Journal of Business Economics and Management
ISSN 1611-1699 print / ISSN 2029-4433 online
2011 Volume 12(1): 69–91
doi:10.3846/16111699.2011.555366

INTERACTIVE MULTICRITERIA DECISION AIDING UNDER RISK – METHODS AND APPLICATIONS

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Received 23 November 2009; accepted 04 October 2010

Abstract. In the paper a discrete multicriteria decision making problem under risk is considered. It is assumed that the set of alternatives consists of a finite number of elements that are explicitly described. The evaluations of alternatives with respect to criteria are represented by distribution functions. The decision maker tries to find a solution preferred to all other solutions. To solve the problem one has to analyze the decision maker's preferences. In the study interactive approach is used. Three interactive methods and its applications in operations management are presented.

Keywords: multicriteria analysis, interactive approach, decision making under risk, stochastic dominance, managerial decisions.

Reference to this paper should be made as follows: Nowak, M. 2011. Interactive multicriteria decision aiding under risk – methods and applications, *Journal of Business Economics and Management* 12(1): 69–91.

JEL Classification: C61.

1. Introduction

Problem solving and decision making are universally considered to be the skills that play the most important role for each manager. The range of problems that present-day manager has to face is extremely wide, including typical tasks that can be solved by standard techniques, as well as unique issues requiring individual approaches. The generally accepted typology of decisions proposed by Simon (1965) includes programmed and non-programmed. Programmed decisions are routine. They rely on some form of predetermined procedures which are invoked when a particular problem occurs. Non-programmed decisions are those for which such procedural guides don't exist. In practice, however, managers often have to face problems that include both typical and unique elements.

One of the most important features of managerial decisions is multidimensionality. In order to make a decision a manager has to consider multiple criteria, including quantitative and qualitative ones. It is also pointed out that decision-making is usually associated with some degree of risk. The process of globalization and fast technological development result in the increasing level of uncertainty that managers have to face. Thus, the need for developing and practical implementation of new decision aiding techniques dedicated for managerial decision making problems appear.

One of the most difficult problem that we have to solve implementing multicriteria techniques is identification of the decision maker preferences. Usually he/she is not able to express precisely and unequivocally his/her expectations with respect to the solution of the problem. In such case interactive techniques can be used. While it is difficult for the decision maker to provide the whole preference information required for constructing the complete ranking of decision alternatives, he/she usually is able to compare selected solutions.

Most of interactive techniques are devoted to decision-making problems under certainty. Unfortunately, as was mentioned above, risk cannot be ignored, when a real-world decision problem is considered. This was the motivation for the author to propose new interactive techniques devoted for decision-making problems under risk. The aim of this paper is to present such techniques and to discuss potential applications in operations management.

The paper is structured as follows. Section 2 provides problem formulation and concise survey of techniques used for solving it. Section 3 presents a brief survey of interactive techniques used for solving decision making problems under certainty. Next section is dedicated to stochastic dominance rules that can be used to compare uncertain projects. In section 5 interactive procedures for discrete multicriteria decision making problems under risk are proposed. Applications of these techniques in managerial decision making problems are discussed in section 6. The last section groups conclusions.

2. A discrete decision-making problem under risk

This paper considers a decision-making problem in which the set of alternatives consists of a finite number of elements that are explicitly described. We assume that up to moderate number of alternatives (not more than one hundred) are considered. Alternatives are evaluated with respect to a finite number of multiple criteria (not less than three and not more than ten). As a decision-making problem under risk is analyzed here, so we assume that the evaluations of alternatives with respect to criteria are described by probability distributions.

The decision situation considered in this paper may be conceived as a problem (**A**, **X**, **E**) where **A** is a finite set of alternatives a_i, $i = 1, 2, ..., m$; **X** is a finite set of criteria X^k, $k = 1, 2, ..., n$; and **E** is a set of evaluations of alternatives with respect to criteria:

$$\mathbf{E} = \begin{bmatrix} X_1^1 & \cdots & X_1^k & \cdots & X_1^n \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ X_i^1 & \cdots & X_i^k & \cdots & X_i^n \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ X_m^1 & \cdots & X_m^k & \cdots & X_m^n \end{bmatrix}. \quad (1)$$

It is assumed that evaluations are expressed numerically. Ordinal scale is used for evaluation alternatives with respect to the qualitative criterion. We also assume that this scale is defined in such a way that a larger value is preferred to smaller ones. As to quantitative criteria we assume that both maximization and minimization is possible.

Numerous techniques for solving decision-making problems under risk are proposed. A bibliographic study can be found in Steuer and Na (2003). Classical approach is based on the multiattribute utility theory proposed by Keeney and Raiffa (1976). They start from the axioms of the utility theory and assume that the set of criteria satisfies the decomposability condition, i.e. the multicriteria comparison of two alternatives can be decomposed to single-criterion comparisons. Such assumption makes it possible to solve the problem by the procedure consisting of four major steps. First, the partial utilities of alternatives with respect to criteria are evaluated. In the second step the parameters of a global utility function are estimated. Next, the global utilities of alternatives are computed, and finally, alternatives are ranked according to global utilities. The simplest form of the global utility function is a linear one. Such function can be used if additive utility independence condition is verified. The multiattribute utility theory played important part in the development of the decision theory. Practical application of this approach, however, is rather inconvenient. The estimation of a utility function is not easy, even if a single criterion is considered. Multicriteria environment requires additionally the identification of the synthesis function and its parameters. Such analysis is again time-consuming and laborious. As a result, the directly implementation of the multiattribute utility approach is difficult. However, numerous approaches using this idea indirectly are proposed. Saaty and Vargas (1987) present a modified version of a well-known Analytic Hierarchy Process (AHP) dedicated to decision-making problems under risk. Huang *et al*. (1978) suggest employing multiattribute stochastic dominance rule for modeling global preferences. According this rule alternative a_i is at least as good as a_j if evaluations of a_i dominate corresponding evaluations of a_j in relation to each criterion. Unfortunately this unanimity condition is very rarely verified, and as a result such approach is useful, when the set of efficient solutions has to be identified, but fails when the final solution of a multicriteria problem is to be identified.

In contrary to techniques based on the multiattribute utility function are the methods that use the concept of outranking relation (Dendrou *et al.* 1980; Martel *et al.* 1986; D'Avignon, Vincke 1988).

Zaras and Martel (1994) combine multiattribute utility approach with outranking relation. They use multiattibute stochastic dominance but suggest weakening the unanimity condition and accepting a majority attribute condition. The procedure uses the idea of multiattribute stochastic dominance for a reduced number of criteria, which is based on the observation that people tend to simplify the multicriteria problem by taking into account only the most important criteria. The ELECTRE I methodology is used to obtain final ranking of alternatives. The procedure that combines this approach with the concept of preference thresholds is proposed in Nowak (2004b).

3. A review of interactive techniques for decision-making problems under certainty

Techniques cited in the previous section divide the solving process into two parts. The first, all preference information required for solving the problem is collected. Next, this information is used for constructing complete or partial order of alternatives. Several criticisms have been expressed against such approach. The assessment of the sufficient a priori preference information is inconvenient and time consuming. If the decision problem is repetitive, this information can be inferred from the past decisions. Usually, however, direct questioning technique has to be employed. Thus, the decision maker is asked to make hypothetical choices between alternatives that often have no practical reality. It is not easy to motivate the decision maker to consider and evaluate such choices. Moreover, as the decision maker is not employed in the second phase of the procedure, when the final solution is generated, so he/she may feel excluded from the important part of the analysis and put little confidence in a final result.

Interactive approach is opposite to techniques based on an a priori basis. Instead of collecting all preference information prior to calculating the final solution, this information is obtained in a stepwise manner. The decision maker is asked to define which criteria influence his/her preferences and to provide preference information with respect to a given solution or a given set of solutions (local preference information).

Various advantages are mentioned for applying interactive techniques. First, the limited amount of preference information is required, as compared to methods in which the decision maker has to provide his/her preferences on an a priori basis. Second, the decision maker does not have to answer hypothetical questions, but is asked to evaluate well-defined solutions, that are known to exist and be feasible. It is also pointed out that the decision maker is closely involved in the whole process of solving the decision problem, thus obtaining more insight into the trade-offs among different criteria. Finally, it is emphasized that the decision makers put much reliance in solutions generated via interactive procedure, and as a result, such solutions have better chances of being implemented.

A variety of interactive techniques have been proposed for the last 35 years. All of them proceed from one solution (or a set of solutions) to another, guided by the requests of the decision maker, which must also be expressed iteratively. While the dialog scenario of each technique is unique, common features can be identified. Usually an initial solution is proposed to the decision maker, and he/she is asked to express his/her preference information with respect to this solution to the analyst. The information articulated by the decision maker is used for generating a new solution. Procedure continues until a satisfactory solution is obtained. Thus, interactive approach corresponds to the Simon's theory of "satisficing". He noticed that managers are usually focused on finding "satisfactory" solution rather than the "optimal" one (Simon 1957).

Interactive techniques use two main paradigms for collecting the preference information: direct and indirect. According to the prior one, the decision maker expresses his/her preferences in relation to the values of criteria. Indirect collection of preferences means that the decision maker has to determine the trade-offs among criteria at each iteration, given the current candidate solution. Methods that combine these approaches are also proposed.

First interactive procedures were proposed in 1970s. In STEM (Step Method) proposed by Benayoun *et al*. (1971) the concept of ideal solution is used. The elements of the ideal solution are the maximum values of the criteria, which are individually attainable within the set of alternatives. STEM is based on the calculation of a candidate alternative, which has a minimal distance to the ideal solution according to the mini-max rule. If the decision maker accepts the proposal, the procedure ends, otherwise the decision maker is asked to define the amounts of relaxation for the criteria, whose values are already satisfactory. Next, a new set of alternatives is generated taking into account the restrictions defined by the decision maker. The procedure continues until an alternative with satisfactory evaluations is found.

A number of techniques based on trade-off ratios are proposed. Geoffrion *et al*. (1972) proposed a method, in which the decision maker has to determine trade-offs between criteria at each iteration, given values of criteria for the considered alternative. It is assumed that the decision maker's preferences can be described by a differentiable, concave and increasing utility function. As, however, this function is unknown, the decision maker is asked to provide the information on trade-offs.

Another class of interactive methods consists of techniques in which the decision maker has to define minimum or maximum values for one or more goal variables at each iteration. These restrictions are used to reduce the feasible region. Such approach is used in interactive multiple goal programming method proposed by Spronk (1981). In his procedure a proposal solution and potency matrix is presented to the decision maker. The solution is a vector of minimum values for the respective goal variables. The potency matrix consists of two vectors representing the ideal and pessimistic solution, respectively. If the proposal solution is not satisfactory for the decision maker, he/she is asked to choose the goal variable to be improved first. The decision maker does not have to specify the amount by which the considered goal variable should be improved. If however he/she is able to define aspiration levels for goal variables, then such information can be used by the procedure.

4. Stochastic dominance rules

Two main approaches are usually used for comparing uncertain alternatives. The first is known as a mean-risk model and is based on two criteria: one measuring expected outcome and the second representing variability of outcomes. Mean-risk analysis is usually used for modeling preferences of a risk-averse decision maker. Although the model of risk-averse preferences is widely exploited in the decision theory, it is not suitable for all situations. Markowitz (1952) noticed the occurrence of risk seeking in choices between negative prospects. This paradox was also justified by experiments conducted by Kahneman and Tversky (1979).

The second approach uses stochastic dominance rules. It is based on axiomatic model of preferences and leads to conclusions which are consistent with the axioms. In stochastic dominance approach random variables are compared by pointwise comparison of some performance functions constructed from their distribution functions.

Stochastic dominance rules are consistent with expected utility maximization rule. If alternative a_i dominates alternative a_j by stochastic dominance rule, then the expected utility of a_i is not less then the expected utility of a_j. However, verifying stochastic dominance relations is less complicated comparing to estimation of alternatives' utilities. To use them we do not need to estimate the utility function. We just have to recognize the decision maker's attitude to risk. Various efficient techniques have been proposed for identifying stochastic dominance relations, especially for discrete distributions, which are used more often (Aboudi and Thon 1994; Vickson and Altmann 1997).

Let us assume that criteria are defined is such a way, that a larger value is preferred to a smaller one (in opposite situations, distribution should be transformed by changing the sign). Let $F_i^k(x)$ and $F_j^k(x)$ be right-continuous cumulative distribution functions representing evaluations of a_i and a_j respectively over criterion X^k:

$$F_i^k(x) = \mathrm{P}\left\{X_i^k \le x\right\}, \tag{2}$$

$$F_j^k(x) = \mathrm{P}\left\{X_j^k \le x\right\}. \tag{3}$$

Definitions of first and second degree stochastic dominance relations are as follows:
Definition 1 – First Stochastic Dominance:
X_i^k dominates X_j^k by FSD ($X_i^k \succ_{\mathrm{FSD}} X_j^k$) in and only if,

$$F_i^k(x) \ne F_j^k(x) \quad \text{and} \qquad F_i^k(x) - F_j^k(x) \le 0 \quad \text{for } x \in R. \tag{4}$$

Definition 2 – Second Stochastic Dominance:
X_i^k dominates X_j^k by SSD ($X_i^k \succ_{\mathrm{SSD}} X_j^k$) in and only if,

$$F_i^k(x) \ne F_j^k(x) \quad \text{and} \quad \int_{-\infty}^{x} \left(F_i^k(y) - F_j^k(y)\right) dy \le 0 \text{ for } x \in R. \tag{5}$$

Hadar and Russel (1969) show that the FSD rule is equivalent to the expected utility rule for all decision makers preferring larger outcomes, while the SSD rule is equivalent to the expected utility rule for risk-averse decision makers preferring larger outcomes. Rules defined above apply to outcomes measured on cardinal scales, such as income, wealth, rates of return and so on, but fail to provide ranking of preferences among variables of ordinal nature. Rules that can be applied in such situations have been proposed by Spector *et al*. (1996). They distinguish two separate ordinal measurements:

1. The alternative outcomes can only be ranked in order of preference.
2. In addition to ranking outcomes, it is also possible to rank the differences between alternative outcomes.

Let us assume that the random variable X_i^k is defined by $\left(e_1^k, \ldots, e_t^k, p_{i1}^k, \ldots, p_{it}^k\right)$, where $e_1^k, \ldots, e_t^k$ are t real numbers, such that $e_l^k < e_{l+1}^k$ for all l = 1, …, t – 1, and $p_{i1}^k, \ldots, p_{it}^k$ are the probability measures. The variable X_j^k is defined similarly with $p_{j1}^k, \ldots, p_{jt}^k$ replacing $p_{i1}^k, \ldots, p_{it}^k$.

If the outcomes can be ranked in order of preferences, i.e. the decision maker prefers e_{l+1}^k over e_l^k for all $l = 1, \ldots, t-1$, Ordinal First Degree Stochastic Dominance (OFSD) rule can be used.

Definition 3 – Ordinal First Stochastic Dominance:
X_i^k dominates X_j^k by OFSD ($X_i^k \succ_{\text{OFSD}} X_j^k$) in and only if,

$$\sum_{l=1}^{s} p_{il}^k \le \sum_{l=1}^{s} p_{jl}^k \text{ for all } s = 1,\ldots,t. \tag{6}$$

Let us assume that the decision maker adds additional information and indicates that the outcome is improved more by switching from e_l^k to e_{l+1}^k than from e_{l+1}^k to e_{l+2}^k for all $l = 1, \ldots, t-2$. In such case Ordinal Second Degree Stochastic Dominance (OSSD) rule can be employed.

Definition 4 – Ordinal Second Stochastic Dominance:
X_i^k dominates X_j^k by OSSD ($X_i^k \succ_{\text{OSSD}} X_j^k$) in and only if,

$$\sum_{r=1}^{s}\sum_{l=1}^{r} p_{il}^k \le \sum_{r=1}^{s}\sum_{l=1}^{r} p_{jl}^k \text{ for all } s = 1,\ldots,t. \tag{7}$$

Spector *et al*. (1996) show that OFSD rule is equivalent to the expected utility rule for all decision makers preferring larger outcomes, while the OSSD rule is equivalent to the expected utility rule for risk-averse decision makers preferring larger outcomes.

Stochastic dominance rules may fail to show dominance in cases where almost everyone would prefer one gamble to another. These rules relate to all utility functions in a given class, even the ones that probably do not characterize the preference of any decision maker. Leshno and Levy (2002) propose modified rules to show how to obtain decisions that reveal a preference for one alternative to another when ordinary stochastic dominance rules fail.

To define almost stochastic dominance let us assume following notation:

$$S_1\left(F_i^k, F_j^k\right) = \left\{t \in [\alpha, \beta] : F_j^k(t) < F_i^k(t)\right\}, \tag{8}$$

$$S_2\left(F_i^k, F_j^k\right) = \left\{t \in S_1\left(F_i^k, F_j^k\right) : \int_\alpha^t F_j^k(x)\,dx < \int_\alpha^t F_i^k(x)\,dx\right\}, \tag{9}$$

$$\left\|F_i^k - F_j^k\right\| = \int_\alpha^\beta \left|F_i^k(t) - F_j^k(t)\right| dt. \tag{10}$$

The definitions of Almost First-Degree Stochastic Dominance (AFSD) and Almost Second-Degree Stochastic Dominance (ASSD) are as follows:
Definition 5 – Almost First Stochastic Dominance:
X_i^k dominates X_j^k by ε-AFSD ($X_i^k \succ_{\text{AFSD}(\varepsilon)} X_j^k$) in and only if,

$$\int_{S_1}\left[F_i^k(t) - F_j^k(t)\right] dt \le \varepsilon \left\|F_i^k - F_j^k\right\|. \tag{11}$$

Definition 6 – Almost Second Stochastic Dominance:
X_i^k dominates X_j^k by ε-ASSD ($X_i^k \succ_{\mathrm{ASSD}(\varepsilon)} X_j^k$) in and only if,

$$\int_{S_2} \left[F_i^k(t) - F_j^k(t) \right] dt \le \varepsilon \left\| F_i^k - F_j^k \right\| \quad \text{and} \quad \mu_i^k \ge \mu_j^k , \tag{12}$$

where: μ_i^k and μ_j^k stand for means of distributional evaluations of a_i and a_j respectively with respect to criterion X^k.

Leshno and Levy (2002) show that ε-AFSD is equivalent to the expected utility rule if the utility function belongs to the class $U_1^*(\varepsilon)$. Similarly, ε-ASSD rule is equivalent to the expected utility rule if the utility function belongs to the class $U_2^*(\varepsilon)$. These types of utility functions do not assign a relatively high marginal utility to very low values or a relatively low marginal utility to large values of x. The value of ε determines the set of utility functions which are permissible. As ε gets smaller the set of permissible utility functions gets larger.

5. Interactive procedures for discrete multicriteria decision problems under risk

Procedures presented in this section are designed for problems with up to moderate number of alternatives (not more than one hundred). Thus we assume that it is possible to compare alternatives pairwisely. The proposed techniques differ in the way in which the dialog process is structured. The information that should be provided by the decision maker is different in each procedure. As a result, techniques can be utilized by various types of decision makers. INSDECM (Nowak 2006, 2008b) is the procedure, which requires the largest amount of preference information. As the decision maker has to define constraints on values of various parameters of distributional evaluations, so this technique should be used by persons familiar with quantitative techniques. STEM-DPR (Nowak 2004a, 2008b) is less requiring. The decision maker has to analyze evaluations of a single proposal and select the criterion that satisfies him/her. Additionally, he/she has to define the limit of concessions on this criterion. Thus, the technique can be used, when the decision maker has a little experience with multicriteria methods. The last technique – ATO-DPR – is least demanding. In this procedure the decision maker is presented a proposal and has just to choose the criterion to be improved and to order other criteria starting from the one, which can be worsen in the first order.

Before starting the dialog, the set of efficient alternatives can be identified. A simple procedure based on pairwise comparisons of alternatives, or a method employing the concept of Quad-tree can be used for this (Nowak 2008b).

5.1. Procedure INSDECM

INSDECM (***IN****teractive* ***S****tochastic* ***DEC****ision* ***M****aking Procedure*) is based on the approach used in Interactive Multiple Goal Programming (Spronk 1981). It is assumed that for each criterion various distribution parameters can be analyzed. The decision maker may examine values of means, standard deviations, lower (upper) semideviations, lower (upper) mean semideviations, probabilities of outcome not less (not exceeding) target values, and other distribution characteristics. In each step the potency matrix is generated. It consists of the worst (pessimistic) and best (optimistic) values of distribution parameters attainable independently within the set of alternatives. The decision maker is asked whether pessimistic values are satisfactory. If the answer is *yes*, he/she is asked to make a final choice between alternatives analyzed in the current phase of the procedure. Otherwise, the decision maker is asked to express his/her requirements by defining a constrain on the value of a selected distributional parameter.

In INSDECM the consistence of constraints defined by the decision maker with stochastic dominance rules is analyzed. Let us assume that the decision maker has defined a constraint on values of a parameter of distributional evaluation with respect to criterion X^k. We say that such constraint is inconsistent with stochastic dominance rules, if following conditions are simultaneously fulfilled:

– the constraint is not satisfied for alternative a_i,
– the constraint is satisfied for alternative a_j,
– $X_i^k \succ_{\text{SD}} X_j^k$,

where $\succ_{\text{SD}}$ stands for stochastic dominance relation appropriate for the decision maker's utility function.

Let us assume the following notation:
$\mathbf{A}^{(l)}$ – the set of alternatives analyzed in iteration l,
$\mathbf{I}^{(l)}$ – the set of indices i such that $a_i \in \mathbf{A}^{(l)}$,
Q – the number of parameters of distributional evaluations analyzed in current phase of the procedure,
$\mathbf{Q}_1$ – the set of indices of parameters, that are defined in such a way that a larger value is preferred to a smaller one,
$\mathbf{Q}_2$ – the set of indices of parameters, that are defined in such a way that a smaller value is preferred to a larger one,
$v_{i\,p}$ – value of parameter number p for alternative a_i, $i = 1, \ldots, m$, $p = 1, \ldots, q$,
$\mathbf{P}^{(l)}$ – potency matrix in iteration l:

$$\mathbf{P}^{(l)} := \begin{bmatrix} \underline{v}_1^{(l)} \cdots & \underline{v}_q^{(l)} & \cdots & \underline{v}_Q^{(l)} \\ \overline{v}_1^{(l)} \cdots & \overline{v}_q^{(l)} & \cdots & \overline{v}_Q^{(l)} \end{bmatrix}, \tag{13}$$

where:

$$\overline{v}_q^{(l)} := \begin{cases} \max\limits_{i \in \mathbf{I}^{(l)}} \left\{ v_{iq} \right\} & \text{for } q \in \mathbf{Q}_1, \\ \min\limits_{i \in \mathbf{I}^{(l)}} \left\{ v_{iq} \right\} & \text{for } q \in \mathbf{Q}_2, \end{cases} \tag{14}$$

$$\underline{v}_q^{(l)} := \begin{cases} \min\limits_{i \in \mathbf{I}^{(l)}} \left\{ v_{iq} \right\} & \text{for } q \in \mathbf{Q}_1, \\ \max\limits_{i \in \mathbf{I}^{(l)}} \left\{ v_{iq} \right\} & \text{for } q \in \mathbf{Q}_2. \end{cases} \tag{15}$$

$\mathbf{R}^{(l)}$ – matrix of solutions analyzed in iteration l:

$$\mathbf{R}^{(l)} := \begin{bmatrix} v_{i_1 1} & \cdots & v_{i_1 Q} \\ \vdots & \cdots & \vdots \\ v_{i_t 1} & \cdots & v_{i_t Q} \end{bmatrix}, \tag{16}$$

where $i_1, \ldots, i_t \in \mathbf{I}^{(l)}$.

INSDECM procedure operates as follows:

Initial phase:

1. $l := 1$, $\mathbf{A}^{(1)} := \mathbf{A}$.
2. Ask the decision maker to specify the parameters of distributional evaluations to be analyzed during the dialog phase of the procedure, calculate values of parameters $v_{i\,q}$, for $i = 1, ..., m$, $q = 1, ..., Q$.

Iteration l

1. Generate potency matrix $\mathbf{P}^{(l)}$.
2. Present potency matrix to the decision maker. Ask him/her whether he/she is satisfied with the information presented. If the answer is *yes* – go to (3), otherwise ask the decision maker to specify the parameters of distributional evaluations to be analyzed during the dialog phase of the procedure, calculate values of parameters $v_{i\,q}$, for $i = 1, ..., m$, $q = 1, ..., Q$, go to (1).
3. Ask the decision maker whether he/she is satisfied with pessimistic values. If the answer is *yes*, go to (13), otherwise – go to (4).
4. Ask the decision maker to specify criterion X^k for which additional requirement will be defined and to express the requirement.
5. Identify the set of alternatives satisfying the constraint formulated by the decision maker – $\mathbf{A}^{(l+1)}$.
6. Generate new potency matrix $\mathbf{P}^{(l+1)}$, present $\mathbf{P}^{(l)}$ and $\mathbf{P}^{(l+1)}$ to the decision maker, ask him/her whether he/she accepts the move from $\mathbf{P}^{(l)}$ to $\mathbf{P}^{(l+1)}$. If the answer is *yes*, go to (7), otherwise – go to (2).
7. For each pair (a_i, a_j), such that $a_i \in \mathbf{A}^{(l)} \setminus \mathbf{A}^{(l+1)}$ and $a_j \in \mathbf{A}^{(l+1)}$, identify stochastic dominance relation between distributional evaluations with respect to criterion X^k. Generate the set of pairs of alternatives, for which the constraint defined by the decision maker is inconsistent with stochastic dominance rules:

$$\mathbf{N}^{(l)} := \left\{ \left(a_i, a_j\right) : a_i \in \mathbf{A}^{(l)} \setminus \mathbf{A}^{(l+1)}, a_j \in \mathbf{A}^{(l+1)}, X_i^k \succ_{\mathrm{SD}} X_j^k \right\}. \tag{17}$$

8. For each pair $(a_i, a_j) \in \mathrm{N}^{(l)}$ calculate:

$$\varepsilon_{ij}^k = \frac{\int_{S_1} \left| F_i^k(t) - F_j^k(t) \right| dt}{\int_{\alpha}^{\beta} \left| F_i^k(t) - F_j^k(t) \right| dt}, \tag{18}$$

where:

$$S_1\left(F_i^k, F_j^k\right) := \left\{ t \in [\alpha, \beta] : F_j^k(t) < F_i^k(t) \right\}. \tag{19}$$

9. If $\mathbf{N}^{(l)} = \varnothing$, assume $l := l + 1$ and go to (1), otherwise – go to (10).
10. Choose the pair $(a_i, a_j) \in \mathbf{N}^{(l)}$, with the lowest value of ε_{ij}^{k}, present distributional evaluations of a_i and a_j with respect to X^k:
   - identify intervals in which distributional evaluations are defined:

$$\alpha_i := \min_{z=1,\ldots,t} \{x_{ikz}\} \quad \beta_i := \max_{z=1,\ldots,t} \{x_{ikz}\}, \tag{20}$$

$$\alpha_j := \min_{z=1,\ldots,t} \{x_{jkz}\} \quad \beta_j := \max_{z=1,\ldots,t} \{x_{jkz}\}; \tag{21}$$

   - according to the decision maker's preferences calculate $\mathrm{P}\{X_i^k \le s_r\}$ and $\mathrm{P}\{X_j^k \le s_r\}$ or $\mathrm{P}\{X_i^k \ge s_r\}$ and $\mathrm{P}\{X_j^k \ge s_r\}$, where:

$$s_r := \min(\alpha_i, \alpha_j) + r\frac{\max(\beta_i, \beta_j) - \min(\alpha_i, \alpha_j)}{R} \tag{22}$$

     and R is the number of observations (determined according to the decision maker's preferences);
   - present the data to the decision maker and ask to choose between following options:
     (a) confirmation of the constraint – a_j should be considered in successive phases of the procedure, while a_i should be ignored;
     (b) the constraint should be weaken – both a_i and a_j should be considered in successive phases of the procedure;
     (c) the constraint should be strengthen – both a_i and a_j should be ignored in successive phases of the procedure.

   If the decision maker chooses (a), go to (11), otherwise – go to (12).
11. Remove pairs (a_r, a_s) such, that $r = i$ or $s = j$ from $\mathbf{N}^{(l)}$, go to (9).
12. Present the ways in which the constraint should be weakened or strengthen to the decision maker. If the decision maker accepts one of the proposals, modify the constraint and go to (5), otherwise – go to (2).
13. Present matrix of solutions $\mathbf{R}^{(l)}$ to the decision maker. Ask him/her whether he/she accepts any of solutions as a final solution. If the answer is *yes* – go to (14), otherwise – go to (2).
14. The end of the procedure.

INSDECM iterates until the decision maker is able to accept one of the considered alternatives as a final solution. Although the procedure does not limit the number of distribution parameters to be presented, the decision maker is usually not able to analyze too many of them. If the number of criteria is large, it is sensible to consider just one parameter for each criterion. Usually, the central tendency measures (mean, median) provide beneficial information. The measures based on the probability of getting outcomes above or below the specified target value are also interesting, as they are intuitively understandable by the decision maker.

The procedure allows the decision maker to define a single constraint at each iteration. Nevertheless, it is also possible to permit him/her formulating multiple restrictions. In particular, if the decision maker has all constraints ready at the beginning of the interactive decision making process, they have to be taken into account. We must remember, however, that in many cases such restrictions cannot be satisfied simultaneously. If none alternative satisfies all constraints, we have to inform the decision maker of that and ask him/her to reformulate his/her restrictions.

The final solution is chosen in step (13). As the worst values of all parameters under consideration are satisfactory for the decision maker, so he/she is asked to make a final choice. The question is what should be done if he/she is not able to do this? In such case we can return to the dialog phase and try to provide additional information to the decision maker presenting values of other distribution characteristics (e.g. probability of meeting another target value).

5.2. Procedure STEM-DPR

STEM-DPR (***STE**p **M**ethod for **D**iscrete Decision Making **P**roblems under **R**isk*) employs the approach similar to the one that was proposed by Benayoun *et al*. (1971) in STEM method. In each step a candidate alternative is generated. If the proposal is satisfactory for the decision maker, the procedure ends. Otherwise, the decision maker is asked to choose the criterion, which has a satisfactory evaluation. Two cases have to be considered then. First, for none criterion the evaluation of the proposal is acceptable for the decision maker. In such instance, the procedure fails in generating a satisfactory solution. If, however, the decision maker is able to choose a criterion which provides a satisfactory evaluation, the decision maker is asked to define the limit of concessions that can be done on this criterion in order to improve evaluations with respect to other criteria. Next, new proposal is identified taking into account requirements expressed by the decision maker.

To describe STEM-DPR technique let us assume additional notation:
$\mathbf{C}_1$ – the set of indices of criteria, that are maximized,
$\mathbf{C}_2$ – the set of indices of criteria, that are minimized,
a_s – new proposal for the decision maker,
ε_{ij}^k – minimal value of ε such, that $X_i^k \succ_{\mathrm{AFSD}(\varepsilon)} X_j^k$.

In STEM-DPR the vector of ideal values of means is employed. It is defined as follows:

$$\bar{\boldsymbol{\mu}}^{(l)} = [\bar{\mu}_1^{(l)}, \ldots, \bar{\mu}_n^{(l)}], \tag{23}$$

where:

$$\bar{\mu}_k^{(l)} := \begin{cases} \max\limits_{i \in \mathbf{I}^{(l)}} \{\mu_i^k\} & \text{for } k \in \mathbf{C}_1, \\ \min\limits_{i \in \mathbf{I}^{(l)}} \{\mu_i^k\} & \text{for } k \in \mathbf{C}_2. \end{cases} \tag{24}$$

STEM-DPR operates as follows:

Initial phase:

1. Identify stochastic dominance relations between distributional evaluations of alternatives with respect to criteria.
2. Calculate μ_i^k for $i = 1, \ldots, m$, $k = 1, \ldots, n$.
3. $l := 1$, $\mathbf{A}^{(1)} := \mathbf{A}$, $\mathbf{K} := \{1, \ldots, n\}$.

Iteration l

1. Identify new proposal:

$$a_s := \arg \min_{a_j \in \mathbf{A}^{(l)}} \max_{k \in \mathbf{K}} \{d_{jk}^{(l)}\}, \tag{25}$$

where $d_{jk}^{(l)}$ is calculated as follows:

$$d_{jk}^{(l)} := \sum_{i \in \mathbf{I}^{(l)}} \varphi_{ij}^k \tag{26}$$

and

$$\varphi_{ij}^k := \begin{cases} 1 & \text{if } X_i^k \succ_{\text{SD}} X_j^k \\ \dfrac{0.5 - \varepsilon_{ij}^k}{0.5} & \text{if } \varepsilon_{ij}^k < 0.5 \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

In the case of a tie choose any a_s minimizing the value of $\max_{k \in \mathbf{K}} \{d_{jk}^{(l)}\}$.

2. Present following data to the decision maker:
   - average evaluations of the proposal a_s with respect to criteria – μ_s^k for k = 1, …, n,
   - values of $d_{sk}^{(l)}$ for k = 1, ..., n,
   - vector of ideal values of means $\bar{\boldsymbol{\mu}}^{(l)}$.
3. Ask the decision maker whether he/she is interested in other parameters of distributional evaluations of the candidate alternative. If the answer is *yes*, ask him/her to specify the data that should be presented, calculate values of parameters and present them to the decision maker.
4. Ask the decision maker whether the evaluations of the proposal are satisfactory. If the answer is *yes*, assume a_i to be the final solution and go to (8).
5. Ask the decision maker whether the proposal is satisfactory with respect to at least one criterion. The answer *no* means that the procedure is not able to generate a satisfactory solution – go to (8).
6. Ask the decision maker to select the criterion with respect to which the proposal is satisfactory – X^k and to define δ_k – the minimal or maximal acceptable value of mean for criterion X^k.
7. Identify the set of alternatives satisfying the requirement expressed by the decision maker in step (6):

$$\mathbf{A}^{(l+1)} := \{a_j : a_j \in \mathbf{A}^{(l)}; \mu_{jk} \geq \delta_k \wedge k \in \mathbf{Q}_1 \vee \mu_{jk} \leq \delta_k \wedge k \in \mathbf{Q}_2\} \tag{28}$$

   assume $l := l + 1$, $\mathbf{K} := \mathbf{K} \setminus \{ k \}$; if $\mathbf{K} = \varnothing$, assume $\mathbf{K} := \{ 1, ..., m \}$, go to (1).
8. The end of the procedure.

K is the set of criteria that are considered when the proposal is generated. Once the decision maker accepts the evaluation of the proposal with respect to X^k, the number of this criterion is removed from **K**. If **K** is empty and satisfactory solution has not been identified, indices of all criteria are again included to **K**.

As the evaluations are represented by probability distributions, so we are not able to generate candidate action in the same way as in STEM method. We apply stochastic dominance rules instead: the distance from the ideal solution is measured by the number of alternatives with evaluations dominating the evaluation of the alternative under consideration according to stochastic dominance rules.

Two types of data are presented to the decision maker during the dialog phase of the procedure: means of distributional evaluations of a_s and values of $d_{sk}^{(l)}$, which measure the distance between the best alternative with respect to criterion X^k and alternative a_s. Thus, the decision maker is able to evaluate the proposal and decide whether he/she accepts its evaluation with respect to X^k.

If for none criterion the proposal is satisfactory, it is not possible to identify the solution of the problem by STEM-DPR. In such case there is no criterion to compromise on.
In order to define the limit of concessions for criterion X^k the decision maker is asked to define minimal (or maximal) value of mean of distributional evaluation with respect to X^k. Obviously, as the decision maker accepts the evaluation of a_i with respect to X^k, so $\delta_k < \mu_s^k$ if $k \in \mathbf{C}_1$, and $\delta_k > \mu_s^k$ if $k \in \mathbf{C}_2$.

5.3. Procedure ATO-DPR

Procedures INSDECM and STEM-DPR use direct paradigm for collecting the preference information. The decision maker defines his/her requirements specifying constraints on values of distribution parameters. Procedure ATO-DPR (***A****nalysis of* ***T****rade-***O****ffs for* ***D****iscrete Decision Making* ***P****roblems under* ***R****isk*) is based on different assumptions. Like in STEM-DPR a candidate alternative is presented to the decision maker. However, instead of defining constraints, the decision maker has to choose the criterion which should be improved and to order other criteria starting from the one that can be weakened in the first order.
ATO-DPR uses point-to-point trade-offs for generating a new proposal. For a pair of alternatives a_i and a_j and a pair of criteria X^p and X^q, a point-to-point trade-off T_{ji}^{pq} is the ratio of a relative value increase in one criterion (X^p) per unit of value decrease in the reference criterion (X^q) when a_i is replaced by a_j:

$$T_{ji}^{pq} = \frac{X_j^p - X_i^p}{X_i^q - X_j^q}. \tag{29}$$

In stochastic case trade-offs are random variables. In ATO-DPR stochastic dominance rules are used for comparing point-to-point trade-offs while identifying new proposal for the decision maker.

The initial proposal is identified in the similar way like in STEM-DPR technique. First, for each criterion stochastic dominance relations between distributional of alternatives are identified. Next, values of d_{jk} coefficients are calculated using the following formula:

$$d_{jk} := \sum_{i=1}^{n} \varphi_{ij}^k, \tag{30}$$

where φ_{ij}^{k} is calculated in the same way, like in STEM-DPR. Finally, alternative a_s for which $\overline{d}_s = \max_{k \in \{1,\ldots,n\}} \{d_{sk}\}$ is minimal is assumed to be the initial proposal.

Before starting the ATO-DPR procedure we assume: $l := 1$, $\mathbf{A}^{(1)} := \mathbf{A}$. Next, successive iterations are realized according to following scenario:

1. Ask the decision maker to specify the data he/she is interested in – the parameters of distributional evaluations (mean, standard deviation, probability of getting a value not less/not greater than ξ, etc.).
2. Compute values of parameters for each alternative under consideration; identify the best value of each parameter.
3. Present the data to the decision maker:
   - the values of parameters for the candidate alternative a_s,
   - best values of parameters attainable within the set of alternatives.
4. Ask the decision maker whether he/she is satisfied with the proposal. If the answer is *yes* – the procedure ends – the proposal is assumed to be the final solution of the problem.
5. Ask the decision maker to specify the criterion to be improved first and to set the order of the remaining criteria, starting from the one that can be decreased first. Let p be the index of the criterion to be improved, while $\{q_1, q_2, \ldots, q_{n-1}\}$ is the order of the criteria that can be decreased.
6. Identify the set of alternatives satisfying the requirements expressed by the decision maker:

$$\mathbf{A}^{(l+1)} = \left\{ a_i : a_i \in \mathbf{A}^{(l)}, a_i \neq a_s, \neg X_s^p \succ_{\text{SD}} X_i^p \right\}. \quad (31)$$

If the set $\mathbf{A}^{(l+1)}$ is empty, notify the decision maker that it is not possible to find an alternative satisfying his/her requirements, unless previous restrictions are relaxed. Next, ask the decision maker whether he/she would like to relax the previous requirements. If the answer is *no*, return to (5). Otherwise, generate the set of alternatives to be considered in the next phase of the procedure as follows:

$$\mathbf{A}^{(l+1)} = \left\{ a_i : a_i \in \mathbf{A}, a_i \neq a_s, \neg X_s^p \succ_{\text{SD}} X_i^p \right\}. \quad (32)$$

7. Assume: $\mathbf{B} = \mathbf{A}^{(l+1)}$, $k = 1$.
8. Generate probability distributions of trade-offs $T_{is}^{pq_k}$ for each i such that $a_i \in \mathbf{B}$.
9. Compare distributions of trade-offs using stochastic dominance rules and identify the set of non-dominated distributions. If the number of non-dominated distributions is equal to 1, assume the corresponding alternative to be the new proposal and go to (13).
10. Identify the alternatives with dominated trade-offs and exclude them from the set **B**.
11. If $k < n - 1$, assume $k := k + 1$ and go to (8).
12. The trade-offs for each pair of criteria have been compared, and the set of potential new proposals **B** still consists of more than one alternative. As the analysis of trade-offs hasn't provided a clear recommendation for the new proposal, analyze the relations between alternatives with respect to criteria. Start from criterion X^p

and identify the set of alternatives with non-dominated evaluations according to stochastic dominance rules. If the number of such alternatives is equal to 1, assume the corresponding alternative to be a new proposal and go to (13). Otherwise exclude from **B** the alternatives that are dominated according to stochastic dominance rules with respect to criterion X^p. Next, analyze relations with respect to other criteria. In this phase use a reversed lexicographic order of criteria: q_{n-1}, $q_{n-2}, \ldots, q_1$. For each criterion identify dominated alternatives using stochastic dominance rules and exclude them from **B**. Continue until **B** consists of one alternative. If all criteria have been considered and **B** still consists of more than one alternative, assume any of them to be a new proposal a_s.

13. Assume $l := l + 1$ and go to 1.

In ATO-DPR the decision maker has to answer very simple questions: are you satisfied with the proposal, and if not: which criterion should be improved and which criteria can be weakened. Trade-offs are used for generating a new proposal.

6. Applications

6.1. Project selection

Various objectives are usually taken into account when investment projects are analyzed. Economic desirability is undoubtedly of primarily importance. Net present value (NPV), internal rate of return (IRR), profitability index (PI), payback period (PP) and other measures are usually employed when financial analysis of a project is performed. In many cases, however, investor's considerations are not limited to economic desirability. Usually objectives reflecting technical, environmental, social, and/or political factors are also taken into account. As the decision maker tries to maximize or minimize outcomes associated with each objective depending on its nature, a multicriteria decision making problem is constituted.

Criteria for project comparison often differ in nature. While financial criteria are quantitative, others are qualitative ones. If, for example, an engineering project is considered, various technical factors of qualitative kind are taken into account, including the level of technological novelty, compatibility with existing facilities, reliability and technical service. A similar situation takes place when social and environmental consequences are examined. While some criteria are quantitative (the volume of pollutants, the area of degraded land, etc.), others are qualitative (changes in landscape, changes in the way of life of the neighboring population, etc.).

When faced with the decision of selecting engineering, construction or R&D project, the decision maker has also to face uncertainty. Project evaluation involves prediction of future outcomes. In the real world, however, not all predictions are known with certainty. Even experts are sometimes wrong in their assessments. In addition, various experts often differ in their opinions on the same project. Thus, risk associated with at least some objectives has to be considered when projects are evaluated.

The project selection problem can be formulated as a discrete multicriteria decision making problem, in which we have:

- the set of projects (decision alternatives) **A**,
- the set of criteria **X**,
- the set of evaluations of projects with respect to criteria **E**.

The set of criteria groups both quantitative and qualitative ones. For example following measures can be employed for evaluating projects:

- Net Present Value (NPV),
- Internal Rate of Return (IRR),
- technical novelty,
- reliability and technical service,
- compatibility with existing facilities.

While NPV and IRR are quantitative criteria, next three are qualitative. A systematic procedure that can be used for solving the problem consists of the following steps:

1. Identification of project proposals.
2. Choosing the criteria.
3. Collecting the data and generating evaluations of alternate projects with respect to criteria.
4. Selecting the project to be realized.

The way in which the evaluations of alternatives are generated depends on the criteria nature. For financial criteria computer simulation can be employed. Various risk factors can be taken into account in a simulation model. For example, when a construction or manufacturing project is analyzed, uncertainties related to availability of resources, market prices, or demand can be considered. On the other hand, in projects with R&D elements activity durations are much more sensitive to incorrect evaluation. In such cases simulation may provide the dates of the milestones of the project, which determine the set of cash-flows during the life cycle of the project. On the other hand, experts' judgments are usually taken into account when projects are evaluated with respect to qualitative criteria. Let's assume that each project a_i is evaluated by l experts with respect to criterion X^k on a specified scale. Such scale can be defined, for example, as a 10-point one, with 1 assigned to the least desirable and 10 to the most desirable output. As a result, l evaluations are obtained for each project. Assuming equal probabilities of each assessment, a distributional evaluation is achieved. Such distribution, however, differs from the one obtained in simulation, as qualitative criteria are measured on ordinal scale.

Once, the knowledge base necessary for evaluating projects has been generated, the last step of the procedure – final selection of the project – can be carried out. Interactive techniques presented in the previous section can be employed for this. In such case FSD/SSD rules should be employed for comparing evaluations of projects with respect to financial criteria, and OFSD/OSSD rules for analyzing relations between alternatives with respect to qualitative measures.

To illustrate the procedure let us consider a manufacturing company operating in a growth market. The management board decided to purchase a new production facility to increase production capacity. Ten alternative projects are considered. All proposals are viable: that is, the output from any of these alternatives meets product specification.

The decision for selecting a project has to be made based on net present value for each project, in addition to three other objectives identified in Step 1 below. The economic life for all projects is assumed to be 5 years. Based on past experience and data provided by the manufacturers of facilities, analysts have determined the probability distributions for: initial investments, salvage values, production costs per unit, fixed costs, demand, market prices.

The decision maker decided to consider the following criteria:
X^1 – net present value,
X^2 – reliability and technical service,
X^3 – technical novelty,
X^4 – compatibility with existing facilities.

Simulation technique has been applied for generating distributional evaluations of projects with respect to attribute X_1. Expert assessments are used for constructing distributional evaluations of the projects with respect to criteria X_2, X_3, X_4. Ten analysts assessed each proposal on the scale from 1 to 10.

FSD/SSD rules are applied for comparing projects with respect to criterion X_1, while OFSD/OSSD rules are employed when projects are analyzed with respect to criteria X_2, X_3 and X_4.

Before starting the dialog procedure, efficient alternatives are identified. Project a_2 is not efficient – its evaluations are dominated by the corresponding evaluations of a_7, a_8, and a_9 with respect to all attributes.

We use INSDECM procedure to identify the final solution of the problem. The dialog with the decision maker goes as follows:

Initial phase:

1. $l := 1$, $\mathbf{A}^{(1)} := \{ a_1, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10} \}$.
2. The decision maker decides that means should be presented during the dialog procedure.

***Iteration 1*:**

1. Potency matrix is constructed and presented to the decision maker (see Table 1).

Table 1. Potency matrix $\mathbf{P}^{(1)}$

$\mathbf{P}^{(1)}$	X^1	X^2	X^3	X^4
worst value	979.66	2.8	3.4	3.9
best value	1432.72	7.9	7.5	8.2

2. The decision maker is satisfied with the information presented.
3. The decision maker is not satisfied with pessimistic values.
4. The decision maker formulates additional requirement:
   The average evaluation with respect to X^1 not less than 1000: $\mu_i^1 \geq 1000$.
5. Alternatives satisfying the requirement are identified:
   $A^{(2)} = \{ a_1, a_3, a_4, a_6, a_7, a_8, a_9, a_{10} \}$.

6. New potency matrix is generated and presented to the decision maker (see Table 2). The decision maker accepts the move from $\mathbf{P}^{(1)}$ to $\mathbf{P}^{(2)}$.

Table 2. Potency matrix $\mathbf{P}^{(2)}$

$\mathbf{P}^{(2)}$	X^1	X^2	X^3	X^4
worst value	1137.93	2.8	3.4	3.9
best value	1432.72	7.9	7.5	8.2

7. For none pair of alternatives inconsistency between stochastic dominance rules and the requirement defined by the decision maker is identified.
8. As $\mathbf{N}^{(l)} = \emptyset$, so $l := 2$ and procedure goes to the next iteration.

Procedure operates until the decision maker is satisfied with pessimistic values of criteria.

6.2. Labor planning

Labor planning is concerned with determining staffing policies that deal with employment stability and work schedules. A staffing plan is a managerial statement of time-phased staff size and labor-related capacities, which takes into consideration customer requirements and machine-limited capacities. Such plan has to balance conflicting objectives involving customer service, work-force stability, cost, and profit.

Various techniques are employed for solving labor planning problems. Linear programming and dynamic programming are used most often. However, these approaches are based on strong assumptions that often are not satisfied. Employees' attainability varies due to planned and unexpected absences. Work-force requirements are not stable as well. Often, considerable fluctuations can be noticed even in short-term. In accounts or payroll departments, for example, work-force requirements are usually higher in the early part of the month than in the latter one.

Let us assume that the decision problem consists in determining the number of full-time and part-time employees for a department in which work-force requirements fluctuate during each month. In order to meet requirements both full-time and part-time employees can be hired. Overtime can also be used to satisfy work-force requirements that cannot be completed in regular time. However, overtime is expensive. According to Polish Labor Code, 50% bonus has to be paid if overtime work is done on working day, while 100% bonus is to be paid for working on Saturdays, Sundays and holidays. Additionally, the number of overtime hours worked by an employee is limited to 150 per year. Moreover, in many cases workers do not want to work a lot of overtime for extended period. Finally, increased utilization of overtime may lead to decreased productivity due to employees' tiredness. If work-force requirements fluctuations are considerable, employees' working hours may not be fully utilized in some periods. Such situation is inconvenient, as it results in the labor costs increase. It is also unfavorable from psychological point of view. Balancing various objectives in order to arrive at an acceptable staffing plan involves consideration of various decision alternatives.

The decision problem considered here consists in determining the number of full-time and part-time employees. The set of decision alternatives groups staffing plans under consideration. Following criteria are used:
X^1 – yearly labor costs,
X^2 – total number of overtime hours worked by all employees in the department during the year,
X^3 – work-force utilization rate measured by the contribution of regular hours effectively worked in the total number of regular hours worked by employees.

In order to solve the problem, alternatives have to be evaluated with respect to criteria. Simulation technique is an efficient and flexible tool for doing this. As a result, distributional evaluations of alternatives with respect to criteria are obtained.

The final solution of the problem can be identified using interactive technique. In Nowak (2008a) INSDECM procedure was used for this. However, STEM-SPR and ATO-DPR can also be used.

6.3. Project planning

Several criteria have to be considered while preparing a project schedule. The completion time and project cost are analyzed in most cases. Additionally, the risk related to the criteria has to be taken into account as well. Thus, project planning problem can be defined as a multicriteria decision problem under risk.

Usually various resources can be used to complete project activities. Let us assume here, that only a finite number of resource allocations can be considered. For example, one, two or three workers can be employed to complete an activity. Thus, we face a discrete decision making problem, in which the decision alternatives are defined by resource allocations.

The completion time depends on the resources allocated to the activity. Let‘s assume that for each activity and for each alternate resource allocation, three completion time estimates are known: optimistic, most probable and pessimistic. We also suppose that the relations between the time and cost are recognized for each activity. For example, knowing the wage per hour paid to a worker and the completion time, we are able to calculate labour cost of the activity. Similarly the cost of other resources can be estimated. As the activity times are uncertain, so the project completion time and project cost are uncertain as well.

The decision situation considered here paper may be conceived as a problem (**A**, **X**, **E**). The set of alternatives **A** consists of alternate resource allocations. Two criteria are used for evaluating alternatives:
X^1 – project completion time,
X^2 – total cost.

As activities‘ completion times are random, so computer simulation can be used for generating distributional evaluations of alternate resource allocations with respect to criteria. As a result, we face a discrete decision making problem under risk. In order to identify the final solution, interactive procedures INSDECM, STEM-DPR or ATO-DPR can be used. In Błaszczyk and Nowak (2009) INSDECM is employed for solving the project planning problem.

6.4. Other applications

Applications of the procedures presented in this paper are not limited to the ones discussed above. Interactive approach can also be imployed for example in aggregate production planning, production process control, inventory management. If only the decision situation can be described as a discrete multicriteria decision making problem under risk with up to moderate alternatives, procedures INSDECM, STEM-DPR and ATO-DPR can provide an effective method for identifying the final solution of the problem.

7. Conclusions

Interactive approach is one of the leading methodologies in multicriteria decision making. Several motivations have been mentioned for implementing this approach. It is usually pointed out that limited amount of a priori preference information is required from the decision maker as compared to other techniques. The interactive procedure may be considered as a learning process. Observing the results of succeeding iterations of the procedure, the decision maker extends his/her knowledge of the decision problem. On the other hand, as the decision maker actively participates in all phases of problem solving procedure, he/she puts much reliance on the final solution. As a result, the solution of the procedure has a better chance of being implemented.

Two main issues have to be considered when an interactive procedure is designed: the way in which the information is presented to the decision maker, and the way in which the decision maker formulates his/her judgments. When only limited information is provided, the decision maker may feel that he/she is not able to analyze important aspects of the problem. Thus, providing the information that the decision maker finds interesting may be the beneficial. On the other hand, however, enabling the decision maker to define his/her requirements in various forms may also be profitable. These issues are especially important in stochastic environment. As the evaluations of alternatives are represented by probability distributions, so the comparison of alternatives is not trivial. On one hand the decision maker is usually interested in maximizing expected outcomes, on the other however, he/she finds the variability of outcomes very important as well. As each decision maker recognizes risk in his own way, so various risk measures should be provided to satisfy his/her demands.

In the paper interactive procedures for discrete stochastic multiple criteria choice problem are suggested. The methodology combines two concepts: interactive approach and stochastic dominance rules used for comparing uncertain evaluations of alternatives with respect to criteria. The interaction process between the decision maker and the decision model includes presentation the information for the decision maker, asking the decision maker for defining additional requirements, and enabling him/her to choose a final solution if he/she is able to do this.

Procedures are designed for various types of decision makers. Those, who are experienced in using multicriteria methods, can use INSDECM technique. It is the most demanding, as it requires defining constraints on values of various parameters of distributional evaluations. STEM-DPR and especially ATO-DPR can be employed by persons, who are less experienced. These methods require a limited amount of preference information from the decision maker. During the dialog phase, he/she has to answer very simple, easy understandable questions.

Procedures presented in this work can also be applied for mixed problems, i.e. problems in which evaluations with respect to some criteria are represented by probability distributions, while the rest are deterministic.

Acknowledgements

This work has been supported by Polish Ministry of Science and Higher Education, as a part of scientific project number NN111 235036.

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INTERAKTYVIŲ DAUGIAKRITERINIŲ SPRENDIMŲ PASISKIRSTYMAS RIZIKOS SĄLYGOMIS: METODAI IR SPRENDIMAI

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Santrauka

Straipsnyje pateikiama diskrečiųjų sprendimų priėmimo problemos analizė apimant rizikos veiksnius. Pasirinkimo alternatyvos suprantamos kaip kompleksas baigtinių elementų ir alternatyvos, atsižvelgiant į kriterijus, yra išreikštos paskirstymo funkcijomis. Priimant sprendimus būtina atrasti išeitį, priimtiną tolesniems sprendimams. Straipsnyje siūloma, kad, norint išspręsti vieną problemą, būtina išanalizuoti sprendimų vertintojo lūkesčius.

Reikšminiai žodžiai: daugiakriterinis vertinimas, interaktyvus metodas, sprendimų priėmimas rizikos sąlygomis, stochastinis pasiskirstymas, valdymo sprendimai.

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