is helpful for extending the operation life of a building, and further improving the living environment level.

The residential whole-decoration is an initiative to develop housing industrialization, and is also a mainstream development trend of the Chinese real estate market (Ministry of Housing and Urban-Rural Development of the People's Republic of China [MOHURD], 2013). Although appropriate strategies and actions have been taken by different levels of government in China to make fit-out activities sustainable, there are still many problems in the current implementation process, such as the high price of furnished housing, the insufficient supply of fit-out materials caused by short-term and large-scale purchasing demands, and the negative effect on living comfort caused by the quality problems of fit-out components (for example, the marble cracking of kitchen operation table and the expansion of cabinet door panel).

Components procurement for housing development is an important process of the implementation of residential whole-decoration. Supplier selection, as the first step of component procurement, is vital for the quality and cost control of residential houses. The selection process is supported by multi-criteria decision making (MCDM) methods, which consist of three major stages: evaluation, prioritization and selection of alternatives (Lam et al., 2010; Luo et al., 2016; Safa et al., 2014; Seth et al., 2018). To the best of our knowledge, few literature considered the MCDM methods on component supplier selection for housing development, which motivates the research question of this paper as how to use an appropriate MCDM model to select the optimal component supplier for housing development.

In MCDM approaches for suppliers' performance evaluation, multiple factors should be taken into consideration to obtain a comprehensive and robust result. One of the new utility-based MCDM methods is the CoCoSo (Combined Compromise Solution) method (Yazdani et al., 2019a, b), which is characterized by three aggrega-

tion strategies to obtain the moderate performances of alternatives. Due to the simple calculation process and the robustness of decision results, the CoCoSo method has been successfully applied in several fields like supply chain finance (Wen et al., 2019) and engineering management (Yazdani et al., 2019a). In addition, when applying MCDM methods, several weight determining methods have been proposed for criteria. Best worst method (BWM), initially introduced by Rezaei (2015), is one of the most effective criteria weighting approaches with less pairwise comparisons and a higher consistency ratio (Mi et al., 2019).

In the evaluation process, usually, a group of experts are invited to give their evaluation information. The experts are not always knowledgeable enough to give precise evaluations, especially in a complicated environment. In this sense, fuzzy methods are flexible to express inaccurate and fuzzy judgments. The hesitant fuzzy linguistic term set (HFLTS) (Rodriguez et al., 2011), which received considerable attention in recent years (Liao et al., 2018), is a powerful qualitative information representation tool since it can provide several linguistic terms in conjunction with a context-free grammar. In addition, traditional aggregation methods of HFLTSs failed to reflect the preferences of a group. It is observed that an interesting aggregation approach named the probability aggregation method was proposed by Wu and Liao (2018) to integrate individuals' opinions. However, such an aggregation method failed to take into account the vagueness of the judgments given by a group. The rough number (Zhai et al., 2008) contains the information that indicates not only the preference of an individual expert but also the diversity of all other experts' opinions. The better the agreements of individual expert's perceptions, the smaller the interval rough boundaries will be. In this sense, to solve the shortcoming of the hesitant fuzzy linguistic aggregation, we propose a novel probability aggregation approach to integrate individuals' evaluation values based on the interval rough boundaries.

Based on the above analysis, this study dedicates to proposing a hybrid MCDM model combining the BWM and CoCoSo method based on interval rough boundaries. The novelty of this study can be summarized as follows:

- 1) We propose a novel probabilistic aggregation approach based on interval rough boundaries, which enables a realistic presentation of collective evaluations by considering the vagueness of the evaluation information given by multi-experts.
- 2) The BWM with interval rough boundaries is developed to determine the weights of criteria for cognitive complex multi-expert decision-making problems.
- 3) The CoCoSo method with interval rough boundaries is proposed, and the calculation procedure of the CoCoSo method under the probabilistic linguistic environment is given.
- 4) A hybrid multi-expert multiple criteria decision-making model by integrating the BWM and CoCoSo method based on interval rough boundaries

³ The promotion of residential whole-decoration strategy in Zhejiang, available at http://www.gov.cn/xinwen/2017-02/02/content_5164705.htm

is developed to help property developers select the “best” construction component suppliers in the cognitive complex decision-making environment, which is conducive to the promotion of residential whole-decoration, and further promoting resource conservation utilization in the construction sector.

The rest of this paper is organized as follows: Section 1 briefly reviews the supplier evaluation problems in the construction industry. Section 2 introduces related concepts and the probabilistic aggregation approach with interval rough boundaries. Section 3 presents a hybrid multi-expert multiple criteria decision-making model to solve cognitive complex decision-making problems. Section 4 provides a case study on a supplier decision-making problem for housing development. The study ends with conclusions in the last section.

1. Literature review

In this section, firstly, we review some literature about the supplier evaluation model in the construction industry. Then, we give a short overview of the MCDM methods involved in this paper.

1.1. Brief review on supplier evaluation in construction industry

To ensure the quality and reduce cost in an increasingly compete construction industry, SCM (Supply Chain Management) has proved to be an indispensable tool. The first step to implement the SCM is supplier selection. An effective and efficient supplier selection model can help construction enterprises select the “best” supplier at the right cost in the right quality (Lam et al., 2010). The existing studies of supplier evaluation in construction industry mainly focused on two types: material or equipment supplier selection, and service supplier selection.

For the supplier selection of construction materials, Lam et al. (2010) constructed a selection model based on the fuzzy PCA (Principal Component Analysis). Safa et al. (2014) developed an integrated model for the efficient procurement of construction materials, primarily through the use of the TOPSIS method. Under the background of housing industrialization in China, Luo et al. (2016) put forward an evaluation index system for supplier selection regarding green housing components by considering 47 indicators. In their evaluation, to reduce the influence of subjective factors, they applied a model combining the Kent index method with the catastrophe theory. Wang et al. (2017) proposed a framework by integrating the building information modeling and geographic information system to select a resilient construction component supplier. Seth et al. (2018) demonstrated the impact of competitive conditions on the supplier evaluation for construction supply chains. For the selection of service suppliers, Eshtehardian et al. (2013) investigated 23 most effective criteria for supplier selection by a questionnaire survey, and integrated

the ANP (Analytic Network Process) with AHP (Analytic Hierarchy Process) to select appropriate suppliers for construction and civil engineering companies. Yin et al. (2017) established 17 criteria, and integrated the interval-valued intuitionistic fuzzy geometric weighted Heronian mean operator with a multi-target nonlinear programming model to obtain a comprehensive evaluation result by considering the influence of constructors’ subjective preferences and objective information on criteria. Matić et al. (2019) presented a combined model for sustainable constructor selection through a full consistency method and a rough COPRAS (Complex Proportional Assessment) method. Yazdani et al. (2019b) extended the CoCoSo method with grey numbers to measure the performances of suppliers, and to achieve the importance of supplier criteria, a combination of two weighting methods, namely, the DEMATEL (Decision Making Trial and Evaluation Laboratory) and BWM, were used in their study.

From the above review of supplier selection in the construction industry, we can find that the MCDM theory is effective for supplier selection in the construction industry. In addition, supplier selection is valued by construction enterprises since it is of great importance in improving their competitiveness. However, the literature that focused on comprehensive MCDM methods for supplier selection in construction industry under the cognitive complex decision-making environment, especially for the construction component suppliers, is limited. Thus, it is urgent to develop a comprehensive and effective MCDM model to select an appropriate construction component supplier for construction enterprises.

1.2. A short overview of MCDM methods

MCDM methods can be divided into two categories: outranking-based methods and utility-based methods. For massive alternatives, the outranking-based methods, such as the ELECTRE (Elimination Et Choice Translating Reality) and PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation), are limited due to complicated calculations. The utility-based methods, such as the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), VIKOR (Vlsekriterijumska Optimizacija I Kompromisno Resenje) and MULTIMOORA (Multiplicative Multi-Objective Optimization by Ratio Analysis), are effective due to their applicability and simplicity (Corrente et al., 2013; Liao & Wu, 2020). The CoCoSo method (Yazdani et al., 2019b), as a utility-based method, considers the compensation effects among criteria by a unique structure that presents the weighted average normalized decision matrix and weighted geometric matrix together. It can give a comprehensive ranking result by using three aggregation strategies, and the calculation process is relatively easy compared with other MCDM methods. This method has been extended to solve practical problems within different cases. For instance, Wen

et al. (2019) extended the CoCoSo method to hesitant fuzzy linguistic context and applied the hesitant fuzzy linguistic CoCoSo method to select third-party logistics service providers for financial institutions; Yazdani et al. (2019a) proposed a hybrid model by extending the CoCoSo method with grey numbers to measure the performance of suppliers in a construction company.

BWM (Rezaei, 2015) is a weighting approach based on linear programming. Compared with the AHP method, it requires less pairwise comparisons, and the obtained weights are more reliable since comparisons in the BWM are carried out with a higher consistency ratio. Due to its advantages, the BWM has received widespread attention in various fields. For example, Pamučar et al. (2018) modified the traditional BWM within the rough set context, and presented the algorithm of the hybrid BWM-MABAC (Multi-Attributive Border Approximation area Comparison) model based on interval-valued fuzzy-rough numbers to evaluate firefighting helicopters; Liao et al. (2019b) extended the BWM to the hesitant fuzzy linguistic environment for hospital performance evaluation; Liao et al. (2019c) integrated the BWM and ARAS method for digital supply chain finance supplier selection; Yazdani et al. (2019a) proposed a structured BWM-CoCoSo model for sustainable supplier selection.

Due to the complexity of numerous indicators and ambiguity in human thinking, there are difficulties in representing evaluation information in accurate values (Pamučar et al., 2018). Since Rodriguez et al. (2011) first proposed the HFLTS, it has been extended to various MCDM methods to solve practical problems, such as the HFL-AHP (Tüysüz & Şimşek, 2017), HFL-BWM (Liao et al., 2019b), HFL-VIKOR (Liao et al., 2014), and HFL-QFD (Quality Function Deployment) (Onar et al., 2016). However, the HFLTS can express only individual evaluations. While some aggregation methods of HFLTSs can be used to integrate hesitant fuzzy linguistic (HFL) evaluations, the information loss is serious (Wu & Liao, 2018). Pang et al. (2016) extended the HFLTS to the probabilistic linguistic term set (PLTS) by assigning probability to each linguistic term. The PLTS has gained many scholars' attention (Liao et al., 2019a; Zhang et al., 2017; Gou et al., 2017; Wu & Liao, 2019; Liao et al., 2020; Mi et al., 2020). Wu and Liao (2018) introduced an aggregation method called the probability aggregation method to integrate the opinions of individuals. However, it failed to consider the vagueness of information given by a group of experts.

Rough numbers were derived from the concept of approximation in the rough theory proposed by Zhai et al. (2008). The boundaries of rough numbers can avoid the subjectivity and measure the vagueness by providing an overall view about the diversity of all experts' opinions. The more diverse of a group's judgments are, the larger the interval rough boundaries of the preferences classes involved will be. Although little research has exploited the interval rough boundaries, the applicability of rough num-

bers has been fully justified in practical decision-making process. Song et al. (2013) proposed the rough group AHP and rough group TOPSIS to optimize the design concept evaluation under the subjective environment. Zhu et al. (2015) developed a novel rough number-based VIKOR to evaluate the design concept alternatives. Pamučar et al. (2017) combined the rough interval DEMATEL with the ANP to enable more objective expert evaluation of criteria in a subjective environment than a crisp approach. Pamučar et al. (2018) modified the traditional BWM and MABAC method by integrating the rough approach, which eliminated the subjectivity that exists when defining the borders of fuzzy sets. For the application of supplier selection in a construction company, Stević et al. (2017) extended the COPRAS and MULTIMOORA method by rough numbers.

Considering the limitation of the probability aggregation method proposed by Wu and Liao (2018) and the usefulness of the interval rough boundaries in representing vague information, this paper proposes a novel probabilistic aggregation approach based on interval rough boundaries. Then, based on the BWM and CoCoSo method, we construct an efficient hybrid MCDM model to select the "best" construction component supplier for construction enterprises under the cognitive complex decision-making environment.

2. A probability aggregation approach based on interval rough boundaries

In this section, we first introduce the concepts of HFLTS and PLTS. After that, we develop a novel probabilistic aggregation approach based on interval rough boundaries to enable a realistic presentation of the collective evaluations.

2.1. Concepts of HFLTS and PLTS

Evaluating qualitative variables by linguistic terms is intuitive and flexible for experts since linguistic terms are in line with human way of thinking and reasoning (Zadeh, 1975). To evaluate a linguistic variable, a linguistic term set (LTS) should be defined first. A general LTS is $S_{(1)} = \{s_\alpha \mid \alpha = 0, 1, \dots, 2\tau\}$, where τ is a positive integer and the subscripts are evenly distributed around the medium one (Herrera et al., 1995). Considering the different thinking mode of experts, the unbalanced semantics for an ordered set of linguistic terms were initially discussed by Torra (1996), and to facilitate understanding and calculation, Xu (2005) defined a subscript-symmetric LTS as $S_{(2)} = \{s_i \mid i = -\tau, \dots, -1, 0, 1, \dots, \tau\}$. Specific LTSs should be chosen according to practical decision-making problems so that the theoretical analyses of these models are not affected by the differences of LTSs.

The HFLTS, which allows people to give more than one linguistic term as the value of a linguistic variable by assigning them with equal importance, has been proved to be an effective tool to express individuals' judgments. It was proposed by Rodriguez et al.

(2011) and redefined in mathematical form by Liao et al. (2015). Let S be an LTS. An HFLTS on X , H_S , is in mathematical form of $H_S = \{ \langle x_i, h_s(x_i) \rangle \mid x_i \in X \}$, where $h_s(x_i) = \{ s_{\varphi_l}(x_i) \mid s_{\varphi_l}(x_i) \in S, l=1,2,\dots,L_i \}$ with L_i being the number of linguistic terms in $h_s(x_i)$ and $s_{\varphi_l}(x_i)$ ($l=1,2,\dots,L_i$) being the continuous terms in S . $h_s(x_i)$ denotes the possible degrees of the linguistic variable x_i to S and is termed as a hesitant fuzzy linguistic element (HFLE).

As the HFLTS was widely used, people found that the linguistic terms are associated with different probabilities in some situations. For example, people might think that the environmental level of an integral kitchen cabinet supplier is “somewhat good” in 70% and “good” in 30%. Here, the numbers “70%” and “30%” can be seen as the probabilities of different linguistic terms. To cope with probabilities, Pang et al. (2016) extended the HFLTS and proposed the probabilistic linguistic term set (PLTS) as $h_s(p) = \{ s^{(k)}(p^{(k)}) \mid s^{(k)} \in S, p^{(k)} \in [0,1], \sum p^{(k)} \in [0,1], k=1,2,\dots,K \}$, where K is the number of different linguistic terms in $h_s(p)$, and $s^{(k)}(p^{(k)})$ is the k -th linguistic term $s^{(k)}$ associated with the probability $p^{(k)}$.

2.2. Rough numbers and interval rough boundaries

Rough set is an effective tool to represent knowledge. In the rough set theory, an information system with decision attributes is called a decision table, denoted by $S = \langle U, Q = (A' \cap D), V, f \rangle$, where $U = \{x_1, x_2, \dots, x_i\}$ is a nonempty, finite set of objects, $A' = \{\alpha_1, \alpha_2, \dots, \alpha_j\}$ is a nonempty, finite set of condition attributes, and $D = \{d_1, d_2, \dots, d_s\}$ is a nonempty, finite set of decision attributes. Without loss of generality, a system with a single decision attribute $D = \{d\}$, is considered in this study. In general, it is assumed that $A' \cap D = \emptyset$ and each attribute $a \in A' \cup D$ forms a mapping $f : U \rightarrow V_a$, where V_a is the value domain of attribute α (Greco et al., 2001; Li et al., 2016). Each nonempty subset $B \subseteq A'$ determines an indiscernibility relation $R_B = \{ (x_n, x_m) \in U \times U \mid f(x_n, a) = f(x_m, a), a \in B \}$. R_B partitions U into a family of disjoint subsets given by $U/R_B = \{ [x]_B \mid x \in U \}$, where $[x]_B$ denotes the equivalence class determined by x with respect to subset B . The equivalence relation causes the vagueness of two classes. The rough set theory offers a means to describe these vague classes through the lower and upper approximations. For any class $C' \subseteq U$ and $B \subseteq A'$, the lower approximation of C' is presented as $\underline{R}_B(C') = \cup \{ [x]_B \mid [x]_B \subseteq C' \}$, and the upper approximation is presented as $\overline{R}_B(C') = \cup \{ [x]_B \mid [x]_B \cap C' \neq \emptyset \}$. The lower approximation is called the positive region of C' and can be denoted alternatively as $POS_B(C')$. C' is called a rough set with respect to B if and only if $\underline{R}_B(C') \neq \overline{R}_B(C')$, and the objects belonging only to the upper approximation but not to the lower approximation compose the boundary region $RB_B(C') = \overline{R}_B(C') - \underline{R}_B(C')$.

Approximations are important in the rough set theory to deal with uncertainty and vagueness. They only rely on the given data and no subjective information is needed.

To use the approximations to represent the vagueness of human assessments, Zhai et al. (2008) defined rough numbers and their corresponding lower and upper limits, as well as rough boundaries. In the following, we define rough number-related algorithms under the HFL environment for MCDM problems on the basis of the definitions proposed by Zhai et al. (2008):

Let $A = \{A_i \mid i=1,2,\dots,m\}$ be a set of alternatives, $C = \{c_j \mid j=1,2,\dots,n\}$ be a set of criteria, and $E = \{e_q \mid q=1,2,\dots,Q\}$ be a set of experts. An expert’s HFL judgment of alternative A_i on criterion c_j is $h_S^{ij(q)} = \{ s_{\varphi_1}^{(q)}, s_{\varphi_2}^{(q)}, \dots, s_{\varphi_{L(q)}}^{(q)} \}$. Then, the collective evaluation values of alternative A_i on criterion c_j can be expressed as $h_S^{ij} = \{ s_{\varphi_1}^{(1)}, s_{\varphi_2}^{(1)}, \dots, s_{\varphi_{L(1)}}^{(1)}, \dots, s_{\varphi_1}^{(q)}, s_{\varphi_2}^{(q)}, \dots, s_{\varphi_{L(q)}}^{(q)}, \dots, s_{\varphi_1}^{(Q)}, s_{\varphi_2}^{(Q)}, \dots, s_{\varphi_{L(Q)}}^{(Q)} \}$.

Suppose that $U^{ij} = \{ \varphi_1^{(1)}, \varphi_2^{(1)}, \dots, \varphi_{L(1)}^{(1)}, \dots, \varphi_1^{(q)}, \varphi_2^{(q)}, \dots, \varphi_{L(q)}^{(q)}, \dots, \varphi_1^{(Q)}, \varphi_2^{(Q)}, \dots, \varphi_{L(Q)}^{(Q)} \}$ contains all the subscripts of the terms in h_S^{ij} and let y be an arbitrary object from U^{ij} . There is a set of Y_{ij} classes $G_{ij} = \{ C_{ij}^{(1)}, C_{ij}^{(2)}, \dots, C_{ij}^{(Y_\beta)}, \dots, C_{ij}^{(Y_j)} \}$ in U^{ij} . If the classes satisfy $C_{ij}^{(1)} < C_{ij}^{(2)} < \dots < C_{ij}^{(Y_\beta)} < \dots < C_{ij}^{(Y_j)}$, then $\forall y \in U^{ij}, C_{ij}^{(Y_\beta)} \in G_{ij}, 1 \leq Y_\beta \leq Y_j$. The lower approximation and upper approximation of $C_{ij}^{(Y_\beta)}$ can be defined as:

$$\underline{Apr}(C_{ij}^{(Y_\beta)}) = \cup \{ y \in U \mid G_{ij}(y) \subseteq C_{ij}^{(Y_\beta)} \}; \tag{1}$$

$$\overline{Apr}(C_{ij}^{(Y_\beta)}) = \cup \{ y \in U \mid G_{ij}(y) \supseteq C_{ij}^{(Y_\beta)} \}. \tag{2}$$

The boundary region of $C_{ij}^{(Y_\beta)}$ is given by

$$RB(C_{ij}^{(Y_\beta)}) = \cup \{ y \in U \mid G_{ij}(y) \neq C_{ij}^{(Y_\beta)} \} = \tag{3}$$

$$\{ y \in U \mid G_{ij}(y) \supset C_{ij}^{(Y_\beta)} \} \cup \{ y \in U \mid G_{ij}(y) \subset C_{ij}^{(Y_\beta)} \}.$$

Then, $C_{ij}^{(Y_\beta)}$ can be represented by a rough number $RN(C_{ij}^{(Y_\beta)})$, which consists of its corresponding lower limit

$\underline{Lim}(C_{ij}^{(Y_\beta)})$ and upper limit $\overline{Lim}(C_{ij}^{(Y_\beta)})$, where

$$\underline{Lim}(C_{ij}^{(Y_\beta)}) = \frac{1}{M_{lo}} \sum G_{ij}(y) \mid y \in \underline{Apr}(C_{ij}^{(Y_\beta)}); \tag{4}$$

$$\overline{Lim}(C_{ij}^{(Y_\beta)}) = \frac{1}{M_{up}} \sum G_{ij}(y) \mid y \in \overline{Apr}(C_{ij}^{(Y_\beta)}); \tag{5}$$

$$RN(C_{ij}^{(Y_\beta)}) = [\underline{Lim}(C_{ij}^{(Y_\beta)}), \overline{Lim}(C_{ij}^{(Y_\beta)})], \tag{6}$$

with M_{lo} and M_{up} being the numbers of objects in $\underline{Apr}(C_{ij}^{(Y_\beta)})$ and $\overline{Apr}(C_{ij}^{(Y_\beta)})$, respectively.

The interval between the lower limit and upper limit is known as the rough boundary, denoted as:

$$RB(C_{ij}^{(Y_\beta)}) = \overline{Lim}(C_{ij}^{(Y_\beta)}) - \underline{Lim}(C_{ij}^{(Y_\beta)}). \tag{7}$$

Example 1. Let $S_{(2)} = \{s_{-3} = \text{very bad}, s_{-2} = \text{bad}, s_{-1} = \text{somewhat bad}, s_0 = \text{normal}, s_1 = \text{somewhat good}, s_2 = \text{good}, s_3 = \text{very good}\}$ be an LTS. Eight experts are invited to evaluate

Table 1. Comparison of rough numbers and their interval rough boundaries (for Example 1)

Class	Rough number	Lower limit	Upper limit	Rough boundary	Proportion of evaluation data
$C^{(1)}$	$\langle 0, 1.21 \rangle$	0.00	1.21	1.21	3/14
$C^{(2)}$	$\langle 0.7, 1.55 \rangle$	0.70	1.55	0.85	1/2
$C^{(3)}$	$\langle 0.92, 2.5 \rangle$	0.92	2.50	1.58	1/7
$C^{(4)}$	$\langle 1.21, 3 \rangle$	1.21	3.00	1.79	1/7

the environmental levels of an integral kitchen cabinet suppliers. Suppose that three experts believe it is “*some-what good*”, expressed as $\{s_1\}$; two consider it is “*between normal and somewhat good*”, expressed as $\{s_0, s_1\}$; two think it is “*at least somewhat good*”, denoted as $\{s_1, s_2, s_3\}$; one holds it is “*normal*”, denoted as $\{s_0\}$. Then, the collective HFLE is $h_S = \{s_1, s_1, s_1, s_0, s_1, s_0, s_1, s_1, s_2, s_3, s_1, s_2, s_3, s_0\}$, and $U = \{1, 1, 1, 0, 1, 0, 1, 1, 2, 3, 1, 2, 3, 0\}$ which has four classes $G = \{C^{(1)}, C^{(2)}, C^{(3)}, C^{(4)}\} = \{0, 1, 2, 3\}$. Take class $C^{(3)}$ whose subscript is “2” to explain the definition of the rough number. According to Eqs. (1)–(3): $Apr(2) = \cup\{y \in U | G(y) \leq 2\} = \{1, 1, 1, 0, 1, 0, 1, 1, 2, 1, 2, 0\}$, $Apr(2) = \cup\{y \in U | G(y) \geq 2\} = \{2, 3, 2, 3\}$, $RB(2) = \cup\{y \in U | G(y) \neq C^{(3)}\} = \{1, 1, 1, 0, 1, 0, 1, 1, 3, 1, 3, 0\}$. By Eqs. (4)–(6), the rough number of $C^{(3)}$ is calculated as: $Lim(2) = \sum G(y) | y \in Apr(2) | M_{lo} = 0.92$, $\overline{Lim}(2) = \sum G(y) | y \in Apr(2) | M_{up} = 2.5$, $RN(2) = [Lim(2), \overline{Lim}(2)] = [0.92, 2.5]$. The corresponding rough boundary is calculated by Eq. (7) as: $RB(2) = \overline{Lim}(2) - Lim(2) = 1.58$. Similarly, the rough number related calculations of other classes can be obtained, which are shown in Table 1.

According to the definition, we can see that each class associated with unique subscript value of an evaluation can be denoted by a unique rough number, whose lower and upper limits are computed based on all experts’ evaluations for an alternative under a criterion. This is because a rough number contains the information indicating not only the preference of an individual expert but also the diversity of all other experts’ opinions. Also, we can find that better agreements of experts’ evaluations will lead to smaller rough boundaries of the involved preference classes. For instance, in the given evaluation values, seven experts involve class $C^{(1)}$, while only two experts involve classes $C^{(3)}$ and $C^{(4)}$. In Table 1, the figure for class $C^{(1)}$ accounts for half of the total number of the given subscripts, while either class $C^{(3)}$ or $C^{(4)}$ accounts for only one-seventh. Thus, class $C^{(1)}$ has smaller rough boundary (0.85) compared with classes $C^{(3)}$ and $C^{(4)}$ (1.58 and 1.79, respectively), which implies that the class with a larger rough boundary is vaguer or less reliable.

2.3. A novel probability aggregated approach

Although the PLTS can express both the flexible qualitative assessment and quantitative probabilistic information,

it is difficult in reality to assign the probabilities of the linguistic terms in an expert’s evaluation since such assignments are determined based on experience, intuition or subjective perception.

In most cases, experts prefer to use HFLTSS to express their evaluations. Compared with the PLTS in which the probability of each linguistic term depends on experts’ subjective judgments, the HFLTSS is more objective. However, the HFLTSS has some shortcomings in representing collective opinions of a group (Wu & Liao, 2018). What’s more, in the collective decision-making matrix, there indeed exists probability distributions of HFLEs to reflect the different preferences of experts. To avoid these defects, Wu & Liao (2018) introduced an aggregation method called the probability aggregation method to calculate the weights of linguistic elements corresponding to a group, which is reliable since the group opinions and the preferences of a group can be presented. The algorithm is shown as follows (Wu & Liao, 2018): let $E = \{e_q | q = 1, 2, \dots, Q\}$ be a set of experts whose weight vector is $\{\gamma^{(1)}, \gamma^{(2)}, \dots, \gamma^{(Q)}\}$ and S be an LTS. Suppose that $h_s^{(q)} = \{s_{\varphi_l}^{(q)}(x) | s_{\varphi_l}^{(q)} \in S, l = 1, 2, \dots, L_q\} (q = 1, 2, \dots, Q)$ are Q HFLTSS on S given by Q experts. Let $h_s = \{s_{\varphi_l} | s_{\varphi_l} \in S, l = 1, 2, \dots, L\}$ be the group HFLE. Then, the weight of $s_{\varphi_l}^{(q)}$ in $h_s^{(q)}$ is defined as:

$$v_{\varphi_l}^{(q)} = \begin{cases} 1/L_q, & \text{if } \varphi_l \in h_s^{(q)} \\ 0, & \text{if } \varphi_l \notin h_s^{(q)} \end{cases}, l = 1, 2, \dots, L_q; q = 1, 2, \dots, Q \quad (8)$$

The weight of $h_s^{(q)}$ given by expert e_q is $\gamma^{(q)}$, for $q = 1, 2, \dots, Q$. Then, the weight of s_{φ_l} in the group HFLE is

$$p^{(\varphi_l)} = \sum_{q=1}^Q v_{\varphi_l}^{(q)} \gamma^{(q)}, l = 1, 2, \dots, L_q \quad (9)$$

Thus, the group judgment can be expressed as the PLTS

$$h_s(p) = \{s_{\varphi_l}(p^{(\varphi_l)}) | s_{\varphi_l} \in S, p^{(\varphi_l)} = \sum_{q=1}^Q v_{\varphi_l}^{(q)} \gamma^{(q)}, l = 1, \dots, L_q\} \quad (10)$$

With Eq. (10), the judgments given by individual experts in HFLEs can be aggregated into a PLTS. If the experts’ judgments are represented by PLTSs, with Eq. (10), the collective PLTS can be obtained when the original probabilities are replaced by $v_{\varphi_l}^{(q)}$. If the weights of the experts are not given, the weight vector can be supposed as $\gamma^{(q)} = 1/Q, q = 1, 2, \dots, Q$.

Example 2. (Continued to Example 1) Suppose that the weight of each expert is $\gamma^{(q)} = 1/8$. Take s_2 as an example. According to Eqs. (8)–(10): we have $p^{(2)} = 0.08$, and

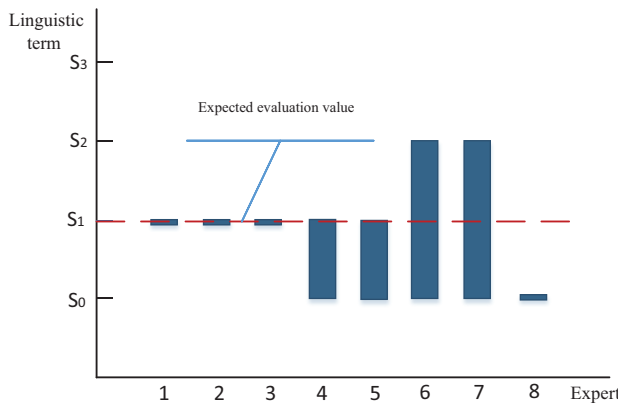


Figure 1. Experts' evaluations in Example 1

then $s_2(p^{(2)})=s_2(0.08)$. The aggregated PLTS of the group is $h_s(p) = \{s_0(0.25), s_1(0.58), s_2(0.08), s_3(0.08)\}$.

However, we may not accept the above result. Based on rational judgment without introducing interval rough boundaries, it can be concluded that the expected evaluation value of the group should be s_1 (as shown in Figure 1) based on the subscripts' arithmetic mean of the eight experts' evaluations. From Figure 1, we can see that the covering range (number) of s_2 is the same as that of s_3 , but the distance to the expected evaluation value of s_2 is closer to s_3 . So, it is unreasonable that s_2 and s_3 are assigned with an equal probability in the aggregated PLTS of the group.

As we discussed in Section 2.2, rough numbers depict the preference information of a given expert. In addition, all other experts' opinions are also considered in the calculation process. Generally, the smaller the boundary is, the crisper the number will be. In Example 1, we calculated the interval rough boundaries of the evaluation values' subscripts as: $RB(1)=1.21$, $RB(1)=0.85$, $RB(2)=1.58$, $RB(3)=1.79$. According to the definition of interval rough boundaries, class "2" is crispier than class "3", so we can conclude that the assigned probability of s_2 should be greater than s_3 in the set of the aggregated PLTS. Here, we define a concept named the accuracy degree to adjust the original aggregated probability, which is expressed as:

$$CD(C_{ij}^{(Y_\beta)}) = \begin{cases} \frac{1/RB(C_{ij}^{(Y_\beta)})}{\sum_{Y_\beta=1}^{Y_\beta} 1/RB(C_{ij}^{(Y_\beta)})}, & RB(C_{ij}^{(Y_\beta)}) \neq 0 \\ 1, & RB(C_{ij}^{(Y_\beta)}) = 0 \end{cases} \quad (11)$$

where: $CD(C_{ij}^{(Y_\beta)})$ denotes the accuracy degree of $C_{ij}^{(Y_\beta)}$. $C_{ij}^{(Y_\beta)}$ refers to the Y_β -th subscript class of all experts' HFL evaluation values of alternative A_i on criterion c_j . Then, the PLTS of the collective judgments can be adjusted as:

$$h_s(p^*) = \{s_{\phi_l}(p^{*(\phi_l)}) \mid s_{\phi_l} \in S, p^{*(\phi_l)} = (p^{(\phi_l)} \times CD(C_{ij}^{(Y_\beta)}))^{1/2} / \sum_{l=1}^L (p^{(\phi_l)} \times CD(C_{ij}^{(Y_\beta)}))^{1/2}, l=1, 2, \dots, L\}, \quad (12)$$

where: $p^{(\phi_l)}$ is the original probability of s_{ϕ_l} in the set of the aggregated PLTS.

Example 3. (Continued to Example 2) According to Eq. (11), we obtain the accuracy degree: $CD(0)=0.26$, $CD(1)=0.37$, $CD(2)=0.2$, $CD(3)=0.18$. According to Eq. (12), we can calculate the adjusted probabilities as: $p^{*(0)}=0.26$, $p^{*(1)}=0.48$, $p^{*(2)}=0.13$, $p^{*(3)}=0.12$. Thus, the PLTS of the collective judgment of the group is adjusted to $h_s(p^*) = \{s_0(0.26), s_1(0.48), s_2(0.13), s_3(0.12)\}$.

With Eq. (12), the judgments given by multiple experts in HFLEs can be aggregated into the adjusted PLTS. Since class "2" is crispier than class "3", from the results yield by our proposed aggregation method, the assigned probability of s_2 is greater than s_3 (0.13 and 0.12, respectively). So, the proposed aggregation method can better integrate individuals' opinions by taking into account the vague degree of the judgments given by a group of experts.

If the rough boundaries of different linguistic terms' subscripts are the same, then, the PLTS of the collective judgments can be directly obtained through the method proposed by Wu and Liao (2018).

Example 4. Let $S_{(2)} = \{s_{-3} = \text{very bad}, s_{-2} = \text{bad}, s_{-1} = \text{somewhat bad}, s_0 = \text{normal}, s_1 = \text{somewhat good}, s_2 = \text{good}, s_3 = \text{very good}\}$ be an LTS. Eight experts are invited to evaluate innovation capabilities of integral kitchen cabinet manufacturers. Suppose that two experts believe it is "somewhat good", expressed as $\{s_1\}$; four consider it is "between somewhat good and good", expressed as $\{s_1, s_2\}$; two think it is "good", denoted as $\{s_2\}$. The subscripts set of the evaluation values can be obtained as $U=\{1,1,1,2,1,2,1,2,2,2\}$, which has two classes $G = \{C^{(1)}, C^{(2)}\} = \{1, 2\}$. From Figure 2, we can see that the distances of different linguistic terms to the expected evaluation value are the same. By Eqs. (4)–(7), the interval rough boundaries of the two subscript classes are calculated as $RB(1)=0.5$, $RB(2)=0.5$. Thus, the adjusted PLTS of the collective judgment is $h_s(p^*)=h_s(p)=\{s_1(0.5), s_2(0.5)\}$, which is the same as that derived by the method of Wu and Liao (2018).

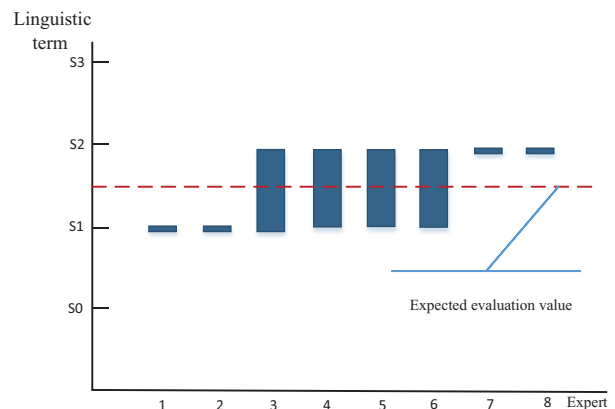


Figure 2. Experts' evaluations in Example 4

3. A hybrid multi-expert MCDM method based on the probability aggregation approach with interval rough boundaries

In this section, the proposed new probabilistic aggregation approach was tested by an MCDM model implemented in two phases: (1) determining the weights of evaluation criteria, and (2) ranking alternatives. To obtain the weights of criteria, the best-worst method (BWM) (Rezaei, 2015) with the probabilistic aggregation approach based on interval rough boundaries was used. After the criteria weights are obtained, a modified combined compromise solution (CoCoSo) method (Yazdani et al., 2019b) based on the probabilistic aggregation approach is applied. The framework of the proposed model is depicted in Figure 3.

3.1. A modified BWM with interval rough boundaries to determine the weights of criteria

BWM is a weight determining approach based on linear programming. It has received widespread attention in various fields (Pamučar et al., 2018; Liao et al., 2019b; Mi et al., 2019; Yazdani et al., 2019a). Motivated by the classical BWM and based on the interval rough boundaries, below we present an algorithm to obtain the weights of criteria.

Suppose that all experts select the best criterion and the worst criterion as c_B and c_w . To obtain a comparison, each expert should determine the preference degree of the best criterion c_B over criterion $c_j (j=1,2,\dots,n)$ and criterion c_j in relation to criterion c_w . The judgment of each pair takes the LTS $S_{(1)} = \{s_0 = \text{equally important}, s_1 = \text{equally very important}, s_2 = \text{moderately important}, s_3 = \text{moderately more important}, s_4 = \text{strongly imp-}$

portant, s_5 = strongly more important, s_6 = very strongly more important, s_7 = extremely strong important, s_8 = extremely more important\}. Each expert's evaluation is expressed using HFLEs. Subsequently, the evaluation results of the Best-to-Others (BO) is obtained as $h_s^{(q)jw}(x) = \{s_{\phi_l}(x) | s_{\phi_l}(x) \in S_{(1)}, l=1,\dots,L^{(q)}, q=1,2,\dots,Q$ and the Others-to-Worst (OW) is obtained as the HFLEs $h_s^{(q)Bj}(x) = \{S_{\phi_l}(x) | s_{\phi_l}(x) \in S_{(1)}, l=1,\dots,L^{(q)}, q=1,2,\dots,Q$.

Next, by the proposed probabilistic aggregation approach based on interval rough boundaries, the aggregated sequences on the preference of c_B over c_j can be expressed as the adjusted PLTS

$$h_s^{Bj}(p^*) = \{s_{\phi_l}^{Bj}(p^{*(\phi_l)}) | s_{\phi_l}^{Bj} \in S_{(1)}, l=1,\dots,L_{Bj}, \sum_{l=1}^{L_{Bj}} p^{*(\phi_l)} = 1\}$$

according to Eqs. (8)–(12). Similarly, the collective evaluations of all experts on the preference of c_j over c_w can be expressed as the adjusted PLTS

$$h_s^{jW}(p^*) = \{s_{\phi_l}^{jW}(p^{*(\phi_l)}) | s_{\phi_l}^{jW} \in S_{(1)}, l=1,\dots,L_{jW}, \sum_{l=1}^{L_{jW}} p^{*(\phi_l)} = 1\}.$$

Then, the BO vector and OW vector can be obtained as:

$$A_{BO} = (E(h_s^{B1}(p^*)), E(h_s^{B2}(p^*)), \dots, E(h_s^{Bn}(p^*))); \quad (13)$$

$$A_{OW} = (E(h_s^{1W}(p^*)), E(h_s^{2W}(p^*)), \dots, E(h_s^{nW}(p^*))), \quad (14)$$

where: $E(h_s^{Bj}(p^*)) = \sum_{l=1}^{L_{Bj}} (\phi_l^{Bj} \cdot p^{*(\phi_l)})$ and $E(h_s^{jW}(p^*)) =$

$\sum_{l=1}^{L_{jW}} (\phi_l^{jW} \cdot p^{*(\phi_l)})$ are the expect values (Pang et al., 2016).

Here, ϕ_l^{Bj} and ϕ_l^{jW} are the subscripts of the PLTSs $h_s^{Bj}(p^*)$ and $h_s^{jW}(p^*)$, respectively.

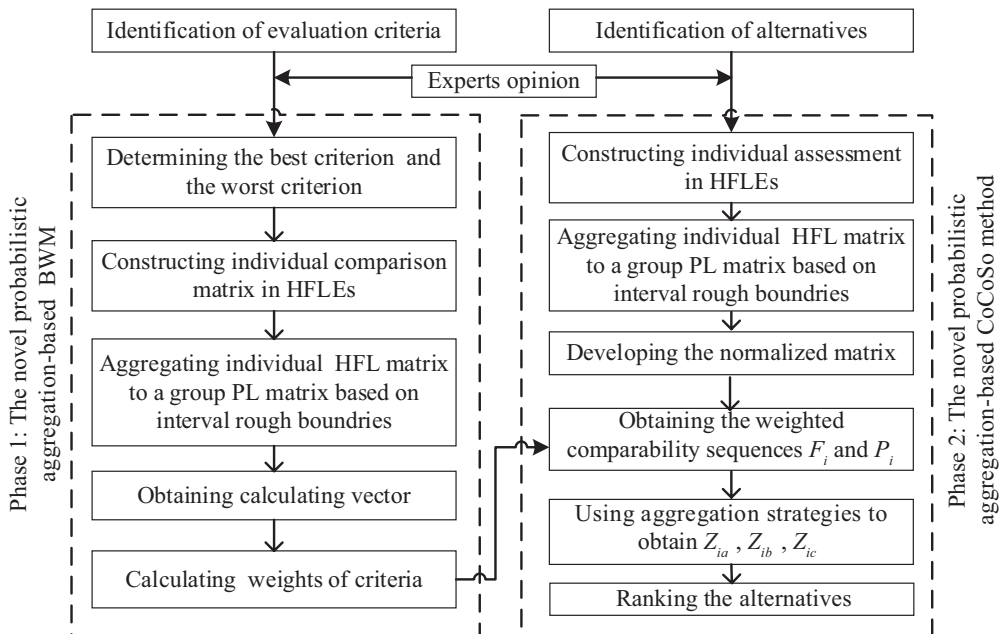


Figure 3. Framework of the proposed model

On the basis of the linear model proposed by Rezaei (2016), Model 1 is constructed as follows:

Model 1

$$\min \xi$$

s.t.

$$\left| \frac{w_B}{w_j} - E(h_S^{Bj}(p^*)) \right| \leq \xi, \text{ for all } j$$

$$\left| \frac{w_j}{w_W} - E(h_S^{jW}(p^*)) \right| \leq \xi, \text{ for all } j$$

$$\sum_{j=1}^n w_j = 1$$

$$w_j \geq 0, \text{ for all } j$$

Solving Model 1, we can obtain the optimal values of criteria weights $(w^*_1, w^*_2, \dots, w^*_j, \dots, w^*_n)^T$ and the corresponding minimum absolute difference ξ^* , based on which, we can check the consistency of the collective preferences $h_S^{Bj}(p^*)$ and $h_S^{jW}(p^*)$. When $\xi^*=0$, the pairwise comparisons are supposed to be consistent. In this sense, ξ^* can be regarded as a consistency measure. For more detail about the consistency checking and repairing of the BWM, please refer to Rezaei (2015).

3.2. A probabilistic aggregation-based CoCoSo method with interval rough boundaries

The CoCoSo method has been utilized widely in applications. In the following, we develop an interval rough boundaries-based probabilistic aggregation CoCoSo method to rank alternatives.

To ensure the flexibility and objectivity of experts' judgments, the HFLEs are used to express experts' assessments of alternatives with a subscript-symmetric LTS $S_{(2)} = \{s_\alpha \mid \alpha = -\tau, \dots, -1, 0, 1, \dots, \tau\}$. Then, an HFL judgment matrix of expert e_q can be constructed as:

$$D^{(q)} = \begin{bmatrix} h_S^{11(q)} & \dots & h_S^{1j(q)} & \dots & h_S^{1n(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_S^{i1(q)} & \dots & h_S^{ij(q)} & \dots & h_S^{in(q)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_S^{m1(q)} & \dots & h_S^{mj(q)} & \dots & h_S^{mn(q)} \end{bmatrix}.$$

To express the group opinions completely, by Eqs. (8)–(12), the individual opinions can be aggregated into a decision matrix $D = (h_S^{ij}(p^*))_{m \times n}$ through the probabilistic aggregation approach based on interval rough boundaries, which is expressed as:

$$D = \begin{bmatrix} h_S^{11}(p^*) & \dots & h_S^{1j}(p^*) & \dots & h_S^{1n}(p^*) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_S^{i1}(p^*) & \dots & h_S^{ij}(p^*) & \dots & h_S^{in}(p^*) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ h_S^{m1}(p^*) & \dots & h_S^{mj}(p^*) & \dots & h_S^{mn}(p^*) \end{bmatrix}.$$

Next, we establish a normalized matrix based on the following compromise normalization equations:

$$y_{ij} = \frac{E(h_S^{ij}(p^*)) - \min_i E(h_S^{ij}(p^*))}{\max_i E(h_S^{ij}(p^*)) - \min_i E(h_S^{ij}(p^*))}, \text{ for benefit criteria} \tag{15}$$

$$y_{ij} = \frac{\max_i E(h_S^{ij}(p^*)) - E(h_S^{ij}(p^*))}{\max_i E(h_S^{ij}(p^*)) - \min_i E(h_S^{ij}(p^*))}, \text{ for cost criteria} \tag{16}$$

where: $E(h_S^{ij}(p^*))$ is the expected value of $h_S^{ij}(p^*) = \{s_{\phi_l}(p^{*(\phi_l)}) \mid s_{\phi_l} \in S_{(2)}, l = 1, 2, \dots, L\}$ with

$$E(h_S^{ij}(p^*)) = \sum_{l=1}^L \left(\frac{\phi_l^{ij} + \tau_2}{2\tau_2} \times p^{*(\phi_l)} \right), \tag{17}$$

where: τ_2 is the scale of the LTS $S_{(2)}$ and ϕ_l^{ij} is the subscript of $h_S^{ij}(p^*)$ (Wu & Liao, 2018).

Next, we compute the weighted average comparability sequences and weighted geometric comparability sequences for each alternative as F_i and P_i , respectively, and obtain two weak rankings through these two weighted methods.

The weighted average operator is a complete compensatory aggregation method, through which the largest comprehensive value with respect to all criteria can be obtained, while the power aggregation operator is characterized by the incomplete compensation, through which the small values cannot be compensated by the large values completely (Liao & Wu, 2020):

$$F_i = \sum_{j=1}^n (w_j y_{ij}); \tag{18}$$

$$P_i = \sum_{j=1}^n (y_{ij})^{w_j}, \tag{19}$$

where: w_j represents the weights of criteria calculated in Section 3.1.

Three appraisal score strategies are used to generate the moderate performances of alternatives, which are presented as:

$$Z_{ia} = \frac{F_i + P_i}{\sum_{i=1}^m (F_i + P_i)}; \tag{20}$$

$$Z_{ib} = \frac{F_i}{\min_i F_i} + \frac{P_i}{\min_i P_i}; \tag{21}$$

$$Z_{ic} = \frac{\lambda F_i + (1-\lambda)(P_i)}{\lambda \max_i F_i + (1-\lambda) \max_i P_i}. \tag{22}$$

In Eq. (22), λ is determined by the preference of an expert on the compensation of criteria values. If the expert considers that the poor performances of an alternative under some criteria can be completely compensated by the good performances of the alternative under other criteria, then F_i should be assigned a big coefficient, namely, $\lambda > 0.5$; if the expert believes that the good performances of an alternative under some

criteria cannot fully compensate for the poor performances under other criteria, P_i should be assigned a big coefficient, namely, $\lambda < 0.5$ (Liao & Wu, 2020).

The final ranking of alternatives is determined based on Z_i in ascending order:

$$Z_i = (Z_{ia}Z_{ib}Z_{ic})^{1/3} + (Z_{ia} + Z_{ib} + Z_{ic})/3. \tag{23}$$

4. Case study: application of the proposed model for supplier evaluation in housing development

Selecting an appropriate construction component supplier for property developers has a vital role in the success for the implementation of the residential whole-decoration strategy. This section presents a case study about supplier selection for construction components in housing development. The proposed multi-expert MCDM method based on the probability aggregation approach with interval rough boundaries is applied to solve the case.

4.1. Case description

Company A is one of the largest and most comprehensive property developers in China. In 2009, the property developer announced its residential whole-decoration strategy, and so far, more than 80% of its residential products delivering to customers have been achieved in whole-decoration. To improve the quality of the delivered residential products and reduce the product fit-out cost through supply chain management, the management committee of the property developer conduct supplier decisions using MCDM methods in their first step of building component supply chain management. Integral kitchen cabinet is one of the most important fit-out components as its expense accounts high proportion in the overall fit-out spending and its important role in quality guarantee of residential products. Thus, this study takes the integral kitchen cabi-

net supplier selection for the property developer as an example.

The proposed hybrid MCDM model is applied to select an optimal supplier from five construction component suppliers $\{A_1, A_2, A_3, A_4, A_5\}$. The decision-making group of A company consists of five members $\{e_1, e_2, e_3, e_4, e_5\}$: the chief executive e_1 , general manager e_2 , contract management manager e_3 , and two experts e_4, e_5 with a minimum of five-year-experience on supply chain management in construction companies. According to the literature review, the fundamental criteria, such as quality, cost, green development and enterprise capability are selected (Luo et al., 2016; Yin et al., 2017). Moreover, since the target company focuses on the long-term relationship with the selected supplier so as to deliver the residential products better, faster and smoother, the cooperation potentiality is defined as strategy related criteria (Lam et al., 2010). The weight of each expert is $\gamma=1/5$. 12 evaluation criteria $\{c_1, c_2, \dots, c_{12}\}$ which are grouped into five clusters are presented in Table 2. Among these criteria, the product price, installation cost and environmental effect are cost criteria, while the others are benefit criteria, and this decision-making group agreed that c_3 is the best criteria, and c_{11} is the worst one.

4.2. Solving the case by the probabilistic aggregation-based CoCoSo method with interval rough boundaries

4.2.1. Determining criteria weights by the BWM with interval rough boundaries

The proposed probabilistic aggregation-based BWM is used here to obtain the weights of criteria for the integral kitchen cabinet supplier selection. Based on the case information given in Section 4.1, each member of the decision-making group is invited to make pairwise comparisons in which the preferences of the criteria

Table 2. The criteria for evaluation over candidate integral kitchen cabinet suppliers

Criterion	Sub-criterion	Form	Target value
Quality	Product sample pass rate (c_1)	Benefit	max
	Level of after-sale service (c_2)	Benefit	max
	Product performance (c_3)	Benefit	max
Cost	Product price (c_4)	Cost	min
	Installation cost (c_5)	Cost	min
Green development	Environmental level (c_6)	Benefit	max
	Environmental effect (c_7)	Cost	min
Enterprise capability	Innovation capability (c_8)	Benefit	max
	Professional skill (c_9)	Benefit	max
	Market position (c_{10})	Benefit	max
Cooperation potentiality	Cooperation intention (c_{11})	Benefit	max
	Supply capability of emergency demand (c_{12})	Benefit	max

c_B and c_w were considered over the remaining criteria from the defined set. The evaluations were represented by HFLEs with the LTS $S_{(1)}$, which are shown as follows:

$$BO = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} & \begin{bmatrix} \{s_3, s_4, s_5\} & \{s_2\} & \{s_0\} & \{s_1, s_2\} & \{s_7\} & \{s_2, s_3\} & \{s_6, s_7\} & \{s_3, s_4\} & \{s_6, s_7\} & \{s_4, s_5\} & \{s_7, s_8\} & \{s_4, s_5\} \\ \{s_4, s_5\} & \{s_2, s_3\} & \{s_0\} & \{s_0, s_1, s_2\} & \{s_7\} & \{s_2, s_3\} & \{s_6, s_7\} & \{s_3, s_4\} & \{s_6\} & \{s_4\} & \{s_8\} & \{s_4, s_5\} \\ \{s_4, s_5\} & \{s_2, s_3\} & \{s_0\} & \{s_0, s_1\} & \{s_7, s_8\} & \{s_3\} & \{s_6\} & \{s_3\} & \{s_7\} & \{s_4, s_5\} & \{s_8\} & \{s_3\} \\ \{s_5\} & \{s_2, s_3\} & \{s_0\} & \{s_1, s_2\} & \{s_7\} & \{s_3\} & \{s_6, s_7\} & \{s_3\} & \{s_6, s_7\} & \{s_4\} & \{s_8\} & \{s_3, s_4\} \\ \{s_5\} & \{s_2\} & \{s_0\} & \{s_0, s_1\} & \{s_7, s_8\} & \{s_1, s_2, s_3\} & \{s_6\} & \{s_3\} & \{s_6\} & \{s_4, s_5\} & \{s_8\} & \{s_4\} \end{bmatrix} \end{matrix};$$

$$OW = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{matrix} & \begin{bmatrix} \{s_4, s_5\} & \{s_6\} & \{s_7, s_8\} & \{s_6, s_7\} & \{s_1\} & \{s_5, s_6\} & \{s_2\} & \{s_4, s_5\} & \{s_1, s_2\} & \{s_3, s_4\} & \{s_0\} & \{s_3, s_4\} \\ \{s_4\} & \{s_6\} & \{s_8\} & \{s_7\} & \{s_1\} & \{s_6\} & \{s_2\} & \{s_4, s_5\} & \{s_1\} & \{s_4\} & \{s_0\} & \{s_3, s_4\} \\ \{s_4\} & \{s_5, s_6\} & \{s_8\} & \{s_7\} & \{s_0, s_1\} & \{s_5, s_6\} & \{s_2\} & \{s_4, s_5\} & \{s_1\} & \{s_4, s_5\} & \{s_0\} & \{s_3\} \\ \{s_4\} & \{s_6\} & \{s_8\} & \{s_6, s_7\} & \{s_0\} & \{s_5, s_6\} & \{s_2\} & \{s_4, s_5\} & \{s_2\} & \{s_3, s_4\} & \{s_0\} & \{s_3, s_4\} \\ \{s_4, s_5\} & \{s_5, s_6\} & \{s_0\} & \{s_7\} & \{s_0, s_1, s_2\} & \{s_5, s_6\} & \{s_2, s_3\} & \{s_5\} & \{s_1, s_2\} & \{s_5\} & \{s_0\} & \{s_4\} \end{bmatrix} \end{matrix}.$$

Then, we aggregate the preferences of the decision-making group on c_B over c_j to PLTSs $\{h_S^{Bj}(p^*)\}_{1 \times 12}$ and c_j over c_w to PLTSs $\{h_S^{jw}(p^*)\}_{1 \times 12}$ by Eqs. (8)-(12). The results are shown as:

$$\{h_S^{Bj}(p^*)\}_{1 \times 12} = \left. \begin{matrix} \left\{ \begin{matrix} c_1 & c_2 & c_3 & c_4 \\ \{s_3(0.12), s_4(0.30), s_5(0.59)\} & \{s_2(0.66), s_3(0.34)\} & \{s_0(1)\} & \{s_0(0.28), s_1(0.43), s_2(0.28)\} \end{matrix} \right\} \\ \left\{ \begin{matrix} c_5 & c_6 & c_7 & c_8 \\ \{s_7(0.76), s_8(0.24)\} & \{s_2(0.34), s_3(0.66)\} & \{s_6(0.66), s_7(0.34)\} & \{s_3(0.76), s_4(0.24)\} \end{matrix} \right\} \\ \left\{ \begin{matrix} c_9 & c_{10} & c_{11} & c_{12} \\ \{s_6(0.59), s_7(0.41)\} & \{s_4(0.66), s_5(0.34)\} & \{s_7(0.13), s_8(0.87)\} & \{s_3(0.29), s_4(0.47), s_5(0.24)\} \end{matrix} \right\} \end{matrix} \right\};$$

$$\{h_S^{jw}(p^*)\}_{1 \times 12} = \left. \begin{matrix} \left\{ \begin{matrix} c_1 & c_2 & c_3 & c_4 \\ \{s_4(0.76), s_5(0.24)\} & \{s_5(0.24), s_6(0.76)\} & \{s_7(0.13), s_8(0.87)\} & \{s_6(0.24), s_7(0.76)\} \end{matrix} \right\} \\ \left\{ \begin{matrix} c_5 & c_6 & c_7 & c_8 \\ \{s_0(0.22), s_1(0.66), s_2(0.12)\} & \{s_5(0.42), s_6(0.58)\} & \{s_2(0.87), s_3(0.13)\} & \{s_4(0.42), s_5(0.58)\} \end{matrix} \right\} \\ \left\{ \begin{matrix} c_9 & c_{10} & c_{11} & c_{12} \\ \{s_1(0.59), s_2(0.41)\} & \{s_3(0.24), s_4(0.62), s_5(0.15)\} & \{s_0(1)\} & \{s_3(0.5), s_4(0.5)\} \end{matrix} \right\} \end{matrix} \right\}.$$

By Eqs. (13) and (14), the BO vector and the OW vector are obtained as:

$$A_{BO} = (4.47, 2.34, 0.00, 1.00, 7.24, 2.66, 6.34, 3.24, 6.41, 4.34, 7.87, 3.95);$$

$$A_{WO} = (4.24, 5.76, 7.87, 6.76, 0.90, 5.58, 2.13, 4.58, 1.41, 3.91, 0.00, 3.50).$$

Then, according to Model 1 in Section 3.1, the optimal weight vector can be obtained as:

$$w^* = (0.066, 0.102, 0.208, 0.131, 0.035, 0.096, 0.041, 0.108, 0.041, 0.069, 0.023, 0.08)^T.$$

4.2.2. Ranking the suppliers by the probabilistic aggregation-based CoCoSo method

Once the weights of criteria are determined, the probabilistic aggregation-based CoCoSo method is applied to evaluate the integral kitchen cabinet suppliers. The evaluations were carried out by the decision-making group with the predefined evaluation criteria in Table 2. The judgments of each member take the LTS $S_{(2)} = \{s_{-3} = \text{very bad}, s_{-2} = \text{bad}, s_{-1} = \text{somewhat bad}, s_0 = \text{normal}, s_1 = \text{somewhat$

$\text{good}, s_2 = \text{good}, s_3 = \text{very good}\}$. The HFL judgments for each expert are shown in Table 3.

Next, we aggregate all individuals' evaluations to group opinions expressed as PLTSs by Eqs. (8)-(12). The results are shown in Table 4.

Afterwards, based on the decision-making matrix, the expected values of the suppliers and the normalized supplier matrix can be obtained by Eqs. (15)-(17), shown in Tables 5 and 6. Then, two aggregation models are computed by Eqs. (18) and (19), and the weak ranking of the suppliers is acquired, listed in Tables 7 and 8.

The values of Z_{ia} , Z_{ib} and Z_{ic} are derived by Eqs. (20)-(22). Here, we suppose that the preference of the decision maker on the criteria compensation is neutral, that is to say, $\lambda = 0.5$. Finally, we calculate the values of Z_i by Eq. (23) for each supplier, which yields the comprehensive performance scores for suppliers. Based on these scores, we can determine the priorities of the suppliers, which are listed in Table 9.

Table 3. Individual evaluations of the suppliers

Alter.		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
A_1	e_1	$\{s_2, s_3\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}, s_0\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_{-1}, s_0\}$	$\{s_2, s_3\}$
	e_2	$\{s_1, s_2, s_3\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_0\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_2, s_3\}$
	e_3	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_2\}$
	e_4	$\{s_2, s_3\}$	$\{s_2\}$	$\{s_2\}$	$\{s_2\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_{-1}\}$	$\{s_{-1}\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_{-1}, s_0\}$	$\{s_2, s_3\}$
	e_5	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_2\}$
A_2	e_1	$\{s_0, s_1\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_{-2}, s_{-1}\}$	$\{s_1, s_2\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_2, s_3\}$
	e_2	$\{s_1\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2, s_3\}$
	e_3	$\{s_1\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_0, s_1\}$	$\{s_2\}$
	e_4	$\{s_0, s_1\}$	$\{s_2\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_{-2}, s_{-1}\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_2, s_3\}$
	e_5	$\{s_1\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_1, s_2\}$	$\{s_0, s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_0, s_1, s_2\}$	$\{s_1\}$	$\{s_1\}$	$\{s_2\}$
A_3	e_1	$\{s_1, s_2\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_1\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_1\}$
	e_2	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}, s_0\}$	$\{s_2\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_1\}$
	e_3	$\{s_2\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_0, s_1\}$	$\{s_0\}$	$\{s_1\}$	$\{s_0\}$	$\{s_{-1}, s_0\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$
	e_4	$\{s_2\}$	$\{s_{-1}\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_{-1}\}$	$\{s_0\}$	$\{s_2\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_1\}$
	e_5	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_1\}$	$\{s_1\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_{-1}, s_0\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_1\}$	$\{s_2\}$
A_4	e_1	$\{s_1, s_2\}$			$\{s_1\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_{-2}, s_{-1}\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_{-1}\}$	$\{s_1, s_2\}$
	e_2	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_1\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_{-2}, s_{-1}\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_{-1}, s_0\}$	$\{s_2\}$
	e_3	$\{s_2\}$	$\{s_2\}$	$\{s_{-1}, s_0\}$	$\{s_1\}$	$\{s_0\}$	$\{s_2\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_1, s_2\}$
	e_4	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_{-1}, s_0\}$	$\{s_2\}$	$\{s_{-2}\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_{-1}\}$	$\{s_1, s_2\}$
	e_5	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_0\}$	$\{s_1\}$	$\{s_0\}$	$\{s_2\}$	$\{s_{-2}, s_{-1}, s_0\}$	$\{s_2\}$	$\{s_0, s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_2\}$
A_5	e_1	$\{s_1, s_2\}$	$\{s_{-1}, s_0\}$	$\{s_0\}$	$\{s_1, s_2\}$	$\{s_{-2}, s_{-1}\}$	$\{s_1, s_2\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_2, s_3\}$
	e_2	$\{s_1\}$	$\{s_{-1}, s_0\}$	$\{s_0, s_1\}$	$\{s_2\}$	$\{s_{-1}\}$	$\{s_1, s_2\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_2\}$
	e_3	$\{s_1\}$	$\{s_{-1}, s_0\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_2, s_3\}$
	e_4	$\{s_1, s_2\}$	$\{s_0\}$	$\{s_0\}$	$\{s_2\}$	$\{s_{-2}, s_{-1}\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_1, s_2, s_3\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_2\}$
	e_5	$\{s_1\}$	$\{s_{-1}, s_0\}$	$\{s_1\}$	$\{s_1\}$	$\{s_{-1}\}$	$\{s_2\}$	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$

Table 4. The collective decision-making matrix

Alter.	c_1	c_2	c_3	c_4	c_5	c_6
A_1	$\{s_0(0.21), s_1(0.26), s_2(0.26), s_3(0.27)\}$	$\{s_1(0.34), s_2(0.66)\}$	$\{s_1(0.24), s_2(0.76)\}$	$\{s_1(0.24), s_2(0.76)\}$	$\{s_1(1)\}$	$\{s_{-1}(0.76), s_0(0.24)\}$
A_2	$\{s_0(0.34), s_1(0.66)\}$	$\{s_2(1)\}$	$\{s_0(0.58), s_1(0.42)\}$	$\{s_1(0.76), s_2(0.24)\}$	$\{s_{-2}(0.24), s_{-1}(0.76)\}$	$\{s_1(0.24), s_2(0.76)\}$
A_3	$\{s_1(0.42), s_2(0.58)\}$	$\{s_{-1}(0.5), s_0(0.5)\}$	$\{s_1(0.59), s_2(0.41)\}$	$\{s_0(0.15), s_1(0.54), s_2(0.3)\}$	$\{s_0(0.6), s_1(0.4)\}$	$\{s_1(0.5), s_2(0.5)\}$
A_4	$\{s_1(0.42), s_2(0.58)\}$	$\{s_1(0.24), s_2(0.76)\}$	$\{s_{-1}(0.14), s_0(0.71), s_1(0.14)\}$	$\{s_1(1)\}$	$\{s_{-1}(0.13), s_0(0.87)\}$	$\{s_1(0.24), s_2(0.76)\}$
A_5	$\{s_1(0.76), s_2(0.24)\}$	$\{s_{-1}(0.42), s_0(0.58)\}$	$\{s_0(0.59), s_1(0.41)\}$	$\{s_1(0.32), s_2(0.68)\}$	$\{s_{-2}(0.24), s_{-1}(0.76)\}$	$\{s_1(0.34), s_2(0.66)\}$
Alter.	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
A_1	$\{s_{-1}(0.5), s_0(0.5)\}$	$\{s_{-1}(0.87), s_0(0.13)\}$	$\{s_1(0.59), s_2(0.41)\}$	$\{s_1(0.34), s_2(0.66)\}$	$\{s_{-1}(0.34), s_0(0.66)\}$	$\{s_2(0.66), s_3(0.34)\}$
A_2	$\{s_0(0.51), s_1(0.49)\}$	$\{s_1(0.66), s_2(0.34)\}$	$\{s_0(0.12), s_1(0.43), s_2(0.44)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.13), s_1(0.87)\}$	$\{s_2(0.66), s_3(0.34)\}$
A_3	$\{s_{-1}(0.5), s_0(0.5)\}$	$\{s_{-1}(0.5), s_0(0.5)\}$	$\{s_1(0.34), s_2(0.66)\}$	$\{s_0(0.4), s_1(0.6)\}$	$\{s_0(0.42), s_1(0.58)\}$	$\{s_1(0.68), s_2(0.32)\}$
A_4	$\{s_{-2}(0.45), s_{-1}(0.44), s_0(0.12)\}$	$\{s_1(0.24), s_2(0.76)\}$	$\{s_0(0.12), s_1(0.5), s_2(0.38)\}$	$\{s_1(0.24), s_2(0.76)\}$	$\{s_{-1}(0.59), s_0(0.41)\}$	$\{s_1(0.34), s_2(0.66)\}$
A_5	$\{s_0(0.34), s_1(0.66)\}$	$\{s_1(0.34), s_2(0.66)\}$	$\{s_1(0.45), s_2(0.44), s_3(0.12)\}$	$\{s_1(0.5), s_2(0.5)\}$	$\{s_0(0.23), s_1(0.46), s_2(0.31)\}$	$\{s_2(0.76), s_3(0.24)\}$

Table 5. The expected values of the suppliers

Supplier	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
A_1	0.764	0.777	0.793	0.793	0.667	0.373	0.417	0.355	0.736	0.777	0.444	0.889
A_2	0.627	0.833	0.570	0.707	0.293	0.753	0.582	0.723	0.720	0.750	0.645	0.889
A_3	0.777	0.417	0.736	0.692	0.567	0.667	0.417	0.417	0.777	0.600	0.596	0.719
A_4	0.763	0.793	0.500	0.667	0.478	0.753	0.279	0.793	0.710	0.793	0.402	0.777
A_5	0.707	0.430	0.569	0.781	0.293	0.721	0.611	0.777	0.778	0.750	0.680	0.873

Table 6. The normalized decision matrix

Supplier	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
A_1	0.912	0.865	1.000	0.000	0.000	0.000	0.585	0.000	0.378	0.917	0.150	1.000
A_2	0.000	1.000	0.240	0.684	1.000	1.000	0.085	0.839	0.143	0.776	0.874	1.000
A_3	1.000	0.000	0.804	0.800	0.268	0.772	0.585	0.141	0.989	0.000	0.698	0.000
A_4	0.905	0.904	0.000	1.000	0.504	1.000	1.000	1.000	0.000	1.000	0.000	0.340
A_5	0.532	0.031	0.235	0.100	1.000	0.916	0.000	0.963	1.000	0.776	1.000	0.906

Table 7. The weighted average comparability sequence and the corresponding ranking results of the suppliers

Supplier	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	F_i	Ranking
A_1	0.060	0.088	0.208	0.000	0.000	0.000	0.024	0.000	0.015	0.063	0.003	0.080	0.543	3
A_2	0.000	0.102	0.050	0.090	0.035	0.096	0.003	0.091	0.006	0.054	0.020	0.080	0.626	2
A_3	0.066	0.000	0.167	0.105	0.009	0.074	0.024	0.015	0.041	0.000	0.016	0.000	0.517	5
A_4	0.060	0.092	0.000	0.131	0.018	0.096	0.041	0.108	0.000	0.069	0.000	0.027	0.642	1
A_5	0.035	0.003	0.049	0.013	0.035	0.088	0.000	0.104	0.041	0.054	0.023	0.072	0.517	4

Table 8. The weighted geometric comparability sequence and the corresponding ranking results of the suppliers

Supplier	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	P_i	Ranking
A_1	0.994	0.985	1.000	0.000	0.000	0.000	0.978	0.000	0.961	0.994	0.957	1.000	7.870	5
A_2	0.000	1.000	0.743	0.951	1.000	1.000	0.904	0.981	0.923	0.983	0.997	1.000	10.482	1
A_3	1.000	0.000	0.956	0.920	0.955	0.975	0.978	0.809	1.000	0.000	0.992	0.000	8.585	4
A_4	0.993	0.990	0.000	1.000	0.976	1.000	1.000	1.000	0.000	1.000	0.000	0.917	8.877	3
A_5	0.959	0.702	0.740	0.740	1.000	0.992	0.000	0.996	1.000	0.983	1.000	0.992	10.104	2

Table 9. Final ranking results of the suppliers

Supplier	Z_{ia}	Z_{ib}	Z_{ic}	Z_i	Final ranking
A_1	0.173	2.127	0.756	1.671	5
A_2	0.228	2.632	0.999	2.129	1
A_3	0.186	2.091	0.815	1.713	4
A_4	0.195	2.460	0.856	1.914	3
A_5	0.218	2.358	0.955	1.966	2

Table 10. Comparative analysis with the PL-VIKOR method

Supplier	CoCoSo (New probability aggregation method)		CoCoSo (Ref. probability aggregation method)		VIKOR (New probability aggregation method)		VIKOR (Ref. probability aggregation method)	
	Z_i	Final ranking	Z_i	Final ranking	R_i	Final ranking	R_i	Final ranking
A_1	1.671	5	1.679	5	0.446	2	0.384	3
A_2	2.129	1	2.139	1	0.314	1	0.280	1
A_3	1.713	4	1.763	4	0.500	3	0.369	2
A_4	1.914	3	1.902	3	0.500	3	0.505	4
A_5	1.966	2	1.925	2	0.658	5	0.780	5

4.3. Results and discussions

From Tables 7 and 8, we can find that if the poor performances of a component supplier under some criteria can be completely compensated by the good performances of the supplier under other criteria, the weak ranking of the suppliers is $A_4 > A_2 > A_1 > A_5 > A_3$; while if the good performances of an supplier under some criteria cannot fully compensate for the poor performances under other criteria, we get another weak ranking of the suppliers as $A_2 > A_5 > A_4 > A_3 > A_1$. When the compensation among the criteria is considered neutral, according to the aggregation strategies of the CoCoSo method, we obtain the strong ranking of the component suppliers as $A_2 > A_5 > A_4 > A_3 > A_1$. Thus, we recommend supplier A_2 as the optimum candidate.

To verify the effectiveness and robustness of the proposed hybrid multi-expert MCDM model, a comparative analysis is conducted. Since we have already analyzed the advantages of our new probability aggregation approach compared with the probability aggregation method proposed by Wu and Liao (2018) in Section 2.2, we will not repeat here but directly conduct a comparative analysis by applying another classical utility value-based ranking method, PL-VIKOR, in this part. The results are listed in Table 10. From the yielded results, we can see that, although the comprehensive scores of two probability aggregation methods are not the same, the final ranking results of the suppliers are the same. However, there are relatively large fluctuations for the ranking results of the VIKOR-based model, which can show the robustness of our proposed hybrid model.

Conclusions

The whole-decoration residential products success of property developers depends on the reliability of a supply network and trustable suppliers. A suitable supplier choosing not only can guarantee the quality of a residential product, but also can reduce its developmental cost and decrease the housing price accordingly. It is beneficial to both property developers and buyers. This paper studied a new multi-expert MCDM method for property developers to select the most suitable construction component supplier in the process of promoting their residential whole-

decoration strategy. Considering experts are not always knowledgeable enough to present information about the criteria in terms of precise values in a qualitative evaluation process, the HFLTS was taken to express the preferences of experts since it is closer to human cognition and perceptions. In addition, to enable a realistic presentation of the collective evaluations of a group, on the basis of interval rough boundaries derived by the rough set theory, we introduced a novel probabilistic aggregation approach. Then, the proposed novel probabilistic aggregation approach was applied to the MCDM problem for property developers to construct a hybrid MCDM model. The novel probabilistic aggregation-based BWM was used to derive the weights of criteria while the novel probabilistic aggregation-based CoCoSo method was developed to find the optimal supplier. The specific operation steps of the multi-expert MCDM model were given.

Considering the effectiveness and practicability of the proposed hybrid MCDM model, our model can also be applied to other group decision-making areas to deal with the uncertainty and vagueness in the decision-making process. In the proposed new probabilistic aggregation approach, we used the geometric average operator to integrate the defined accuracy degree to the original aggregated probability. Although the diversity of experts' perceptions were presented, the small probabilities of the PLTS in the aggregated judgment was increased, which seems with some defects. In the future, we shall research a more suitable operator for integration.

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Author contributions

Zhiying ZHANG and Huchang LIAO proposed the original idea and conceived the study. Zhiying Zhang and Huchang LIAO were responsible for developing the method, collecting and analyzing the data. Abdullah AL-BARAKATI, Edmundas Kazimieras ZAVADSKAS and Jurgita ANTUCHEVIČIENĖ were responsible for data

interpretation. Zhiying Zhang and Huchang LIAO wrote the first draft of the article. Abdullah AL-BARAKATI, Edmundas Kazimieras ZAVADSKAS and Jurgita ANTUCHEVIČIENĖ revised the paper.

Disclosure statement

The authors have no competing financial, professional, or personal interests from other parties that are related to this paper.

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