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A DYNAMIC PERFORMANCE-BASED PAYMENT MECHANISM FOR PUBLIC-PRIVATE PARTNERSHIP PROJECTS: AN INTEGRATED MODEL FOR PRINCIPAL-AGENT AND MULTI-OBJECTIVE OPTIMIZATION MODELS

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Abstract. Performance-based payment mechanism is one of the key issues to ensure all stakeholders' benefits in infrastructure Public-Private Partnership (PPP) projects. However, most existing research on performance-based payment with a fixed incentive coefficient can't play a good incentive role. This study aims to the intrinsic mechanism between the performance appraisal score and the performance-based payment structure, so as to design the dynamic performance-based payment mechanism for infrastructure PPP projects. Firstly, the multi-objective optimal method is used to calculate the unit-payment. Second, principal-agent theory is used to construct the performance-based payment model with a changeable incentive coefficient. The findings of this study show that, the performance-based payment mechanism can effectively motivate participants to provide high-quality and efficient services, because their remuneration directly depends on their performance. When the outcome does not meet expectations, the amount paid can be adjusted accordingly, thus ensuring the maximum protection of public resources as well as the private sector's profits. They serve a dual purpose, on one hand, they offer insights to rectify the shortcomings in the current unsatisfactory payment structure. On the other hand, the study provides a theoretical reference for the public sector to effectively incentivize the private sector in enhancing project performance.

Keywords: multiple-objective programming, payment structure, performance-based payment, PPP, principal-agent theory.

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1. Introduction

In recent decades, the externalization of infrastructure and public services by the government has significantly increased with the rapid development of Public-Private Partnership (PPP) projects (Grout, 1997; Hodge & Greve, 2007; Leviäkangas et al., 2018; Narbaev, 2022). Within PPP projects, the public sector enters agency agreements with the private sector to procure infrastructure and public services over several years or even decades, contingent upon the fulfillment of specific performance conditions by the private sector (Grout, 1997). The payment mechanism stands as the central provision for government payment based on the private sector's performance level. Simultaneously, it plays a crucial role in attracting private sector participation in projects and ensuring profitability (Li et al., 2022). More specifically, the public sector remunerates for services (Yescombe, 2007), typically spanning 20–35 years from the private finance initiative (PFI) contract. Payments from the public sector, aligned with the contractual provisions, offer financial security to the private sector (European PPP Expertise Centre, 2012). Consequently, an effective payment mechanism is indispensable for both the public and private sectors to ensure the success of the contract.

While increased attention has been devoted to the design of payment mechanisms, the predominant approach comprises a combination of fixed and performance payments (Sharma, 2012; Shi et al., 2020; Su et al., 2023, 2024). The fixed payment remains constant over a brief period, independent of performance, whereas the performance payment is contingent upon the level of provided public services (Shi et al., 2020; Shang & Aziz, 2020). In essence, the prevailing payment structure in most infrastructure PPP projects integrates fixed payments with performance payments (Shen et al., 2014), typically characterized by a fluctuating incentive or subsidy coefficient (Li et al., 2022; Su et al., 2023). Given the profit-driven nature of the private sector, a performance payment structure featuring an unalterable incentive or subsidy coefficient may not effectively enhance performance. In cases where the private sector receives limited income despite increased efforts in project operation and maintenance (Li et al., 2020, 2022), a fixed incentive or subsidy coefficient could hinder performance improvement.

The general payment curve of the Australian Defense Contract standard (ASDEFCON) series contract template (Jacopino, 2018) is instrumental in addressing performance payment with the same incentive or subsidy coefficient. This framework encompasses three primary types of payment curves: all or none payment curves, linear payment curves, and non-linear payment curves. Each type possesses distinctive features. The first type, all or none payment curves, introduces varying degrees of risk for all parties engaged in PPP projects. In this context, the private sector is entitled to receive will get all of the entire performance payment once it attains the performance level stipulated by the contract. Conversely, the basic cost becomes irrecoverable should the private sector fail to meet the contract requirements. This circumstance prompts the private sector to potentially prioritize immediate performance over the long-term benefits of the projects (Su et al., 2024). Consequently, the risk to the public sector is elevated.

In comparison, the linear payment curves determine varying performance payments based on the different performance level (Shi et al., 2020; Shang & Aziz, 2020). While this approach mitigates risks to the public sector, it exhibits its shortcomings as the linear payment curves lack tailored specifically to different performance levels. Consequently, the different payments to the private sector become challenging, and the unchangeable incentive or subsidy coefficient within such curves fails to effectively stimulate the private sector towards achieving higher performance level. Consequently, the type of non-linear payment curves is very effective to overcome the insufficiencies of the other two types payment curves (Feng et al., 2023). With the non-linear payment curves, when the performance level is higher, the private sector will have to expend more efforts to reach it, and the public sector will thus have to compensate them with a higher payment (Jacopino, 2018). The risks to both parties are therefore minimized.

From the existing literatures, research has seldom concentrated on the performance-based payment structure involving a changeable incentive or subsidy coefficient, resulting in two consequences: (1) Excessive incentives may occur in the initial stage of operation and maintenance. Specifically, the private sector can obtain high profits with a relatively low performance appraisal score; (2) Incentive deficiency may manifest over the extended concession period (Li et al., 2022). In other word, the private may not obtain effective incentives throughout the entire lifecycle of the projects. Therefore, this study aims to enhance the performance-based payment structure based on the aforementioned non-linear payment curves. The payment structure in this study comprises two components: fixed payment and performance payment, with the latter characterized by non-linear payment curves.

The objective of this study is to answer the following research questions: (1) How to characterize the different payment with varying performance appraisal scores for the private sector? and (2) How to design a dynamic performance-based payment mechanism with a changeable performance payment for infrastructure PPP projects? To tackle the first question, a new term, unit-payment, is introduced, with its detailed description in this study referring to the government payment for each performance appraisal score obtained by private sectors at different performance levels.

Specifically, the primary innovations of this study consists of two points in the performance-based payment mechanism.

(1) A stepped-payment structure with distinct payment curves is developed, in which the payment from the public sector varies with the performance appraisal score of the private sector across different performance levels. This payment structure not only facilitates the equitable distribution of benefits among all parties, but also enables the fair allocation of risks in PPP projects.

(2) An optimal unit-payment model is formulated to characterize performance-based payments. This model serves as a theoretical reference for the public sector, providing guidance to encourage the private sector to actively enhance their performance level.

The remainder of this study is organized as follows: Section 2 presents a comprehensive review, encompassing research on payment mechanisms and the performance-based payment structure for PPP projects. Section 3 outlines the methodology for constructing the performance-based payment model. Section 4 presents a numerical simulation analysis of the model results, with discussions and implications presented in Section 5. Finally, Section 6 offers the conclusions of this study.

2. Literature reviews

Infrastructure has emerged as a significant factor in gauging national economic progress and social benefits, often financed through government subsidies, private investment, and PPP. Nevertheless, PPP have generated considerable public attentions due to their reduced profitability and intricate operational requirements (Ma et al., 2023), and suitable payment models make up for this very well (Shang & Aziz, 2020).

Availability payment projects are typical in PPP projects, and the payment depends on the assets or service availability of infrastructure projects throughout the concession period (Giglio & Friar, 2017; Sharma & Cui, 2012). Strategic management is crucial for significant infrastructure PPP projects, characterized by heavy and irreversible investments over long periods (Zhang et al., 2020). In instances where infrastructure or services are inaccessible or fail to meet stipulated output criteria in terms of unavailable or their quality, safety and service levels, the government reserves the right to reduce the payment paid to the private sector. And the payment types for availability payment will reflect the government's goal of adopting the PPP mode (Aziz & Abdelhalim, 2017).

Recently, many scholars have paid more attention to the research on payment mechanisms. Wu et al. (2013) constructed a game model between the public and private sectors, formulating an optimal compensation mechanism by incorporating the concepts of incentive and the induction mechanism of Stackelberg game. Employing multi-cases comparison approach, Aziz and Abdelhalim (2017) examined payment mechanism structures through an analysis of the implementation of availability payment projects on expressways in the United States and Canada. These encompassed maximum availability payments, performance deductions, and other (incentive) payments. Dai et al. (2018) proposed a principal-agent model that considered pollution tax and the incentive mechanism, which can be used to resolve the conflict of interest between the government and enterprises in water pollution control projects. This study demonstrated that incentive policies and pollution taxes positively influence enterprises in reducing water pollution, with an illustrative example validating the effectiveness of the model. Employing the traditional principal-agent model, Wang et al. (2018) established an optimal incentive mechanism for infrastructure projects. This involved integrating reciprocal preference theory and analyzing risk sharing. This showed that the government can use reciprocal preference to encourage investors to make more effort, mitigating moral hazards in the collaborative process. Drawing on agency theory, Soliño and Albornoz (2021) developed a payment mechanism for transport PPP projects, which combined fixed payment to the contractor, service-based payment, and payment according to the number of users, offering a comprehensive approach to aligning incentives and compensations in the context of transport PPP projects. Su et al. (2024) constructed the multi-stage dynamic programming model to design the payment model for the public sector in whole operation and maintenance period. Furthermore, through constructing an optimal proportion of the performance-based payment in the total payment, Su et al. (2023) designed a performance-based payment structure for infrastructure PPP project.

Summarizing the existing research, prevailing payment structures exhibit a predominantly hybrid nature, mostly comprising multiple components. Also, the majority of existing studies has concentrated on how to guarantee the fundamental interests of both public and private sectors. Essentially, payment mechanisms are designed to incentivize the private sector to maximize the social benefits of a project, concurrently enabling it to attain its anticipated income and judiciously distribute risks (Ding & Li, 2020). Furthermore, they seek to curb speculative activities within the private sector while concurrently enhancing the performance level of projects (An et al., 2018; Liu et al., 2016).

The majority of performance-based payment structures typically comprise a fixed payment and a performance payment (Liu et al., 2016; Luo et al., 2016). The fixed payment remains constant in the short term, while the performance payment is subject to adjustments in the long term. Modifications to the performance payment are often due to changes in the incentive or subsidy coefficients.

For a constant incentive or subsidy coefficient throughout the entire concession period of infrastructure PPP projects, the performance payment may not effectively incentivize the private sector or serve the needs of the public sector. As mentioned above, according to the Australian Defense Contract standard (ASDEFCON) series of contract templates used by the Australian Department of defense (Jacopino, 2018), three types of performance-based payment curves are identified: all or none payment curve, linear payment curve, and non-linear payment curve, as illustrated in Figures 1–3. Each payment curve exhibits unique characteristics. Specifically, the all or none payment curve entails the entirety of the performance payment being either awarded or deducted based on the stipulated performance requirements in the contract. In the case of the linear payment curve, the performance payment from the government increases or decreases linearly. Using the principal agent theory, Li et al. (2022) established a multi-period dynamic incentive mechanism, in which the performance payment is linear and varies with different incentive coefficients during different performance appraisal periods. Additionally, Lv et al. (2022) proposed a credit-based demand side incentive mechanism, incorporating identical coefficients for load reduction-related and credit-related prices in the compensation structure, respectively.

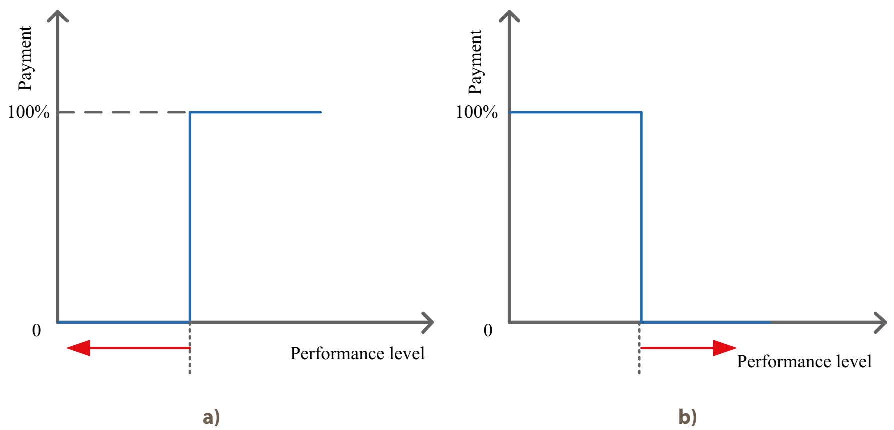
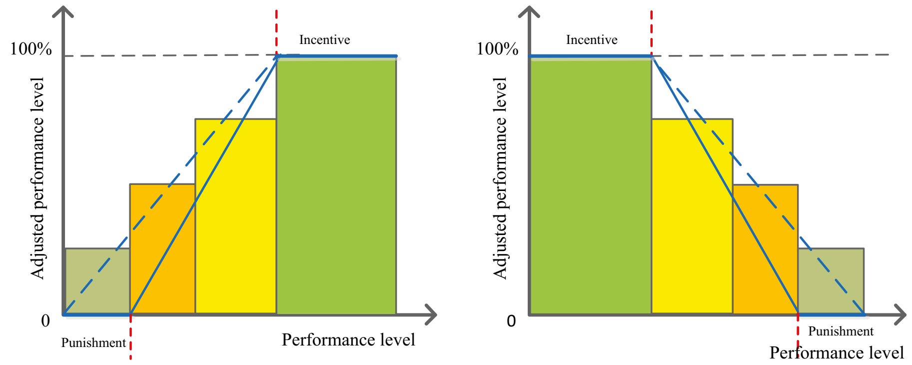
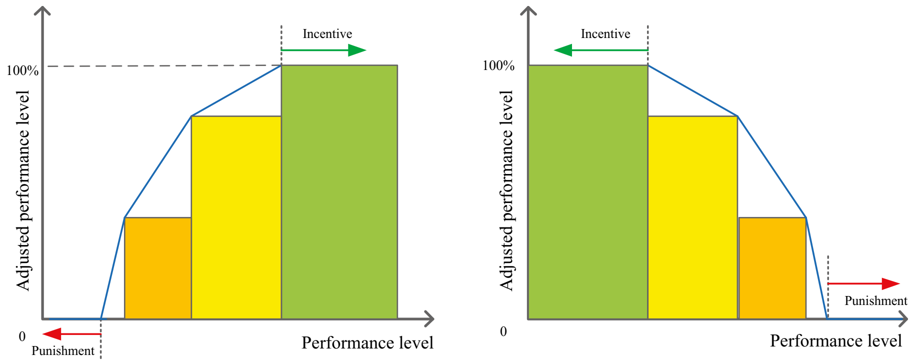


Figure 1. All or none payment curves



a) b)

Figure 2. Linear payment curves



a) b)

Figure 3. Non-linear payment curves

In recent literature on PPP projects, the predominant focus of existing studies has centered on performance payments throughout the entire operation and maintenance period. Nevertheless, different performance appraisal scores are essential to evaluate distinct levels of effort exerted by the private sector within the same operation and maintenance period. Unfortunately, there has been a notable neglect of the dynamic adjustments of contracts during the concession period for infrastructure PPP projects.

3. Methodology

The essence of PPP projects lies in harnessing advanced management and governance strategies from the private sector through collaborative efforts with the public sector. This collaboration aims to deliver high-quality products or efficient services to the public sector, thereby maximize social benefits. The private sector, in turn, can receive profit from government payment. In the context of infrastructure PPP projects, the main force driving the private sector to participate in projects is the prospect of obtaining a relatively stable profit through the establishment of a long-term cooperative relationship with the public sector. Also, a central concern in this context is the design of the payment mechanism. This section elaborates on the development of a rational payment mechanism to enhance the payment contract. And the general theoretical framework is illustrated in Figure 4.

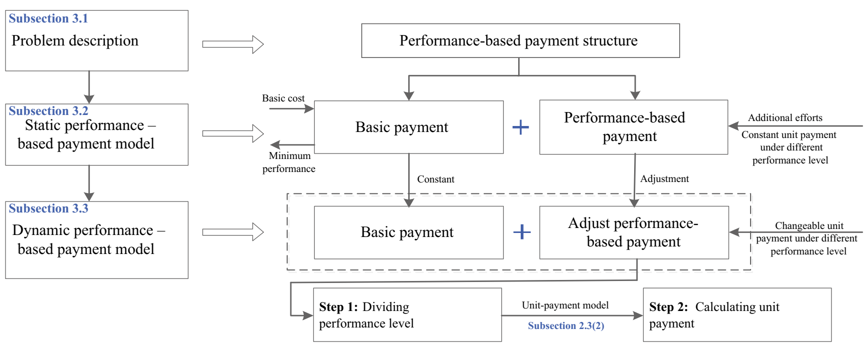


Figure 4. The general theoretical framework

3.1. Problem description for the design of the payment mechanism

Throughout the entire concession period of infrastructure PPP projects, the payment structure includes both the basic payment and the performance-based payment at each performance appraisal stage. Specifically, the basic payment is the payment given by the government to offset the private sector's basic cost, ensuring the minimum performance level in each performance appraisal stage. Conversely, the performance-based payment from the government is contingent upon the performance appraisal score and the performance grade level in the corresponding performance appraisal stage.

In general, there exist five performance grade levels: minimum low, low, medium, good, and excellent, as depicted in Figure 5. The horizontal axis represents the performance appraisal score of the private sector, and the vertical axis represents the total government payment.

In Figure 5, the performance appraisal score within the interval $(Q_1, Q_2]$ corresponds to the low performance level, and the payment from the government is represented as the difference between M_1 and M_0, i.e. $M_1 - M_0$. Here, the low performance level indicates that the performance appraisal score reaches Q_1, and its corresponding basic payment from government is M_0. The intervals $(Q_2, Q_3]$, $(Q_3, Q_4]$ and $(Q_4, Q_5]$ are the medium, good, and excellent performance levels, respectively, and their corresponding payments are $M_2 - M_1$, $M_3 - M_2$, and $M_4 - M_3$. In this way, the stepped-payment structure is formed. Furthermore, within different performance levels, the government payment varies in accordance with the appraisal score according to the different curves. In essence, the performance appraisal score is categorized into different intervals, i.e. the different performance levels, and payment differs by performance level, since the unit-payment following different curves varies by different performance level.

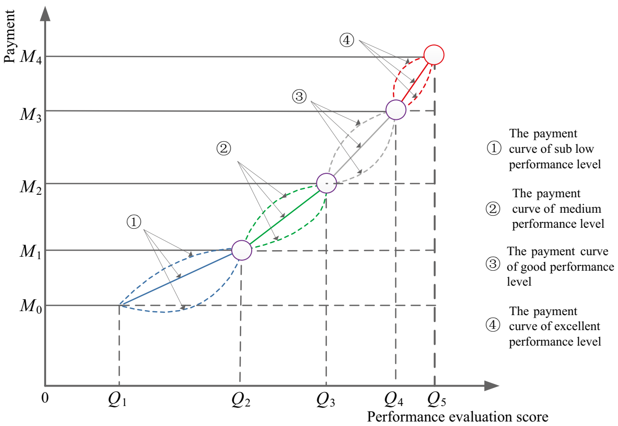


Figure 5. The performance-based payment curves for infrastructure PPP projects

3.2. Construction of the static performance-based payment model

In infrastructure PPP projects, the government commissions the private sector to carry out the operation and maintenance of projects, and the private sector can obtain profits by diligently executing and enhancing project performance throughout the operation and maintenance period. However, the government faces a challenge as it cannot directly observe or control the actions of the private sector throughout the entire operation and maintenance process (Su et al., 2023, 2024). The government can only measure the efforts made by the private sector through observable variables, such as the performance appraisal score. In other words, this is a classic problem of information asymmetry. Also, opportunism or moral hazards may exist for the private sector to gain more profit. The principal-agent theory, renowned for its efficacy in addressing information asymmetry, includes participation and incentive compatibility constraints. The former is that the expected utility of the private sector entering into the contract surpasses the maximum expected utility achievable if the private sector doesn't sign the contract. While the latter asserts that the expected utility derived from the private sector's chosen action exceeds that obtained when any alternative action is chosen. In essence, this implies that the expected utility associated with the private sector choosing a certain action is surpasses the expected utility obtained from any other actions chosen by the private sector.

(1) Model assumptions

Before giving model assumptions, the symbol interpretation is as shown in Table 1.

Table 1. Basic symbols and interpretation

No.	Symbol	Interpretation of symbols
1	q	The unit-payment
2	x	The performance appraise score
3	θ_n	The input level of private sector
4	e_n	The effort level of private sector
5	γ	The output coefficient of social benefit
6	μ_n	The exogenous random variable
7	ρ	The absolute risk aversion coefficient of private sector
8	σ_n^2	The variance of the payment paid to private sector
9	c	The cost coefficient from private sector
10	α_n	The basic payment
11	β_n	The payment coefficient

According to Holmstrom and Milgrom's principal agent model (Holmstrom & Milgrom, 1991), some assumptions for developing the payment mechanism are given as follows:

Assumption 1: Assume that the performance output in the nth performance appraisal period for infrastructure PPP projects is as follows (Li et al., 2022):

$$\pi_n = \gamma x_n + \mu_n, \tag{1}$$

where $x_n = \theta_n e_n$ is the performance appraisal score corresponding to the nth performance appraisal period; θ_n and e_n are the input and effort level of the private sector, respectively; γ is the output coefficient of the social benefit; $\mu_n \sim N\left(0, \sigma^2\right)$ indicates the exogenous random variable.

Assumption 2: Let the principal (the government) be risk-neutral and the agent (the private sector) be risk-averse, then the risk cost of the private sector in the nth performance appraisal period is (Li et al., 2022):

$$RC = \rho\sigma_n^2/2, \tag{2}$$

where $\rho(\rho > 0)$ is the absolute risk aversion coefficient of the private sector; σ_n^2 is the variance of government payment paid to the private sector.

Assumption 3: Let e_n and θ_n be the effort level and input of the private sector in the nth performance appraisal period, respectively. Then, the cost function of the private sector (Gill & Stone, 2010) is:

$$C\left(e_n, \theta_n\right) = c\left(e_n^2 + \theta_n^2\right)/2, \tag{3}$$

where c is the cost coefficient from the private sector.

(2) Model development with the multi-stage game approach

Assume that α_n denotes the basic payment when the performance score of the private sector is $x_0^n = \theta_0^n e_0^n$, which means that the private sector has attained the minimum performance appraisal score according to the performance standard. If $q_n \in [0,1]$ is the unit-payment set by the government according to the performance output of the projects, then the payment contract S_n in the nth performance appraisal period is linear as follows (Gibbons, 1992):

$$S_n = \alpha_n + q_n\left(x_n - x_0^n\right), \tag{4}$$

where $x_n = \theta_n e_n$ is the performance score of the private sector in the nth performance appraisal period.

Therefore, the expected utility functions of the government and private sector in the nth performance appraisal period are:

$$\Phi_G = E\left(\pi_n - S_n\right) = E\left(\gamma x_n + \mu_n - \alpha_n - q_n\left(x_n - x_0^n\right)\right) =$$

$$\gamma x_n - \alpha_n - q_n\left(x_n - x_0^n\right) = \gamma\theta_n e^n - \alpha_n - q_n\left(\theta_n e^n - \theta_0^n e_0^n\right) \tag{5}$$

and

$$\Phi_P = E\left(S_n - C_n\right) = E\left(\alpha_n + q_n\left(x_n - x_0^n\right) - \rho\sigma_n^2/2 - ce_n^2/2\right) =$$

$$\alpha_n + q_n\left(x_n - x_0^n\right) - \rho\sigma_n^2/2 - ce_n^2/2 =$$

$$\alpha_n + q_n\left(\theta_n e_n - \theta_0^n e_0^n\right) - \rho\sigma_n^2/2 - ce_n^2/2. \tag{6}$$

From Equations (5) and (6), the maximum expected utility function of the government is obtained by determining the basic payment and the unit-payment in each performance level. The expected utility function of the private sector can be maximized by choosing the appropriate effort e_n. Thus, the payment contract should simultaneously satisfy the maximum utility functions of the government and the private sector. Meanwhile, according to the principal agent theory, the incentive compatibility and participation constraints should also be satisfied. Thus, the payment contract can be described as follows:

$$\max_{\beta_n}\left\{\gamma\theta_n e^n - \alpha_n - q_n\left(\theta_n e^n - \theta_0^n e_0^n\right)\right\}, \tag{7}$$

$$\begin{cases} (IR) \ \ \alpha_n + q_n\left(\theta_n e_n - \theta_0^n e_0^n\right) - \rho\sigma_n^2/2 - ce_n^2/2 \ge \Phi_0 \\ (IC) \ \ \max\limits_{e_n}\left\{\alpha_n + q_n\left(\theta_n e_n - \theta_0^n e_0^n\right) - \rho\sigma_n^2/2 - ce_n^2/2\right\} \end{cases}, \tag{8}$$

where Φ_0 is the reservation utility.

3.3. The dynamic performance-based payment model

The traditional payment model for infrastructure PPP projects includes a basic payment and a performance-based payment. In this framework, the performance-based payment is paid according to the performance appraisal score, utilizing the same unit-payment. However, it is imperative to garner different performance appraisal scores obtained by the private sector within different performance levels are needed to ensure that different levels of effort are compensated appropriately. Employing the same unit-payment for all appraisal scores within a performance level fails to provide sufficient motivation for the private sector to enhance their performance further. Therefore, a stepped-payment structure, incorporating different unit-payments within different performance levels, is more suited to addressing practical problems.

Therefore, to realize the objective of this study, this section will first construct the unit-payment model.

(1) The unit-payment model for the dynamic performance-based payment model

As analyzed previously, the optimal unit-payment should first be solved. To do so, the unit-payment model will be constructed using the multi-objective optimization method.

The performance appraisal score will be divided into M different performance levels based on performance appraisal standards. If it is assumed that the performance appraisal score of the private sector is $x \in [x_j, x_{j+1}]$, $j = 0,1,2, \ldots, M-1$, and its probability density function in the performance level interval $[x_j, x_{j+1}]$ is $f(x)$, then the expected utility function of the government and the income function of the private sector can be expressed as follows:

$$U = \int_{x_j}^{x} f\left(x\right) q_j \mathrm{d}x \tag{9}$$

and

$$\Phi = \left(x - x_j\right) q_j - \int_{x_j}^{x} C\left(x\right) \mathrm{d}x, \tag{10}$$

where $C(x)$ represents the corresponding cost curve of the performance evaluation score x; q_j is the unit-payment determined by the government in the $j + 1$th performance level $[x_j, x_{j+1}]$.

In the operation and maintenance process of infrastructure PPP projects, the government aims to maximize the performance appraisal score of the private sector. Conversely, the private sector seeks to obtain greater benefits with minimal effort cost. Therefore, the unit-payment should satisfy the public and private sectors' objectives, simultaneously. If Equations (9) and (10) are considered as the objectives of the government and the private sector, respectively, and the weights of them are w_1 and w_2, where w_1, $w_2 \in [0,1]$ and $w_1 + w_2 \in [0,1]$, it can be seen as a multi-objective optimal problem. And the objective function is:

$$\max_{\left(x, q_j\right)} \left\{ w_1 \int_{x_j}^{x} f\left(x\right) q_j \mathrm{d}x + w_2 \left(\left(x - x_j\right) q_j - \int_{x_j}^{x} C\left(x\right) \mathrm{d}x \right) \right\}, \tag{11}$$

where $q_j \geq 0$, $x_j \in (0,100]$.

It is assumed that the performance appraisal score of the private sector in performance level $[x_j, x_{j+1}]$ is uniformly distributed, and the probability is $p \in [0,1]$. Additionally, the cost curve is assumed to be the parabolic curve $C\left(x\right) = \bar{u}x^2$, where $\bar{u}$ is the corresponding coefficient in this curve.

Since the cost curve is supposed to be the parabolic curve $C\left(x\right) = \bar{u}x^2$, the objective function is obtained through substituting it into Equation (11):

$$\max_{\left(x, q_j\right)} \mathrm{F}_1 = \max_{\left(x, q_1\right)} \left\{ w_1 \left(\int_{x_j}^{x} p x q_j \mathrm{d}x \right) + w_2 \left(\left(x - x_j\right) q_j - \int_{x_j}^{x} \bar{u} x^2 \mathrm{d}x \right) \right\}. \tag{12}$$

To find the optimal unit-payment, the first-order derivatives of Equation (12) with respect to unit-payment q_j and performance appraisal score x are set to zero, leading to the following equations:

$$\frac{\partial \mathrm{F}_1}{\partial x} = q_j w_2 - w_2 \bar{u} x^2 + w_1 p q_j x = 0$$

and

$$\frac{\partial \mathrm{F}_1}{\partial q_j} = \left(x - x_j\right) w_2 + \frac{1}{2} w_1 p x^2 - \frac{1}{2} w_1 p x_j^2 = 0,$$

thus

$$x = \frac{-w_2 + \sqrt{w_2^2 + w_1 p \left(2 w_2 x_j + w_1 p x_j^2\right)}}{w_1 p}, \tag{13}$$

$$q_j = \frac{2 w_2^3 \bar{u} + 2 w_1 w_2^2 x_j p \bar{u} + w_1^2 w_2 p^2 x_j^2 \bar{u} - 2 w_2^2 \bar{u} \sqrt{w_2^2 + w_1 p \left(2 w_2 x_j + w_1 p x_j^2\right)}}{\sqrt{w_2^2 + w_1 p \left(2 w_2 x_j + w_1 p x_j^2\right)}}. \tag{14}$$

From Equation (14), the unit-payment is primarily associated with the private sector's cost curve, the performance appraisal score, and the performance level covering this performance appraisal score. The conclusions for them are as follows:

1) According to $\partial q_j / \partial p > 0$, there exist positive correlations between the unit-payment q_j and the probability of performance level $[x_j, x_{j+1}]$ covering the performance appraisal score x. Essentially, the higher the probability p is, the greater the possibility of the performance appraisal score falling within the interval $[x_j, x_{j+1}]$ is. This implies that the government can set a relatively large unit-payment, since the risks taken by both the government and the private sector will be small. Conversely, a smaller probability p indicates increased risks for both parties, necessitating a correspondingly smaller unit-payment from the government.

2) According to $\partial q_j / \partial x_j > 0$, there are positive correlations between unit-payment q_j and lower bound x_j of performance level $[x_j, x_{j+1}]$ covering performance appraisal score x. The unit-payment q_j goes up with a relatively lower bound x_j. Obviously, the higher the lower bound x_j is, the greater the cost will be to the private sector to improve their performance appraisal score is. Only the government can set a relatively large unit-payment, the private sector can only make a greater efforts to improve performance. In turn, the smaller the lower bound x_j is, the lower the cost will be to the private sector to improve their performance, so only a relatively low unit-payment is needed from the government.

3) According to $\partial q_j / \partial \bar{u} > 0$, there are positive correlations between the unit-payment q_1 and the coefficient $\bar{u}$ of the cost curve $C\left(x\right) = \bar{u}x^2$. That is, the larger the coefficient $\bar{u}$ is, the greater the unit-payment q_1 is. This shows that a large coefficient $\bar{u}$ means that the cost curve $C\left(x\right) = \bar{u}x^2$ is steeper.

Under this circumstance, a relatively large unit-payment set by the government will not only enhance social benefits, but also enable the private sector to obtain reasonable profits. Conversely, a smaller the coefficient of the cost curve implies a smoother cost curve, indicating relatively lower costs for the private sector. Consequently, the government should set a relatively low unit-payment.

(2) Construction of the dynamic performance-based payment model

Based on the assumptions in Subsection 3.2, another version of the performance output is presented in terms of the performance appraisal score. If q_j^n is the unit-payment in the jth performance level interval $[x_j, x_{j+1}]$ for the nth performance appraisal period, then the payment contract from Equation (4) in the nth performance appraisal period for infrastructure PPP projects is expressed as follows:

$$S_n = \alpha_n + \sum_{j=0}^{l'-1} q_j^n \left(x_{j+1}^n - x_j^n\right), \tag{15}$$

where the α_n denotes the basic payment when the private sector has the minimum performance appraisal score meeting the performance standard in the nth performance appraisal period, and the performance appraisal score is $x^n = x_1^n + \left(x_2^n - x_1^n\right) + \cdots + \left(x_{l'+1}^n - x_{l'}^n\right)$. For example, if the performance appraisal score of the private sector is x^n = 80, and the minimum performance appraisal score for the performance standard in the nth performance appraisal period is $x_0^n = 60$, then the performance appraisal score is denoted as 80 = 60 + (70 – 60) + (80 – 70). Here each performance level is contained ten possible scores. The performance appraisal score can also denoted as 80 = 60 + (80 – 60) with twenty possible scores in a performance level.

Therefore, the expected utility functions of the government and the private sector in the nth performance appraisal period are:

$$\Psi_G = E\left(\pi_n - S_n\right) = E\left(\gamma x_n + \mu_n - \alpha_n - \sum_{j=1}^{l'} q_j^n \left(x_j^n - x_{j-1}^n\right)\right) =$$

$$\gamma \sum_{j=1}^{l'} \theta_j^n e_j^n - \alpha_n - \sum_{j=1}^{l'} q_j^n \left(\theta_j^n e_j^n - \theta_{j-1}^n e_{j-1}^n\right) \quad (16)$$

and

$$\Psi_P = E\left(S_n - C_n\right) =$$

$$E\left(\alpha_n + \sum_{j=1}^{l'} q_j^n \left(x_j^n - x_{j-1}^n\right) - \rho\sigma_n^2/2 - \sum_{j=1}^{l'} c\left\{\left(e_j^n\right)^2 + \left(\theta_j^n\right)^2\right\}\Big/2\right) =$$

$$\alpha_n + \sum_{j=1}^{l'} q_j^n \left(x_j^n - x_{j-1}^n\right) - \rho\sigma_n^2/2 - \sum_{j=1}^{l'} c\left\{\left(e_j^n\right)^2 + \left(\theta_j^n\right)^2\right\}\Big/2 =$$

$$\alpha_n + \sum_{j=1}^{l'} q_j^n \left(e_j^n \theta_j^n - e_{j-1}^n \theta_{j-1}^n\right) - \rho\sigma_n^2/2 - \sum_{j=1}^{l'} c\left\{\left(e_j^n\right)^2 + \left(\theta_j^n\right)^2\right\}\Big/2. \quad (17)$$

From Equations (16) and (17), and through the analogous analysis, the dynamic payment model for the government to the private sector can be expressed as follows:

$$\max_{q_j^n} \left\{\gamma \sum_{j=1}^{l'} \theta_j^n e_j^n - \alpha_n - \sum_{j=1}^{l'} q_j^n \left(\theta_j^n e_j^n - \theta_{j-1}^n e_{j-1}^n\right)\right\}, \quad (18)$$

$$\begin{cases} (IR)\ \alpha_n + \sum_{j=1}^{l'} q_j^n \left(e_j^n \theta_j^n - e_{j-1}^n \theta_{j-1}^n\right) - \rho\sigma_n^2/2 - \\ \sum_{j=1}^{l'} c\left\{\left(e_j^n\right)^2 + \left(\theta_j^n\right)^2\right\}\Big/2 \geq \bar{\Phi}_0 \\ (IC)\ \max_{e_j^n} \left\{ \begin{array}{l} \alpha_n + \sum_{j=1}^{l'} q_j^n \left(e_j^n \theta_j^n - e_{j-1}^n \theta_{j-1}^n\right) - \rho\sigma_n^2/2 - \\ \sum_{j=1}^{l'} c\left\{\left(e_j^n\right)^2 + \left(\theta_j^n\right)^2\right\}\Big/2 \end{array} \right\} \end{cases}, \quad (19)$$

where $\bar{\Phi}_0$ represents the reservation utility.

Obviously, based on the aforementioned analysis, the optimal unit-payment and effort level need to be determined within the dynamic performance-based payment model for infrastructure PPP projects. In the next subsection, the unit-payment model will be constructed using the multi-objective optimization method.

(3) Optimal effort level for the private sector in the performance-based payment model

(I) Analysis of the private sector's optimal effort level in the performance-based payment model

According to Equations (7) and (8), the private sector's optimal effort level aims to maximize its utility function. Taking the first-order derivative with respect to the effort level in Equation (6), the optimal effort level e_n^* can be obtained.

Since

$$\frac{\partial \Phi_P}{\partial e_n} = q_n \theta_n - c e_n = 0, \quad (20)$$

the optimal effort level e_n^* is as follows:

$$e_n^* = \frac{q_n \theta_n}{c}. \quad (21)$$

It is evident that the optimal effort level e_n^* in Equation (21) and the unit-payment q_n (input θ_n) exist a positive correlation with increases in the unit-payment q_n. The optimal effort level e_n^* and the cost coefficient c are negatively correlated with increases in the cost coefficient c. Some related conclusions on the optimal effort level are as follows:

1) The optimal effort level e_n^* and the unit-payment q_n demonstrate a positive correlation increases in the unit-payment q_n. Notably, the unit-payment is directly proportional to the risk shared by the private sector. Throughout the operation and maintenance of projects, the private sector will receive more profits when actual performance surpasses the target performance. Consequently, the private sector will actively increase its efforts to improve performance, given the prospect of obtaining higher payments from the government.

2) The optimal effort level e_n^* and the private sector's input θ_n are positively correlated with increases in the private sector's input θ_n. If the input is fixed, then the performance appraisal score will increase as the optimal effort level increases. In other words, the input is positively proportional to the performance income as well as the effort.

3) The optimal effort level e_n^* and the private sector's cost coefficient c are negatively correlated with increases in the private sector's cost coefficient c. The greater the cost is, the larger the risk undertaken by the private sector. Therefore, the private sector will select the effort level to reduce its risk in the operation and maintenance process of projects.

(II) Analysis of the private sector's optimal effort level in the dynamic performance-based payment model

The method similar to that used in the above analysis is adopted below. According to Equations (18) and (19), the private sector's optimal effort level will maximize the private sector's utility function. Taking the first-order derivative with respect to effort level in Equation (17), the optimal effort level $\tilde{e}_n^*$ can be obtained.

Similarly, by

$$\frac{\partial \Psi_P}{\partial e_j^n} = \sum_{j=1}^{l'} q_j^n \theta_j^n - \sum_{j=1}^{l'} c e_j^n = 0, \quad (22)$$

the optimal effort level $\tilde{e}_n^*$ is obtained in terms of the optimal effort level $\left(\tilde{e}_j^n\right)^*$ in the jth performance appraisal period as follows:

$$e_n^* = \sum_{j=1}^{l'} \left(\tilde{e}_j^n\right)^* = \sum_{j=1}^{l'} \frac{q_j^n \theta_j^n}{c}. \quad (23)$$

Obviously, the optimal effort level $\tilde{e}_n^*$ in Equation (23) is positively related to the unit-payment q_j^n (input θ_n) with increases in the unit-payment q_j^n. Meanwhile the optimal effort level $\tilde{e}_n^*$ is negatively related to the cost coefficient c. Similar conclusions on the optimal effort level $\tilde{e}_n^*$ are summarized to the optimal effort level e_n^* in Equation (21), which are hereby excluded for simplicity.

(III) Comparative analysis of the private sector's optimal effort levels in two proposed performance-based payment models

In this subsection, a comparative analysis of two optimal effort levels for the private sector in the two different performance-based payment models is carried out.

Based on the analysis in Subsection 3.3, the primary difference between the two models is whether the unit-payment is constant. For a certain performance appraisal period of infrastructure PPP projects, the unit-payment is unchanging in the first performance-based payment model, and varies according to different performance levels in dynamic performance-based payment model.

From the optimal effort level in Equation (23), if the performance appraisal score x^n is divided into l' performance level intervals according to performance appraisal standard, such as when the interval $[x_j, x_{j+1}]$ is the jth performance level interval in the nth performance appraisal period and e_j^n is the jth effort level corresponding to the jth performance level interval, then the unit-payment q_j^n and the effort level e_j^n in the jth performance level interval satisfy $q_{l'}^n > q_{l'-1}^n > \cdots > q_2^n > q_1^n > 0$ with $\sum_{j=1}^{l'} q_j^n = q_n$ and $e_{l'}^n > e_{l'-1}^n > \cdots > e_2^n > e_1^n > 0$ with $\sum_{j=1}^{l'} e_j^n = e_n$, respectively. Obviously, the private sector will has more has enthusiasm to continually improve performance through a stepped payment model.

Compared to the above analysis, when the unit-payment is constant during a certain performance appraisal period, it will lead to two consequences. On the one hand, if the performance standard is low, private sector can gain more profit from the relatively low performance appraisal scores. On the one hand, if the performance standard is high, the private sector is a little less likely to improve performance. Since the relatively high performance appraisal scores require more efforts.

4. Numerical simulation and case study analysis

In this section, a simulation analysis is conducted based on a water environment governance and ecological restoration PPP project in China, with the total investment of 2162.6205 million CNY. The estimated cooperation period of this project is 20 years, consisting of a 2-years construction period and an 18-year operation and maintenance period. To assess the project's performance, the government entrusts some relevant departments, such as the Water Conservancy Bureau, as the implementation agencies. These agencies conduct performance appraisal during the operation and maintenance period. The government pays the private sector according to performance appraisal scores, and the amount paid is determined by the designed payment contract. Consequently, the private sector obtains economic benefits through the operation and maintenance of the project (Li et al., 2022).

Subsequently, a numerical simulation is presented to analyse the influences of key parameters on the unit-payment, including probability, lower bound of the performance level, and coefficient of the cost curve. For clarity, the values in different scenarios are shown in Table 2.

Table 2. Values of all parameters in different situations

(1) Influence of the probability of performance level covering performance appraise score on unit-payment			
Figure 6a	$w_1 = w_2 = 0.5$	$\bar{u} = 0.5$	$x_j = 60$; $x_j = 70$; $x_j = 80$; $x_j = 90$; $x_j = 100$
Figure 6b		$x_j = 70$	$\bar{u} = 0.1$; $\bar{u} = 0.3$; $\bar{u} = 0.5$; $\bar{u} = 0.7$; $\bar{u} = 0.9$
(2) Influence of the lower bound of performance level covering performance appraise score on unit-payment			
Figure 7a	$w_1 = w_2 = 0.5$	$\bar{u} = 0.5$	$p = 0.1$; $p = 0.3$; $p = 0.5$; $p = 0.7$; $p = 0.9$
Figure 7b		$p = 0.7$	$\bar{u} = 0.1$; $\bar{u} = 0.3$; $\bar{u} = 0.5$; $\bar{u} = 0.7$; $\bar{u} = 0.9$
(3) Influence of the coefficient of cost curve on unit-payment			
Figure 8a	$w_1 = w_2 = 0.5$	$x_j = 70$	$p = 0.1$; $p = 0.3$; $p = 0.5$; $p = 0.7$; $p = 0.9$
Figure 8b		$p = 0.7$	$\bar{u} = 0.1$; $\bar{u} = 0.3$; $\bar{u} = 0.5$; $\bar{u} = 0.7$; $\bar{u} = 0.9$

(1) Influence of the probability of performance level covering performance appraise score on unit-payment

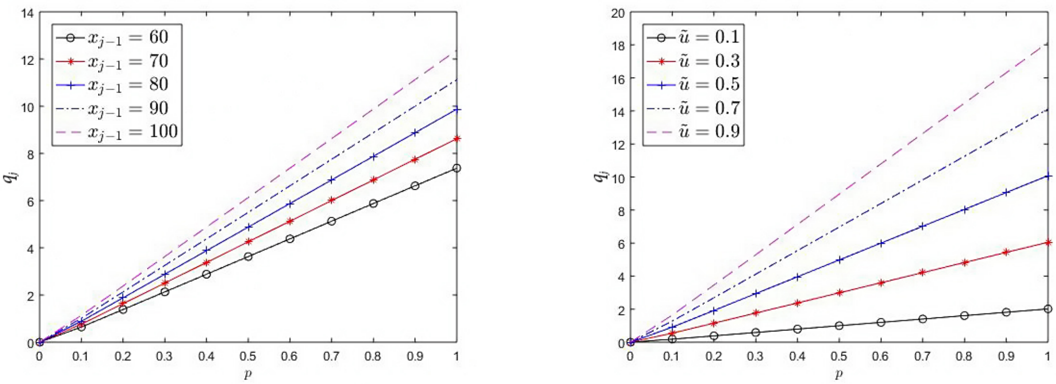
In Equation (14), it is assumed that $w_1 = w_2 = 0.5$, $\bar{u} = 0.5$, and the influence of probability p of performance level $[x_{j-1}, x_j]$ covering performance appraisal score x on unit-payment is shown in Figure 6a, where the different lower bounds are $x_{j-1} = 60$, $x_{j-1} = 70$, $x_{j-1} = 80$, $x_{j-1} = 90$ and $x_{j-1} = 100$. As shown in Figure 6a, at a certain cost coefficient, there are positive correlations between the unit-payment q_j and the probability p with the different lower bound x_{j-1} of performance level $[x_{j-1}, x_j]$. Furthermore, for a certain probability, the larger the lower bound x_j is, the larger the unit-payment q_j is.

Similarly, it is assumed that $w_1 = w_2 = 0.5$, $x_{j-1} = 70$, and the influence of probability p of performance level $[x_{j-1}, x_j]$ covering performance appraisal score x on unit-payment is shown in Figure 6b. The different coefficient values of the private sector's cost curve are $\bar{u} = 0.1$; $\bar{u} = 0.3$; $\bar{u} = 0.5$; $\bar{u} = 0.7$ and $\bar{u} = 0.9$. As shown in Figure 6b, at a certain coefficient of the cost curve, there are positive correlations between the unit-payment q_j and the probability p of the performance level covering the performance appraisal score with the different coefficient $\bar{u}$ of the cost curve. Furthermore, for a certain probability, the higher the coefficient $\bar{u}$ is, the larger the unit-payment q_j is.

Based on the above analysis, it is evident that the unit-payment will increase with the rise in probability and the lower bound. The performance level achieved by the private sector should be thoroughly considered when the public sector sets the unit-payment, given that the private sector's marginal cost will increase with a high lower bound. Based on the principles of equity and rationality, the increasing of unit-payment will promote an improvement spiral for project performance.

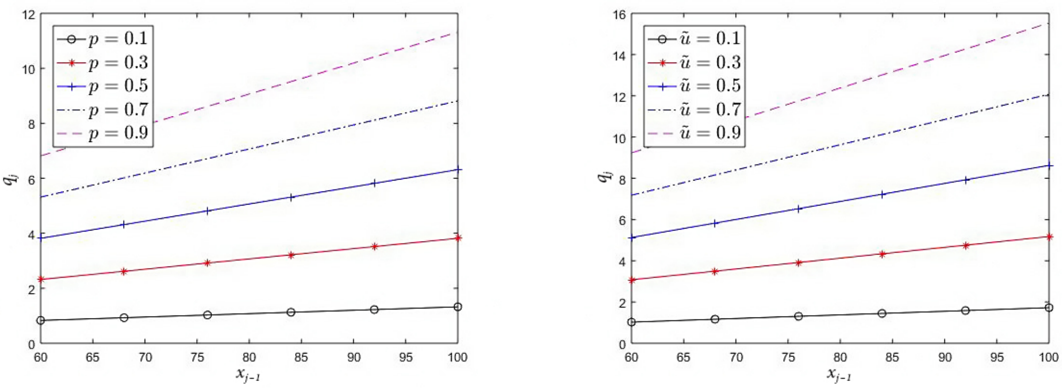
(2) Influence of the lower bound of performance level covering performance appraisal score on unit-payment

In Equation (14), it is assumed that $w_1 = w_2 = 0.5$, $\bar{u} = 0.5$, and the influence of the lower bound x_{j-1} of performance level $[x_{j-1}, x_j]$ covering performance appraisal score x on the unit-payment is shown in Figure 7a, where the different probability values are $p = 0.1$, $p = 0.3$, $p = 0.5$, $p = 0.7$, and $p = 0.9$. As shown in Figure 7a, at a certain coefficient of the cost curve, there are positive correlations between the unit-payment q_j and the lower bound x_{j-1} of performance level $[x_{j-1}, x_j]$ covering the performance appraisal score with the different probability p. Furthermore, for a certain lower bound x_{j-1}, the larger the probability p is, the larger the unit-payment q_j is.



a) b)

Figure 6. The influence of the probability on unit-payment under the different lower bounds and coefficients of the cost curve



a) b)

Figure 7. The influence of the lower bound on unit-payment under the different probabilities and coefficients of the cost curve

Similarly, it is assumed that $w_1 = w_2 = 0.5$, $p = 0.7$, and the influence of the lower bound x_{j-1} of performance level $[x_{j-1}, x_j]$ covering performance appraisal score x on unit-payment is shown in Figure 7b, where the different coefficient values of the private sector's cost curve are $\bar{u} = 0.1$; $\bar{u} = 0.3$; $\bar{u} = 0.5$; $\bar{u} = 0.7$ and $\bar{u} = 0.9$. As shown in Figure 7b, at a certain probability, there are positive correlations between the unit-payment q_j and the lower bound x_{j-1} of performance level $[x_{j-1}, x_j]$ covering performance appraisal score x with the different cost coefficient $\bar{u}$ of the private sector's cost curve. Moreover, for a certain lower bound x_{j-1}, the larger the coefficient $\bar{u}$ is, the larger the unit-payment q_j is.

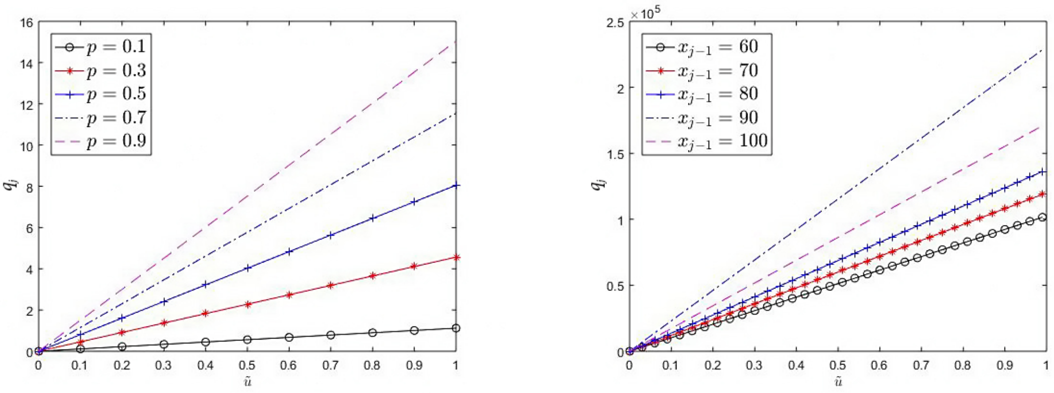
Based on the above analysis, it is evident that the unit-payment increases with the lower bound of the performance level, and a higher lower bound corresponds to a greater cost for the private sector to improve performance. Only when the government sets a relatively large unit-payment will the private sector exert significant efforts to improve performance. Conversely, for a small lower bound, the private sector's cost to improve performance is relatively low, requiring only a relatively small unit-payment from the government. From the sustainability perspective of the contract, for the high performance standards, the private sector will exert greater efforts to achieve a high performance level, incurring a large marginal cost. To stimulate the private sector to achieve a relatively higher performance level, the public sector should reasonably increase the unit-payment. This not only guarantees reasonable income for the private sector, but also ensures the maximization of social benefits, ensuring the steady execution of the contract.

(3) Influence of the coefficient of the cost curve on unit-payment

In Equation (14), it is assumed that $w_1 = w_2 = 0.5$, $x_{j-1} = 70$, and the influence of the cost coefficient of the private sector on the optimal unit-payment is illustrated in Figure 8a, in which the different probability values are taken as $p = 0.1$, $p = 0.3$, $p = 0.5$, $p = 0.7$, and $p = 0.9$. As depicted in Figure 8a, under the condition of the lower bound $x_{j-1} = 70$, there exist positive correlations between the unit-payment q_j and the coefficient $\bar{u}$ of the private sector's cost curve $C(x) = \bar{u}x^2$ for different values of the probability p. Specifically, for a certain lower bound x_{j-1}, the larger the coefficient $\bar{u}$ of the private sector's cost curve $C(x) = \bar{u}x^2$ is, the larger the unit-payment q_j is.

Similarly, it is assumed that $w_1 = w_2 = 0.5$, $p = 0.7$. The influence of the different coefficients $\bar{u}$ of the private sector's cost curve on the unit-payment is demonstrated in Figure 8b, in which the different values of the private sector's cost curve coefficient are $\bar{u} = 0.1$; $\bar{u} = 0.3$; $\bar{u} = 0.5$; $\bar{u} = 0.7$ and $\bar{u} = 0.9$. As shown in Figure 8b, under the condition of different lower bound values x_{j-1}, there exist positive correlations between the unit-payment q_j and the coefficient $\bar{u}$ of the private sector's cost curve. Moreover, for a certain probability, the larger the coefficient $\bar{u}$ of the private sector's cost curve is, the larger the unit-payment q_j is.

Therefore, it is clear that the unit-payment increases with the coefficient of the cost curve, and a higher the coefficient implies a steeper the cost curve. This means that the private sector incurs higher costs to improve their performance appraisal score. With a relatively large unit-payment set by the government, the private sector is incentivized to exert greater effort to improve their performance. Conversely, for a determined cost, a higher lower bound results in a larger unit-payment. Given the profit-driven nature of the private sector, there is a possibility of speculation to obtain substantial profits. To avoid this phenomenon, the public sector must strengthen supervision to ensure both of the private sector's profits and the social benefits.



a) b)

Figure 8. The influence of the coefficients of the cost curve on unit-payment under the different probability and lower bounds

5. Discussion and implications

Based on the above analysis, it can be seen that a clear and reasonable payment structure is crucial to attract the profit-driven private sector to improve performance in Infrastructure PPP projects. To solve the unchangeable incentive or subsidy coefficient, this study establishes an optimal model to calculate unit-payment under the different performance levels, which is obtained by constructing and solving an optimization function, and the different results influenced by some main parameters are also obtained as follows.

(I) The unit-payment are positively correlated with the private sector's costs. The larger the cost curve coefficient, the higher the cost paid by the private sector to improve the level of performance. This requires the government to set up a relatively large unit-payment to improve the enthusiasm of social capital efforts. Since the private sector is profit-driven, and it can make huge profits by speculating. To avoid this phenomenon, the public sector must strengthen regulation to ensure the private sector's reasonable profits and the public's social benefits.

(II) The unit-payment is positively correlated with the lower bound of the performance level. For the larger lower bound of the performance level, it means that the private sectors' costs to improve the performance level is relatively large, and only by setting a relatively large unit-payment can the government encourage the private sector to actively strive to improve the performance. From the perspective of contract sustainability, for the high performance standards, the large marginal costs will be incured when it strives to achieve a relatively high level of performance. Therefore, to stimulate the private sector to achieve a relatively high level of performance, the government needs to set a relatively high unit-payment. On the one hand, the reasonable returns for the private sector are guaranteed, and, on the other hand, it ensures the maximization of social benefits.

(III) When the unit-payment is unchanged in a certain performance appraisal period, the private sector can make more profit from a relatively low performance appraisal score for a lower performance standard. Conversely, if the performance standard is relatively high, the private sector is less likely to actively improve the performance of the project since the relatively high performance appraisal scores require more effort from the private sector.

Therefore, the public sector should enhance project supervision and encourage private sector performance, ensuring their fair income. When the performance level has a high lower bound, improving it becomes costlier, necessitating higher unit-payments. However, excessively high unit-payments can lead to private sector speculation. So, for high performance standards, the public sector should reasonably increase unit-payments to motivate the private sector while ensuring contract stability and maximizing social benefits.

Additionally, pricing in PPP projects is complex, with many influential parameters. Given the long concession periods and unexpected situations, payments should vary based on performance factors (Chen & Nozick, 2016). Strategies for price adjustments are needed. Cooperation between public and private sectors is essential to leverage their strengths and compensate for weaknesses, enhancing stakeholder satisfaction and maximizing social benefits during project operation.

The implications of this study arise as follows: (1) The government should prioritize flexibility in pricing mechanisms and establish price adjustment protocols for emergencies. Setting income boundaries for the private sector is imperative to ensures fairness and prevents excessive profit-taking, thereby fostering a mutually beneficial, win-win situation. (2) The private sector must commit to the long-term, safe, and efficient project operation and maintenance of projects spanning decades. To secure both income and social benefits, they should enhance organization design, system development, and information-based maintenance platforms. Continuous optimization and cost reduction are paramount to guarantee sustained income. The payment mechanism is fundamentally a comprehensive system, not just a mere pricing mechanism. Its design significantly impacts project success and warrants heightened attention from both the public and private sectors (Shi et al., 2020; Kweun et al., 2017).

6. Conclusions

In the operation and maintenance process, the establishment of a reasonable payment mechanism holds paramount significance for the public and private sectors. However, the efficacy of fixed incentive or subsidy coefficients within current payment structures in fulfilling their incentivizing role is notably limited. To address this inadequacy, the current study undertakes the development of a dynamic multi-stage performance-based payment model, employing the framework of principal-agent theory.

The main procedures of this study are as follows: (1) A multi-stage performance-based payment model is developed based on principal-agent theory, wherein the incentive or subsidy coefficient remains constant; (2) An optimal unit-payment model is constructed, proving to be instrumental in the development of a multi-stage dynamic performance-based payment model, incorporating the performance appraisal score of private sector; (3) A multi-stage dynamic performance-based payment model is formulated, featuring a changeable incentive or subsidy coefficient; (4) A series of results is derived through numerical simulation analysis, illustrating the impact on payment associated with different key parameters. These findings offer a theoretical reference for the formulation and implementation of PPP contracts.

The design of the payment structure for infrastructure PPP projects should comprehensively consider the project characteristics to align with the actual needs of the projects. The contributions of this study primarily encompass two aspects: (1) The study reveals the mechanism of unit-payment, characterizing the payment of the per each score, which can provide a theoretical support for the reasonable distribution of income within projects.

(2) The study introduces a stepped dynamic performance-based payment structure, facilitating the rational allocation of risks among all project participants, which enriches the existing research on the performance-based payment mechanism. In essence, the findings of this study serve a dual purpose, on one hand, they offer insights to rectify the shortcomings in the current unsatisfactory payment structure. On the other hand, the study provides a theoretical reference for the public sector to effectively incentivize the private sector in enhancing project performance.

Of course, there are some limitations associated with this study. In the proposed dynamic performance-based payment mechanism, the alterations in performance payment predominantly revolve around the performance appraisal score, and the effort level stands out as a primary parameter influencing unit-payment. Nevertheless, the design of a performance-based payment mechanism necessitates the consideration of various other crucial factors, including the private sector's reputation and the ratchet effect observed in long-term incentive processes. These factors exert significant influences on performance payment. Due to space constraints, this study can only highlight the major factors. Moreover, some factors like effort level, reputation, and the ratchet effect pose challenges in terms of quantification.

Certainly, some future research endeavors will delve into several aspects. Firstly, optimal contracts ought to mitigate certain demand risks, particularly those existing in dimensions observable by users but not verified by authorities, thereby influencing service demand in practical scenarios. Secondly, there is a need to prioritize performance evaluation in the design of performance-based contracts, enabling efficient benchmarking between appraisal results and unit-payment. Lastly, there is an opportunity to investigate smart contract design by incorporating digital techniques like blockchain and digital twins when devising payment mechanisms.

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Author contributions

Yongchao CAO and Limin SU wrote the first draft of the article. Yongchao CAO, Huimin LI and Limin SU conceived the study and were responsible for the design and development of the data analysis. Yonghcao CAO were responsible for data collection and analysis. Huimin LI and Limin SU were responsible for data interpretation.

Data availability

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure statement

The authors declare that they have no conflicts of interest.

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