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## LAYERED APPROACH FOR LONG-TERM DEFLECTIONS ANALYSIS OF REINFORCED CONCRETE BEAMS

D. Bačinskas, G. Kaklauskas, P. Vainiūnas

Vilnius Gediminas Technical University

### 1. Introduction

With the present trend of use of high-strength concrete and reinforcement, leading to longer spans and smaller depths, serviceability (deformations, deflections and crack width), but not the strength requirements often are the governing structural design criterion.

A new statistically verified constitutive model [1] has been recently developed for deformational analysis of flexural reinforced concrete members subjected to short-term loading. The model (called the *integral constitutive model*) has been developed by means of innovative method [1, 2] aimed at deriving constitutive relationships from flexural tests of reinforced concrete members. An important part of the model is a constitutive relationship for cracked tensile concrete which greatly influences numerical results. The proposed constitutive relationship in a simple averaging manner (convenient for finite element formulation) includes concrete cracking and tension stiffening effects. For the first time, a quantitative relationship relating the shape of the constitutive relationship with the reinforcement ratio has been proposed [1, 2].

Along with cracking, non-linear time effects of creep and shrinkage provide the major concern to the structural designer because of the inaccuracies and unknowns that surround them.

In this paper, an attempt has been made to extend application of the integral constitutive model to long-term deformational analysis of flexural reinforced concrete members. Constitutive relationships for compressive concrete and cracked tensile concrete used in the analysis are presented. A deformation calculation technique based on age-adjusted effective modulus method [3], a relaxation procedure [4] and layered approach is briefly described. Statistical deflection calculation analy-

sis has been carried out for a large number of experimental beams.

### 2. Constitutive relationships

#### 2.1. Concrete in compression

Many stress-relationships for compressive concrete based on short-term uniaxial tests has been proposed by different authors. Among some widely used expressions reviewed in [1], one proposed by Hognestad [5] was developed in an experimental and analytical study on eccentrically loaded short reinforced concrete columns.

In this study, a modified Hognestad's relationship shown in Fig 1 was used for modelling long-term behaviour of compressive concrete. The ascending branch of the relationship has the form:

$$f_c(t) = f_{c,\max}(t) \left[ \frac{2\varepsilon_c(t)}{\varepsilon_0(t)} - \left( \frac{\varepsilon_c(t)}{\varepsilon_0(t)} \right)^2 \right], \quad (1)$$

where  $f_{c,\max}(t)$  is the maximum compressive stress and  $\varepsilon_0(t)$  is the corresponding strain.

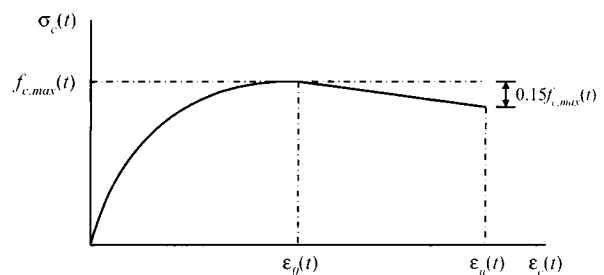


Fig 1. Stress-strain relationship for concrete in compression under long-term loading

It is well known that strength of compressive concrete decreases in the case of sustained [6] or cyclic [7] loading. In the present study, this effect is

accounted for by factor  $\eta$  proposed by Yashin [6]:

$$\eta = f_{c,\max}(t)/f_c(t_0) = 0.92 - 0.04 \log(t - t_0), \quad (2)$$

where  $t_0$  is time at loading (days);  $t$  is time under consideration;  $f_c(t_0)$  is short-term compressive strength at time  $t_0$ .

The strain  $\varepsilon_0(t)$  corresponding to the maximum stress and the limit strain  $\varepsilon_u(t)$  (see Fig 1) are determined as follows:

$$\varepsilon_0(t) = \frac{2f_{c,\max}(t)}{\bar{E}_e(t, t_0)}, \quad (3)$$

$$\varepsilon_u(t) = 1.9\varepsilon_0(t), \quad (4)$$

where  $\bar{E}_e$  is the age-adjusted effective modulus discussed in section 3.

## 2.2. Concrete in tension (tension stiffening)

Due to bond with reinforcement, cracked tensile concrete between cracks carries a certain amount of tensile force normal to the cracked plane. The concrete adheres to the reinforcement bars and contributes to overall stiffness of the structure. The phenomenon called tension stiffening has significant influence on numerical results of short-term deformational analysis. Based on a variety of assumptions, many constitutive models for cracked concrete in tension have been proposed for case of short-term loading.

Degradation of tension stiffening takes place with increasing time under sustained loading. However, experimental investigations [8] have shown that this effect should not be neglected, particularly for members with small reinforcement ratios.

Very few constitutive relationships of cracked tensile concrete have been proposed for case of long-term loading. Collins and Mitchell offered [9] the average stresses of cracked tensile concrete to calculate by the relationship taken from the *Modified Compression Field Theory* [10] using the reduction factor 0.7. Cypinas [11] modified the constitutive relationship proposed by Prakhya and Morley [12] for short-term analysis. The linear ascending branch of the relationship is taken according to the classical theory of linear creep whereas the descending branch is assumed the same as for the short-term analysis. Alwis et al. [13] used bilinear stress-strain relationship proposed by Bazant and Oh [14].

In the present study, a modified constitutive relationship proposed by Kaklauskas [1, 2] was used for modelling long-term behaviour of cracked tensile concrete. The ascending branch of the relationship shown in Fig 2 has the following expression:

$$f_t(t) = 0.625f_{cr}(t) \left( 1 - \frac{\bar{\varepsilon}_t(t)}{\beta(t)} + \frac{1 + 0.6\beta(t)}{\beta(t)\varepsilon_t(t)} \right), \quad (5)$$

$$\bar{\varepsilon}_t(t) = \frac{\varepsilon_t(t)}{\varepsilon_{cr}(t)}, \quad \varepsilon_{cr}(t) = \frac{f_{cr}(t)}{\bar{E}_e(t, t_0)}. \quad (6)$$

Here  $f_{cr}(t)$  is the strength of tensile concrete at time  $t$ ;  $\varepsilon_{cr}(t)$  is the cracking strain of tensile concrete at time  $t$ ;  $\sigma_t(t)$ ,  $\varepsilon_t(t)$  are the stress and strain of tensile concrete at time  $t$ , respectively;  $\beta(t)$  is the factor describing the length of the descending branch of the constitutive relationship, see Fig 2.

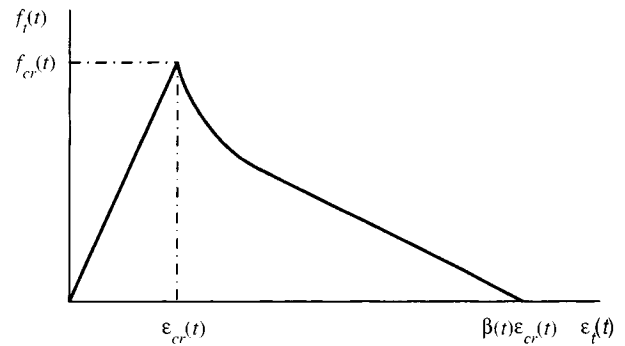


Fig 2. Modified stress-strain relationship for cracked concrete in tension under long-term loading

Reduction in concrete tensile strength due to sustained loading is assessed by factor  $\eta_{cr}$  proposed by Shkoukani and Walraven [15]:

$$\eta_{cr} = \frac{f_{cr}(t)}{f_{cr}(t_0)} = 0.794 - 0.06 \log(t - t_0), \quad (7)$$

where  $f_{cr}(t_0)$  is the short-term strength of tensile concrete.

Factor  $\beta(t)$  is taken as

$$\beta(t) = \frac{\beta(t_0)}{\eta_{cr}^2(1 + \chi(t, t_0)\phi(t, t_0))}, \quad (8)$$

where  $\chi(t, t_0)$  is the ageing factor; and  $\phi(t, t_0)$  is the creep factor.

Factor  $\beta(t_0)$  describing the length of the descending branch for case of short-term loading [1] is expressed as

$$\beta(t_0) = 32.8 - 27.6p + 7.12p^2 \quad (9)$$

$$(\beta(t_0) = 5, \text{ if } p \geq 2\%),$$

where  $p$  is the reinforcement percentage.

### 2.3. Reinforcement

For reinforcement material idealization, a bilinear, trilinear or more complex stress-strain relationship can be adopted. The stress-strain curve for long-term analysis is assumed to be similar to that for short-term analysis, ie no creep is assumed in steel.

### 3. Method of calculation long-term deflections

A powerful approach for the time analysis of the cracked cross-section is adopted. It is based on a relaxation procedure proposed by Bresler and Selna [4]. The method of analysis has been described in more detail by Ghali and Favre [16] or Gilbert [17], and makes use of age-adjusted effective modulus method [3] to model the effects of creep in concrete.

The section contains one or more levels of non-prestressed reinforcement, and is subjected to a constant sustained bending moment applied at time  $t_0$ . The cross-section analysis is performed on the so-called transformed section. The area of bonded steel reinforcement is transformed into an equivalent area of concrete. Normally, section properties are calculated about the centroidal axis of the transformed section. However, when the effects of creep are considered and time analysis is undertaken, the effective modulus for concrete varies with time and so too does the area of the transformed section and the position of the neutral axis. It is convenient therefore to develop expressions for strain and curvature in terms of reference point, which does not vary with time. For this analysis, the top fibre of the section is taken to be the reference level.

The basic equations can be employed in eight step procedure to determine the immediate and long-time strain (Fig 3), stress and deflections (curvatures). The eight steps are:

1. The section curvature  $\kappa_i$  and the extreme compressive fibre strain  $\epsilon_{0i}$  due to short-term loading are calculated by the technique based on classical expressions of strength of materials extended to application of layered approach and full material diagrams [1].

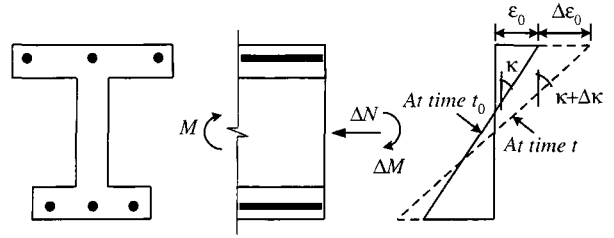


Fig 3. Deformations under long-term loading

2. Age-adjusted modulus of elasticity of concrete for gradually varying stresses is calculated

$$\bar{E}_e(t, t_0) = \frac{E_c(t_0)}{1 + \chi(t, t_0)\phi(t, t_0)}, \quad (10)$$

where  $\chi(t, t_0)$  is the ageing coefficient which depend on the magnitude of creep coefficient  $\phi(t, t_0)$ , the age of concrete at first loading  $t_0$  and the time under the load  $t - t_0$ . The graphs showing the relationship between the ageing coefficient and the age of concrete at loading, based on the CEB-FIP Model 1978 creep function are given in reference [17].

3. If creep were not restrained in any way, the top fibre strain and curvature during the time interval  $t - t_0$  would increase to  $\phi(t, t_0)\epsilon_{0i}$  and  $\phi(t, t_0)\kappa_i$ , respectively. If shrinkage is uniform over the depth of the section and completely unrestrained, the shrinkage induced top fibre strain, which develops during the time interval  $t - t_0$  is  $\epsilon_{sh}(t, t_0)$  and the curvature is zero. The total restraining forces required to prevent these deformations are obtained from Eqs 11:

$$\begin{aligned} \Delta N &= \bar{E}_e \left[ \phi(t, t_0)(A_c \epsilon_{0i} + B_c \kappa_i) + \epsilon_{sh} A_c \right] \\ \Delta M &= \bar{E}_e \left[ \phi(t, t_0)(B_c \epsilon_{0i} + I_c \kappa_i) + \epsilon_{sh} B_c \right] \end{aligned} \quad (11)$$

where  $A_c, B_c, I_c$  are geometrical characteristics of concrete gross section (ignoring reinforcement).

These restraining forces in reversed direction are applied to the age-adjusted transformed section (Fig 3).

4. The increments of top fibre strain and curvature produced by the axial force  $\Delta N$  and the moment  $\Delta M$ , gradually applied about the top reference level, may be obtained from the following equations

$$\Delta \epsilon_0 = \frac{\bar{B}_e \Delta M - \bar{I}_e \Delta N}{\bar{E}_e (\bar{B}_e^2 - \bar{A}_e \bar{I}_e)}, \quad (12)$$

$$\Delta \kappa_0 = \frac{\bar{B}_e \Delta N - \bar{A}_e \Delta M}{\bar{E}_e (\bar{B}_e^2 - \bar{A}_e \bar{I}_e)}, \quad (13)$$

**Table 1.** Main characteristics of beams

Author of experiment	Number of beams	Age at loading [days]	Duration of loading [days]	Span [m]	Height [mm]	Width [mm]	Reinforcement ratio [%]	150 mm cube strength [MPa]
Figarovskij [18]	24	28	90–258	3.00	249–254	179–181	0.25–1.26	9.56–32.8
Prokopovich and Temnov [19]	21	7–90	410–1052	2.00	160	100	0.56–2.34	12.8–29.3
Gilkey and Ernst [20]	16	90–194	144–500	3.05	152	76	0.58–1.23	29.1–45.5
van Nieuwenburg et al. [21]	15	28	772–1610	2.80	280	150	0.82–3.53	35.1–40.3
Dajun et al. [22]	11	30	531–2190	2.88	99–200	62–300	0.74–3.43	24.6–29.9
Clarke et al. [23]	4	28	180	2.10	152–154	100	1.19–1.21	30.5
Corley and Sozen [24]	3	28	700	1.83	110–152	76	1.37–3.04	28.4
Bakoss et al. [25]	1	28	500	3.75	150	100	1.74	46.0

where  $\bar{A}_e$  is area of the age-adjusted transformed section;  $\bar{B}_e$  and  $\bar{I}_e$  are first and second moments of the area of the age-adjusted transformed section about top surface. The age-adjusted effective modulus  $\bar{E}_e$  is used in the above equations because the forces  $\Delta N$  and  $\Delta M$  are gradually applied.

5. Total strain due to short-term and long-term effects is calculated for each layer:

$$\varepsilon = \varepsilon_{0i} + \Delta\varepsilon_0 + (\kappa_i + \Delta\kappa)y. \quad (14)$$

6. For the assumed material diagrams of steel and concrete (Figs 1 and 2), stress  $\sigma_i$  corresponding to strain  $\varepsilon_i$  is obtained. A secant deformation modulus  $\bar{E}_i = \sigma_i / \varepsilon_i$  is determined.

7.  $\bar{E}_i$  values obtained for every layer are compared with the previously assumed or computed ones. If the agreement is not within the assumed error limits, a new iteration is started from step 3.

8. After convergence of the deformation modulus  $\bar{E}_i$  for all the layers, final values of strains, stresses and curvature are assessed. For deflection calculation which is performed by Mohr's integral technique, analogous computations are carried out for other sections of the beam.

#### 4. Comparison of analysis with experimental results

This section presents a comparison between the computed deflections (using the described procedure) and the measured deflections reported by Figarovskij [18], Prokopovich and Temnov [19], Gilkey and Ernst [20], van Nieuwenburg et al. [21], Dajun et al. [22], Clarke et al. [23], Corley and Sozen [24], Bakoss et

al. [25] (totally 95 beams). The beams were tested under a four-point loading system, which gave a constant moment zone and two equal shear spans. Main characteristics of the beams indicating variations in age at loading, duration of loading, span, cross-section parameters and concrete strength are presented in Table 1. Most of the beams had a rectangular (84 beams), but some an inverted T section (9 beams) and T section (2 beams).

Accuracy of the predictions made has been assessed using basic statistical parameters such as mean value and standard deviation calculated for relative deflections  $f_{th} / f_{exp}$  where  $f_{th}$  is the calculated and  $f_{exp}$  is the experimental deflection. According to the calculation results presented in Table 2, reasonable accuracy has been achieved giving 14.3% standard deviation and 1.02 mean value.

**Table 2.** Statistical parameters for relative deflections,  $f_{th} / f_{exp}$ , estimated by the proposed method

Author of experiment	Mean	Standard
Figarovskij [18]	1.03	0.088
Prokopovich and Temnov [19]	1.00	0.134
Gilkey and Ernst [20]	1.11	0.221
van Nieuwenburg et al. [21]	1.06	0.073
Dajun et al. [22]	0.99	0.140
Clarke et al. [23]	0.93	0.095
Corley and Sozen [24]	0.84	0.034
Bakoss et al. [25]	1.05	0.105
<b>Total</b>	<b>1.02</b>	<b>0.143</b>

## 5. Concluding remarks

In this paper, an attempt has been made to extend application of the *integral constitutive model* to long-term deformational analysis of flexural reinforced concrete members. Constitutive relationships for compressive concrete and cracked tensile concrete used in the analysis are presented. A deformation calculation technique is based on the age-adjusted effective modulus method, a relaxation procedure and layered approach. Statistical deflection calculation analysis carried out for a large number of experimental beams has shown reasonable accuracy.

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## SLUOKSNIŲ MODELIS GELŽBETONINIŲ SIJŲ ILGALAIKIŲ ĮLINKIŲ ANALIZEI

D. Bačinskas, G. Kaklauskas, P. Vainiūnas

### Santrauka

Straipsnyje integralinis fizikinis deformatyvumo modelis [1, 2] pritaikytas lenkiamųjų gelžbetoninių elementų ilgalaikėms deformacijoms nustatyti. Pasiūlytas inžinerinis skaičiavimo metodas pagrįstas senėjančios medžiagos efektyviojo modulio (*age-adjusted modulus*) [3] ir sluoksnių metodais bei relaksacijos procedūra, kurią pasiūlė Bresleris ir Selna [4]. Svarbi pasiūlytojo modelio dalis yra modifikuotos gniuždomojo bei tempiamojo betono įtempimų-deformacijų diagramos. Gniuždomajam betonui modeliuoti taikoma modifikuota Hognestado [5] diagrama, nustatyta trumpalaikio apkrovimo atvejui. Ši diagrama pateikta 1 pav., o jos kylančioji dalis aprašoma (1) priklausomybe. Ilgalaikio stiprumo ribos sumažėjimas įvertinamas išraiška (2), pasiūlyta Jašino [6]. Maksimalius įtempimus atitinkančios deformacijos bei ribinės gniuždomojo betono deformacijos nustatomos atitinkamai pagal (3) ir (4) priklausomybes. Tempiamojo betono modeliavimui taikoma modifikuota integralinė įtempimų-deformacijų diagrama (2 pav.). Krintančioji šios diagramos dalis aprašoma (5) priklausomybe. Tempiamojo betono ilgalaikio stiprumo sumažėjimas įvertinamas faktoriumi  $\eta_{cr}$  pagal (7), o krintančiosios dalies ilgį apibūdinantis koeficientas  $\beta(r)$  – pagal (8) išraišką. Armatūros darbui modeliuoti gali būti taikoma dvišesė, trišesė arba kita faktinė armatūros įtempimų-deformacijų diagramą atitinkanti kreivė.

Ilgalaikėms deformacijoms nustatyti taikoma paprasta inžinerinė metodika, detaliau aprašyta darbuose [16, 17]. Nagrinėjamas ilgalaikė apkrova veikiamas vienpusio arba dvišio armavimo lenkiamasis gelžbetoninis elementas (3 pav.). Skerspjūvį sudalijus į sluoksnius, trumpalaikių bei ilgalaikių deformacijų (3 pav.), įtempimų bei įlinkių (kreivių) skaičiavimas atliekamas iteracijomis tokiais žingsniais:

1. Pagal darbe [1] pateiktą algoritmą apskaičiuojamas kreivis bei viršutinio sluoksnio deformacija nuo trumpalaikės apkrovos.
2. Pagal (10) išraišką nustatomas senėjančios medžiagos efektyvusis modulis.
3. Pagal (11) formulę nustatomi vidinių jėgų pokyčiai, atsirandantys dėl laisvų valkšnumo bei susitraukimo deformacijų suvaržymo. Kad nebūtų pažeista pusiausvyros sąlyga, tokio pat dydžio, bet priešingo ženklo jėgomis yra apkraunamas senėjančios medžiagos redukuotas skerspjūvis (3 pav.).
4. Pagal (12) ir (13) išraiškas apskaičiuojami kreivio bei viršutinio sluoksnio deformacijų pokyčiai, sukelti įrašų  $\Delta N$  ir  $\Delta M$ .
5. Pagal (14) priklausomybę nustatomos suminės kiekvieno sluoksnio deformacijos nuo trumpalaikės bei ilgalaikės apkrovos.
6. Gautai kiekvieno sluoksnio deformacijai, taikant atitinkamą medžiagos diagramą, apskaičiuojami įtempiai bei kirstinis deformacijų modulis.

7. Kiekvienam sluoksniui gauta kirstinio deformacijų modulių reikšmė palyginama su ankstesnėje iteracijoje apskaičiuota reikšme. Jei šios reikšmės nėra lygios priimtos paklaidos ribose, nuo 3-iojo žingsnio pradedama nauja iteracija.

8. Sukonvergavus visų sluoksnių kirstinių deformacijų modulių reikšmes, Moro integralu apskaičiuojami įlinkiai.

Siekiant įvertinti pasiūlytojo metodo tikslumą buvo atlikta 8 autorių [18–25] 95 eksperimentinių sijų statistinė įlinkių analizė. Visos sijos buvo laisvai atremtos bei apkrautos ilgalaikė dviejų koncentruotų jėgų apkrova. Pagrindiniai sijų duomenys pateikti 1 lentelėje. Vertinant skaičiavimo tikslumą, santykiniams įlinkiams  $f_{th} / f_{exp}$  buvo nustatyti svarbiausieji statistiniai dydžiai – vidurkis bei vidutinis kvadratinis nuokrypis. Atlikus apskaičiavimus (2 lent.), gautas geras teorinių bei eksperimentinių įlinkių sutapimas (vidurkis – 1,02, vidutinis kvadratinis nuokrypis – 14,3%).

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**Darius BAČINSKAS.** PhD student. Dept of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-2040 Vilnius, Lithuania. E-mail: Darius.Bacinskas@st.vtu.lt

BSc (1995) and MSc (1997) in Civil Engineering at Vilnius Gediminas Technical University. Co-author of 4 research papers. Research interests: deformations of reinforced concrete structures.

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**Gintaris KAKLAUSKAS.** Dr Habil, Professor. Dept of Reinforced Concrete and Masonry Structures, Vilnius Gediminas Technical University (VGTU), Saulėtekio al. 11, LT-2040 Vilnius, Lithuania. E-mail: Gintaris.Kaklauskas@st.vtu.lt

Graduate of Vilnius Civil Engineering Institute (presently Vilnius Gediminas Technical University, 1982 (civil engineer). PhD (1990). Dr Habil (2000). Research visits: Aalborg University (Denmark), 1991, University of Glamorgan (UK, 1994/1995, 1998), University of Illinois, Urbana-Champaign (USA, 1996). Author and co-author of 2 monographs, 1 invention and a number of papers. Research interests: development of constitutive relationships for concrete and numerical simulation of reinforced concrete structures.

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**Povilas VAINIŪNAS.** Doctor, Associate Professor. Dean of Civil Engineering Faculty at Vilnius Gediminas Technical University (VGTU), Saulėtekio al. 11, LT-2040 Vilnius, Lithuania. E-mail: povva@rasa.vtu.lt

PhD (1970) from Kaunas Polytechnical Institute (presently Kaunas Technological University). Chairman of national group of International Association for Bridge and Structural Engineering (IABSE). Former vice-president (1992–95) and board member (since 1995) of Association of European Civil Engineering Faculties (AECEF). Chairman of scientific committee of biennial intern. conference "Modern building materials, structures and techniques" held at VGTU, Lithuania. Author and co-author of over 40 research papers. Research interests: mechanics of reinforced concrete, theory of durability and reliability, design of buildings, development of territory planning and building code systems of Lithuania and real estate assessment.