JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
2008
14(3): 177–182

CRACKING SHEAR STRENGTH OF RC SLENDER BEAMS WITHOUT STIRRUPS

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Received 6 May 2008; accepted 14 July 2008

Abstract. This study presents alternative cracking shear strength equations for slender reinforced concrete (RC) beams without stirrups. More than 80 data has been obtained from existing sources of RC beam shear test results covering a wide range of beam properties and test methods. The proposed cracking shear strength equations are applied to existing test data for normal strength concrete (NSC) and high-strength concrete (HSC) slender beams and the results are compared with those predicted by the ACI 318 equations. It can be also noted that the test results are in better agreement with proposed cracking shear strengths. However, because the test data for high-strength concrete members are very limited, further research is required to verify these equations.

Keywords: compressive strength, reinforced concrete, cracking, shear strength, slender beam, dowel action, diagonal tension.

1. Introduction

The shear strength in steel-reinforced concrete members has been the subject of many controversies and debates since the beginning of the 20th century. Still, the question of shear strength prediction of reinforced concrete members without stirrups is far from being settled (Rebeiz 1999). Many equations have been proposed to estimate the ultimate and cracking shear strength of reinforced concrete (RC) beams. This study presents alternative cracking shear strength equations for RC slender beams without stirrups. A significant scatter exists between the ACI 318 Building Code equation for predicting cracking shear strength and experimental cracking shear strength values (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003).

The ACI 318 Building Code contains equation for shear strength of slender beam without stirrups, subject only to shear and flexure, ACI 318 Building Code equation (*ACI Committee 318* 2002):

$$v_c = \frac{1}{7}\left(\sqrt{f_c} + 120\rho\frac{V_u d}{M_u}\right) \le 0.3\sqrt{f_c}, \quad (\text{MPa}) \quad (1)$$

$$(a/d \ge 2.5).$$

In ACI 318 Building Code equation, the cracking shear strength of RC beams without stirrups is mainly dependent on the compressive strength of concrete, longitudinal reinforcement ratio and shear span-to-depth ratio. The shear capacity of a beam without stirrups is divided into a concrete contribution and a longitudinal reinforcement contribution. Eq (1) is typically simplified into the following:

$$v_c = \frac{V_c}{b_w d} = \frac{1}{6}\sqrt{f_c}, \ (\text{MPa}) \ (a/d \ge 2.5). \quad (2)$$

with v_c in MPa; f_c in MPa; b_w and d in mm. The current ACI 318 Building Code assumes that shear strength is essentially proportional to $f_c^{0,5}$.

ACI 318 Building Code simplified Eq (2) is the basic expression for shear strength, while Eq (1) is an approximation often used in design practice. Both equations are based on the assumption that the useful shear strength of a beam without stirrups is exhausted when inclined cracking first develops.

With respect to various empirical formulas, a considerable differences exist as a result of the following factors: the uncertainty in assessing the influence of complex parameters in a simple formula; the scatter of the selected test results due to inappropriate tests being considered; a poor representation of some parameters in tests; and finally, the concrete tensile strength is often not being evaluated by control specimens. These issues limit the validity of empirical formulas and increase the necessity for rational models and theoretically justified relationships (*ASCE-ACI Committee 445* 1998).

The main original contribution of this study is to present briefly a simplified cracking strength equation of RC slender beams without stirrups. Zsutty's equations (Zsutty 1968) are only valid for the ultimate shear strength; they do not apply to the cracking shear strength. In the same way, the ACI 318 Building Code equations are only valid for the cracking shear strength; they should not apply to the ultimate shear strength. In this respect, the proposed equation is verified by the test data for cracking shear strength of RC beams without stirrups reported in the literature (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003) and compared with ACI 318 Building Code equation.

ISSN 1392–3730 print / ISSN 1822–3605 online
DOI: 10.3846/1392-3730.2008.14.14

In short beams where a/d is shorter than 2.0~3.0, applied loads are transmitted directly to the supports by arch action. The main factors influencing this action are shear span-to-depth ratio, compressive strength of concrete and area of tension reinforcement. According to Khuntia and Stojadinovic (2001), a cross-section plane before loading remains plane under load. This means that the longitudinal strain distribution at a cross-section is linear. Thus, this model is not applicable to very deep members (with shear span-to-depth ratio of less than 2), for which the longitudinal strain distribution is not linear.

In this study, for a slender RC beam where a/d is greater than 2.0, the cracking shear force at section is mainly resisted by the tensile strength of concrete and dowel action. The proposed equations compare favourably with the results of high-strength concrete (HSC) beams with compressive strength of $f_c \geq 41.4$ MPa (6000 psi), and normal-strength concrete (NSC) beams with lower f_c values reported in the literature. The experimental cracking shear strength values (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003) show that the mean values for the ratio of the ACI 318 Building Code simplified equation to the experimental cracking shear strength values (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003) are 0.73 for NSC and 0.76 for HSC, respectively. Since current design provisions were found to be conservative in predicting the cracking shear strength capacity of RC slender beams without stirrups, new equations are presented to allow a more accurate estimate of cracking shear capacity of such beams (Arslan 2005).

2. Development of new cracking shear strength prediction equations

It is generally accepted that the shear failure of RC members without stirrups initiates, when the principal tensile stress within the shear span exceeds the tensile strength of concrete and a diagonal crack propagates through the beam web (Khuntia and Stojadinovic 2001). Therefore, the diagonal tensile cracking strength depends directly on the tensile strength of concrete. In the experiments, a diagonal crack is defined as a major inclined crack, extending from the level of the longitudinal reinforcement towards the application point of the load and the load at the growth of this first inclined crack is termed as the diagonal tension-cracking load. Taylor (1960) indicated that the diagonal cracking stage was not clearly defined in the experimental beams, where the crack formed close to the applied load because the development of the inclined cracks was gradual. Since the diagonal cracking load is very sensitive to the observer's judgment of and the location of the initiating flexural crack there is a large scatter of the values experimentally determined (Bazant and Kazemi 1991). Therefore, it is difficult to determine the value of the diagonal tension stress and the cracking load in a reinforced concrete beam because the distribution of shear and flexural stresses is not known with certainty. Furthermore, the crack initiation load is not proportional to the failure load and it can be much smaller or only slightly smaller depending on the beam size and other factors (Bazant and Kazemi 1991).

In order to calculate the cracking shear strength of a reinforced concrete beam without stirrups, we must answer the following questions: What is the magnitude of shear resistance over the effective cross-section and how can we determine the effective shear depth and shear stress distribution?

According to Khuntia and Stojadinovic (2001), the shear stress distribution is modeled as parabolic over the effective shear depth with the maximum value at the neutral axis. Thus, the magnitude of shear resistance over the effective cross-section equals $\tau_{max} = V_{cr}/(2/3 b_w k_1 d)$, in which b_w is the width of section, $k_1 d$ is the effective shear depth and τ_{max} is the shear stress at the neutral axis. The shear failure of RC members without stirrups initiates when the principal tensile stress within the shear span exceeds the tensile strength of concrete and a diagonal crack propagates thorough the beam web. Mathematically,

$$\tau_{max} = \frac{V_{cr,t}}{\frac{2}{3} b_w k_1 d} = f_t , \qquad (3)$$

in which f_t and $V_{cr,t}$ are the tensile strength of concrete and diagonal tension cracking shear force, respectively.

$$V_{cr,t} = \frac{2}{3} f_t b_w k_1 d = v_{cr,t} b_w d . \qquad (4)$$

The proposed simplified procedure for shear strength of reinforced concrete members without stirrups, at the design section under the factored bending moment M_u and axial load P_u, calculate the effective shear depth $k_1 d$, using the method of satisfaction of strain compatibility and equilibrium conditions (Khuntia and Stojadinovic 2001):

$$k_1 d = kd\left(1 + \frac{\varepsilon_{cr}}{\varepsilon_c}\right), \qquad (5)$$

in which kd is the depth of neutral axis, ε_c is the compressive strain in concrete and taken as 0.003.

The longitudinal reinforcement ratio has a pronounced effect on the basic shear transfer mechanisms. An important factor that affects the rate, at which a flexural crack develops into an inclined one, is the magnitude of shear stresses near the tip of that crack. The intensity of principal stresses above the flexural crack depends on the depth of crack penetration. The greater the value of ρ, the less the flexural crack penetration, the less the principal stresses for a given applied load, and consequently the greater must be the shear to cause the principal stresses that will result in diagonal tension cracking (Elzanaty *et al.* 1986).

According to the classical bending theory of reinforced concrete beams with only tensile reinforcement and a negligible tensile capacity of concrete,

$$k = (n^2\rho^2 + 2n\rho)^{1/2} - n\rho , \qquad (6)$$

where $n = E_s / E_c$ is a ratio of elastic modulus of steel and concrete. Eq (6) is, however, unnecessarily complicated and may be replaced by following simpler expression (Kim and Park 1994):

$$k = 0.82(n\rho)^{0.36} . \quad (7)$$

Within the practical range, i. e., $5 \le n \le 10$ and $0.005 \le \rho \le 0.035$; consequently, $0.025 \le n\rho \le 0.35$.

During the formation of primary cracks and for a reinforcing ratio ρ less than a limiting value ρ_{stbl} , the average strains increase until a stabilized cracking state is reached tension-softening stress in concrete at cracking. If ρ is the reinforcement ratio, and $n = E_s / E_c$ is the modular ratio, the minimum reinforcement ratio required to maintain constant strain at the crack, when the cracking load is applied to the member and held constant, is (Massicotte *et al.* 1990):

$$\rho_{stbl} = \frac{1}{6n} . \quad (8)$$

According to Massicotte *et al.* (1990), Eq (8) can be also interpreted as the minimum steel ratio needed for a test setup to measure accurately the tension-softening branch in a plain concrete tension test under a load-controlled procedure.

Since ρ_{stbl} is expressed by the minimum reinforcement ratio required to maintain constant strain at the crack, the corresponding limit value for the shear strength capacity of the diagonal tension crack of slender beams can also be interpreted as ρ_{stbl} equal to ρ (in Eq (8)).

The cracking strain, ε_{cr} , in concrete is taken as the ratio of the tensile strength of concrete f_t to its modulus of elasticity E_c :

$$\varepsilon_{cr} = \frac{f_t}{E_c} . \quad (9)$$

The tensile strength of plain concrete f_t ranges from about 0.25 to $0.50\sqrt{f_c}$ (Carreira and Chu 1986; Nilson and Winter 1991; Paulay and Priestley 1992). In this study, the direct tensile strength is accepted as $0.50\sqrt{f_c}$ for normal strength concrete and $0.40\sqrt{f_c}$ for high strength concrete. The modulus of elasticity E_c is taken as $4750\sqrt{f_c}$ (Ersoy and Özcebe 2001).

Substituting Eqs (9), (8), (7) and (5) into Eq (4), we obtain:

for normal strength concrete (NSC) $v_{cr,t} = 0.15\sqrt{f_c}$, (10a)

for high strength concrete(HSC) $v_{cr,t} = 0.12\sqrt{f_c}$. (10b)

The question of what mechanisms of shear transfer will contribute most to the resistance of a particular beam is difficult to answer (*ASCE-ACI Committee 445* 1998). Recent works (Vintzeleou and Tassios 1986; Vintzeleou and Tassios 1987) have reaffirmed the well-known work by Baumann and Rüsch on the resistance of dowels near a surface (*ASCE-ACI Committee 445* 1998).

Normally, dowel action is not very significant in members without transverse reinforcement, because the maximum shear in a dowel is limited by the tensile strength of the concrete cover supporting the dowel (*ASCE-ACI Committee 445* 1998). According to Vintzeleou and Tassios (1986, 1987), the dowel force is represented as

$$V_{cr,d} = k_2 b_{ct} d_b f_t , \quad (11)$$

where b_{ct} is the net width of section, d_b is the bar diameter and k_2 is a constant. Assuming the number of bars is not changed, the dowel force can be expressed approximately by more general terms ρ as follows (Kim and Park 1994):

$$V_{cr,d} = k_3 (f_c^{'})^{0,5} \rho^r b_w d , \quad (12)$$

where k_3 is a constant and r varying from 0.3 to 0.5 in practical range, is a parameter that depends on spacing of reinforcement. Substituting Eq (8), and $n = E_s / E_c$ into Eq (12) and assuming that the modulus of elasticity of reinforcement is $E_s = 200$ GPa and $r = 0.3$, the dowel strength can be expressed approximately by more general terms f_c :

$$v_{cr,d} = 0.02(f_c)^{0.65} . \quad (13)$$

As previously discussed, the shear strength of a diagonal tension crack of slender beams may be represented by Eq (14):

$$v_{cr} = v_{cr,t} + v_{cr,d} . \quad (14)$$

Substituting Eq (10) and Eq (13) into Eq (14), we obtain:

for NSC $v_{cr} = 0.15(f_c)^{0.5} + 0.02(f_c)^{0.65}$; and (15a)

for HSC $v_{cr,t} = 0.12\sqrt{f_c}$. (15b)

3. Evaluation of proposed equation

The effects of compressive strength, shear span-to-depth ratio and the percentage of longitudinal tension reinforcement ratio on the proposed cracking shear strength and ACI 318 Building Code equations are discussed as follows.

Fig. 1 compares the proposed cracking shear strength obtained from Eq (15) with the experimental cracking shear strength values obtained from tests (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003). It can be also noted that the test results are in better agreement with a proposed cracking shear strength. However, because the test data for high-strength concrete members are very limited, further research is required to verify the proposed equations.

Provisions for shear design in the ACI 318 Building Code equations are based mainly on experimentally derived equations. Tests, providing the basic data for these equations, were conducted on members with concrete strength members with concrete strength mostly below 41.4 MPa (6000 psi) (Elzanaty *et al.* 1986).

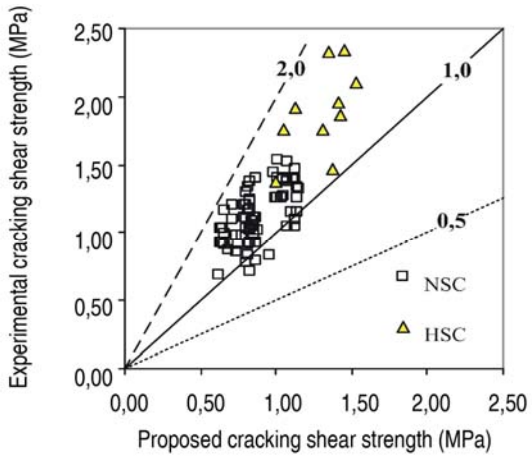

Fig. 1. Proposed cracking shear strength values using Eq (15) versus experimental cracking shear strength values

Fig. 2 compares the cracking shear strength obtained from ACI 318 Building Code Eq (2) with the experimental cracking shear strength values. Most of the values, obtained from the tests, are larger than the cracking shear strength values obtained from ACI 318 Building Code Eq (2). The mean values for the ratio of the ACI 318 Building Code equation to the experimental cracking shear strength values (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003) and standard deviations are 0.73 and 0.12 for NSC, 0.76 and 0.13 for HSC, respectively.

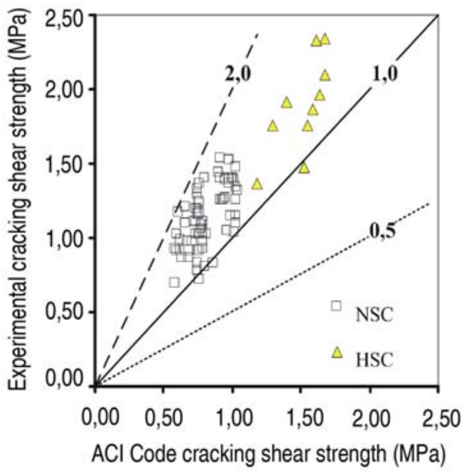

Fig. 2. Cracking shear strength values using ACI Code versus experimental cracking shear strength values

Fig. 3–5 compares the proposed cracking shear strength obtained from Eq (15) with those derived from the ACI 318 Building Code cracking shear strength for various values of shear span-to-depth ratio, flexural reinforcement ratio and concrete compressive strength. It can be observed that the ACI 318 Building Code cracking shear strength prediction is much more conservative than the proposed equations, particularly at high values of shear span-to-depth ratio (Fig. 3) and the values of flexural reinforcement ratio (Fig. 4). In addition, the difference between the ACI 318 Building Code shear strength prediction values and the proposed cracking shear strength seem to be more pronounced and more erratic in the case of NSC, as shown in Fig. 5. However, more data on HSC are needed to verify this point.

The strength of members with low reinforcing ratios was rarely investigated in the past and is often overestimated in the present codes (*ASCE-ACI Committee 445* 1998). The test results of the cracking shear strength of slender beams with low reinforcing ratios are very limited ($\rho < 1.0\%$), consequently, further research is required to verify the proposed equations.

Figs 6–8 show the errors which can be induced by the discrepancy of ρ, f_c and a/d between test and proposed cracking shear strength, the comparison was made with test results. The ratio of experimental to proposed cracking shear strength is not significantly influenced by increasing ρ, f_c and a/d, but shear test data are not homogeneous in respect of slender beams.

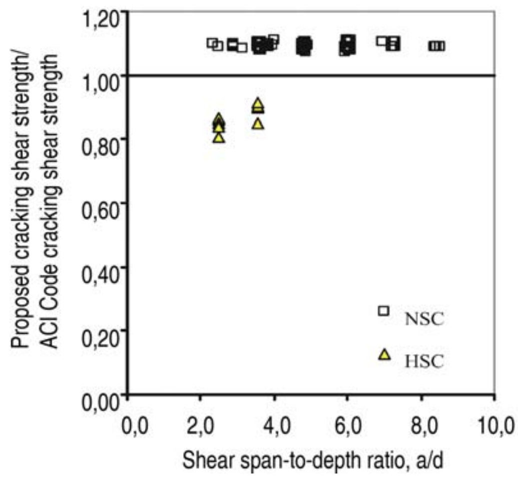

Fig. 3. Comparing proposed cracking shear strength of Eq (15) with ACI Code for various shear span-to-depth ratios

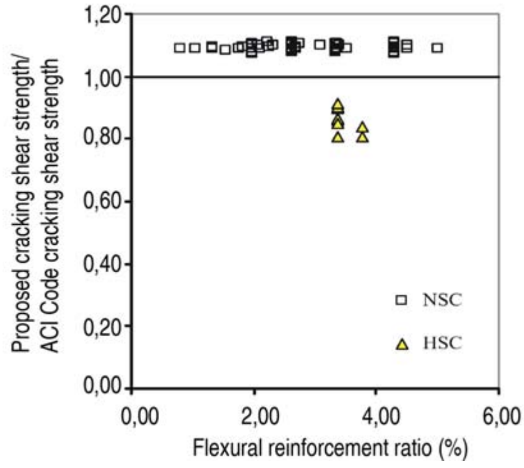

Fig. 4. Comparing proposed cracking shear strength of Eq (15) with ACI Code for various flexural reinforcement ratios

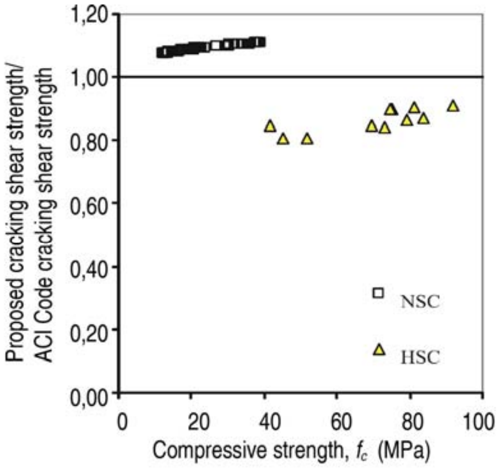

Fig. 5. Comparing proposed cracking shear strength of Eq (15) with ACI Code for various compressive strength values

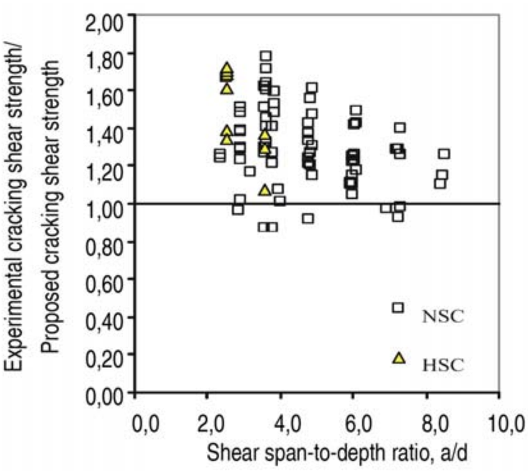

Fig. 6. Comparing experimental cracking shear strength values with proposed cracking shear strength of Eq (15) for various shear span-to-depth ratios

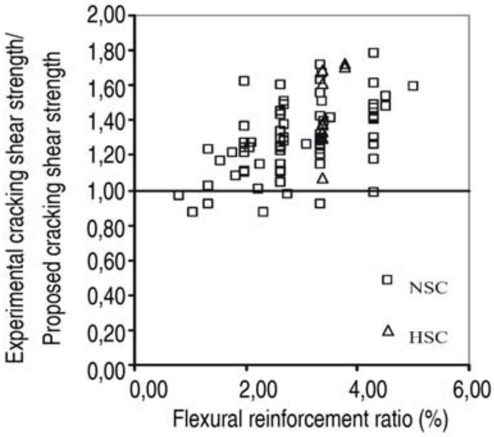

Fig. 7. Comparing experimental cracking shear strength values with proposed cracking shear strength of Eq (15) for various flexural reinforcement ratios

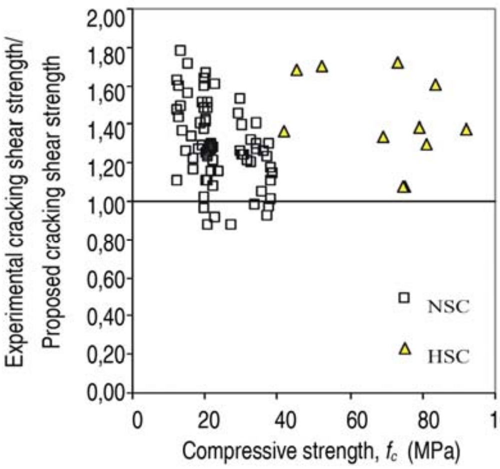

Fig. 8. Comparing experimental cracking shear strength values with proposed cracking shear strength of Eq (15) for various compressive strength values

4. Conclusions

The main purpose of this study is to present briefly a simplified equation of cracking shear strength of RC beams and to compare test results reported in the literature (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003) with the current ACI Code provisions. The most important conclusions may be summarized as follows:

1. The comparison of calculated and experimental results (Bresler and Scordelis 1963; Krefeld and Thurston 1966; Mphonde and Frantz 1984; Cho 2003) shows that the proposed shear strength equations can predict with satisfactory accuracy cracking shear strength capacity of RC slender beams without stirrups. However, because the test data for high-strength concrete members are very limited, further research is required to verify the proposed equations.

2. The ratio of experimental to proposed cracking shear strength is not significantly influenced by increasing ρ, f_c and a/d, but shear test data are not homogeneous in respect of slender beams.

3. In the mode of cracking shear strength, proposed equations performed almost as well as ACI 318 Building Code simplified equation (according to the coefficient of variation) and the ACI 318 Building Code cracking shear strength prediction values tend to be more conservative than obtained from proposed equations (mean value of 0.80 for the ratio proposed / test data and mean of 0.73 for the ratio ACI Code proposed / test data).

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Notations

List of symbols

ρ	longitudinal reinforcement ratio,
ε_c	compressive strain in concrete,
ε_{cr}	cracking strain value in concrete,
$\tau_{\max}$	maximum shear stress in effective shear depth, (MPa),
a	shear span, (mm),
a/d	shear span-to-depth ratio,
b_{ct}	net width of section, (mm),
b_w	width of web, (mm),
d	effective depth, (mm),
d_b	bar diameter, (mm),
E_c	modulus of elasticity of concrete, (MPa),
E_s	modulus of elasticity of steel, (MPa),
f_c	compressive strength of concrete, (MPa),
f_t	tensile strength of concrete, (MPa),
$k_1,\ k_2,\ k_3$	constants,
$k_1 d$	effective shear depth, (mm),
kd	depth of neutral axis, (mm),
M_u	ultimate bending moment, (Nmm)
n	ratio of elastic modulus of steel to concrete,
v_{cr}	cracking shear strength, (MPa),
$v_{cr,d}$	dowel strength, (MPa),
$v_{cr,t}$	diagonal tension cracking shear strength, (MPa)

SANKABOMIS NEARMUOTŲ LIAUNŲ GELŽBETONINIŲ SIJŲ ATSPARUMAS SKERSINEI JĖGAI

G. Arslan

S a n t r a u k a

Pateikta alternatyvi sankabomis nearmuotų liaunų gelžbetoninių sijų pleišėjimo stiprio įstrižajame pjūvyje skaičiavimo formulė. Surinkta daugiau nei 80 gelžbetoninių sijų eksperimentinių duomenų, kurie apima platų geometrinių matmenų, medžiagų savybių ir bandymo metodų spektrą. Šiai duomenų imčiai atliktas lyginamasis statistinis pleišėjimo stiprio įstrižajame pjūvyje skaičiavimas taikant pasiūlytąją formulę bei ACI 318 projektavimo normų priklausomybę. Derėtų pabrėžti, kad pasiūlyta išraiška gautas geresnis teorinių ir eksperimentinių skaičiavimo rezultatų sutapimas. Vis dėlto aukšto stiprio betono sijų eksperimentinių duomenų kiekis yra ribotas. Todėl pasiūlyta priklausomybė turėtų būti tobulinama, tiksliau įvertinama betono stiprio įtaka.

Reikšminiai žodžiai: gniuždomasis stipris, gelžbetonis, pleišėjimas, atsparumas skersinei jėgai, liauna sija, kaiščio efektas, įstrižasis tempimas.

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