

Use of Galerkin Technique to the Rolling of a Plate in Deep Water

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Abstract. The classical problems of surface water waves produced by small oscillations of a thin vertical plate partially immersed as well as submerged in deep water are reinvestigated here. Each problem is reduced to an integral equation involving horizontal component of velocity across the vertical line outside the plate. The integral equations are solved numerically using Galerkin approximation in terms of simple polynomials multiplied by an appropriate weight function whose form is dictated by the behaviour of the fluid velocity near the edge(s) of the plate. Fairly accurate numerical estimates for the amplitude of the radiated wave at infinity due to rolling and also for swaying of the plate in each case are obtained and these are depicted graphically against the wave number for various cases.

Keywords: rolling motion, partially immersed and submerged plate, integral equation, Galerkin approximation, amplitude of radiated wave.

AMS Subject Classification: 74F10.

1 Introduction

Study of water wave problems wherein waves are produced by rolling motion of a partially immersed as well as a submerged plate which executes simple harmonic oscillations about a horizontal axis was considered by a number of researchers. The rolling motion of a ship modelled as a partially immersed thin

plate in infinitely deep water was first investigated by Ursell [31] who obtained the explicit expression for the amplitude at infinity of the wave motion. Later, Haskind [11] obtained the exact values for general coefficients of damping and mass coupling for partially immersed plate. Evans [6] used Green's integral theorem to derive a simple expression for the amplitude of the radiated waves at infinity for the same problem considered by Ursell [31] and also deduced the wave amplitude for swaying of the plate. Later, Mandal [19] also used Green's integral theorem to obtain the amplitude of the radiated wave produced by small oscillations of a thin vertical plate submerged in deep water and also considered the case of swaying of the plate. The two-dimensional problem of the small-amplitude forced rolling motion of a thin vertical plate in finite depth water was considered by Evans and Porter [9]. They used two complementary formulations in conjunction with an accurate and efficient Galerkin method to solve the problem. Later, Banerjea and Mandal [2] considered the generation of waves due to rolling of a thin vertical plate which was solved by Green's integral theorem.

The problem of the generation of waves due to small rolling oscillations of a thin vertical plate partially immersed in uniform finite-depth water is investigated by Banerjea et al. [1] using eigenfunction expansion of the velocity potentials and also by hypersingular integral equation formulation. Mandal et al. [20] used dual integral equations involving trigonometric functions as kernel to reinvestigate the classical rolling ship problem due to small rolling oscillations of a thin vertical plate partially immersed in deep water. Later, Rakshit and Banerjea [26] investigated the effect of a small bottom undulation of the sea bed on the surface waves generated due to a rolling oscillation of a vertical plate either partially immersed or completely submerged in water of variable depth using Green's integral theorem.

The Galerkin approximation is a useful method to obtain an approximate solution of integral equations. The advantage of this method is that the set of functions need not be an orthogonal set or not necessary be complete. The Galerkin method converges faster than the collocation method (cf. Kanoria and Mandal [13]). Previously, Evans and Morris [8], Porter and Evans [25] employed Galerkin's approximation technique followed by the Havelocks [12] expansion of water wave potential to obtain numerical estimates of reflection coefficient for oblique incidence of the wave train on a single vertical barrier in deep as well as finite depth water. Evans and Fernyhough [7] investigated about the numerical evidence of the existence of edge waves travelling along a periodic coastline using Galerkin approximation technique.

The water wave scattering by vertical thin porous barriers has been solved by Li et al. [15] considering a surface piercing barrier and a submerged bottom-standing barrier using a multi-term Galerkin method. In recent times, Meng et al. [24] presented a hybrid element-free Galerkin (HEFG) method for solving three dimensional wave propagation problem. Also Liu and Cheng [16], Liu et al. [17] and Cheng et al. [3] employed element-free Galerkin method in the study of some problems involving swelling of polymer gels, elastoplastic deformation problems and 3D elasticity problems. Gupta and Gayen [10] used multi-term Galerkin approximation in terms of Chebyshev polynomials to analyse the effect

of two submerged unequal permeable plates in the propagation of water waves and also Roy et al. [29] used multi-term Galerkin approximation technique with Chebyshev polynomials for water wave scattering by unequal partially immersed vertical barriers in water of uniform finite depth.

Previously, Roy et al. [27, 28, 30] considered different geometrical configurations of multiple barriers in infinite depth water and formed a system of singular integral equations for each problem which were solved by using single term Galerkin expansion wherein the single-term is the exact solution of the corresponding integral equation for single barrier due to normal incidence. To avoid the complicacy of calculations using single term Galerkin approximation some researchers use Galerkin approximation in terms of simple polynomials multiplied by an appropriate weight function to solve the singular integral equations. Das et al. [4] investigated the problem of oblique scattering by thin vertical barriers in deep water by multiterm Galerkin technique using simple polynomials as basis functions. Kaur et al. [14] also used the method of polynomial approximation to solve the resulting singular integral equations for double trench problem. Recently, Das et al. [5] also used multiterm Galerkin technique with simple polynomials as basis functions to solve the integral equation arising in the problem of thick barrier in deep water.

In this paper, we have reinvestigated the problem of wave generation due to rolling of a partially immersed plate as well as a rolling submerged plate in infinitely deep water by using Galerkin approximation in terms of simple polynomials multiplied by an appropriate weight function whose form is dictated by the behaviour of the fluid velocity near the submerged edge(s) of the plate. The numerical estimate of the amplitude of the radiated wave produced by the rolling motion of the plate is obtained and it is depicted against the wave number for different sizes of the plate in a number of figures. The results given previously by Ursell [31] and Mandal [19] for partially immersed and submerged plate are recovered. Lastly, the case of swaying of the both types of plate is considered and the amplitude is obtained and depicted against the wavenumber in a number of figures and the results given by Evans [6] have been recovered.

2 Formulation

A Cartesian rectangular co-ordinate system is chosen in which the y -axis is taken vertically downwards into the fluid region and xz -plane is the rest position of the mean free surface $y = 0$. The plate whose equilibrium position is $x = 0$, $y \in L$ where $L \equiv L_1 = (0, b)$ for partially immersed plate and $L \equiv L_2 = (a, b)$ for submerged plate, makes small oscillations of circular frequency ω about a fixed point $(0, c)$ which need not to be in L . The geometry of the two positions of the plate is best described in Figure 1.

Assuming linear theory and the motion in water to be irrotational, time harmonic and independent of the co-ordinate z , the motion in the fluid region can be described by a velocity potential $Re\{\phi(x, y)e^{-i\omega t}\}$, where ϕ satisfies

$$\nabla^2\phi = 0$$

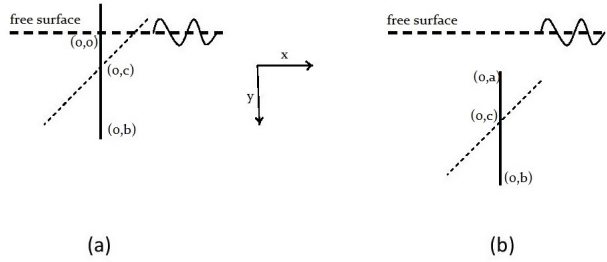


Figure 1. Diagram of rolling of (a) partially immersed plate and (b) submerged plate.

in the fluid region, the free surface condition

$$\frac{\partial \phi}{\partial y} + K\phi = 0 \text{ on } y = 0, \quad -\infty < x < \infty,$$

where $K = \frac{\omega^2}{g}$, g being the acceleration due to gravity, the condition due to small oscillations of the plate

$$\frac{\partial \phi}{\partial x} = g(y) \quad \text{for } y \in L, x = 0, \tag{2.1}$$

where

$$g(y) = -i\omega\theta_0(c - y) \tag{2.2}$$

is a known function given by Evans [6] if the plate is hinged at $(0, c)$ and makes small oscillation of amplitude θ_0 . The bottom conditions for deep water is

$$\nabla \phi \rightarrow 0 \text{ as } y \rightarrow \infty, \quad 0 < x < \infty,$$

while the edge condition is

$$r^{\frac{1}{2}} \nabla \phi \text{ is bounded as } r \rightarrow 0,$$

where r is the distance from a submerged edge of the plate, and finally for $|x| \rightarrow \infty$

$$\phi(x, y) = \begin{cases} A_- e^{-Ky - iKx} & \text{as } x < 0, \\ A_+ e^{-Ky + iKx} & \text{as } x > 0, \end{cases}$$

where A_- and A_+ denote the amplitude (unknown) of the water wave created by the rolling of the plate in the negative direction and positive direction of the x -axis respectively which is to be obtained for different sizes of the plate.

3 Reduction to integral equation

Use of Havelock's [12] expansion for water wave potentials produces

$$\phi(x, y) = A_- e^{-Ky - iKx} + \int_0^\infty A(k) S(k, y) e^{kx} dk \quad \text{for } x < 0, \quad 0 < y < \infty$$

and

$$\phi(x, y) = A_+ e^{-Ky+iKx} + \int_0^\infty B(k)S(k, y)e^{-kx} dk \quad \text{for } x > 0, 0 < y < \infty,$$

where $S(k, y) = k \cos ky - K \sin ky$ and $A(k)$ and $B(k)$ is an unknown functions.

The continuity of velocity at $x = 0, 0 < y < \infty$ gives

$$\phi_x(+0, y) = \phi_x(-0, y), \quad 0 < y < \infty,$$

which gives for $0 < y < \infty$

$$ikA_+ e^{-Ky} - \int_0^\infty kB(k)S(k, y)dk = -ikA_- e^{-Ky} - \int_0^\infty kA(k)S(k, y)dk,$$

so that by using Havelock’s inversion formula(cf. Mandal and Chakrabarti [23])

$$-iKA_- = iKA_+ = 2K \int_0^\infty F(u)e^{-Ku} du, \tag{3.1}$$

$$kA(k) = -kB(k) = \frac{2}{\pi} \frac{1}{k^2 + K^2} \int_0^\infty F(u)S(k, u)du, \tag{3.2}$$

wherein we have used

$$\frac{\partial \phi}{\partial x} = F(y) = \begin{cases} g(y) & \text{for } y \in L, x = 0, \\ f(y) & \text{for } y \in \bar{L}, x = 0, \end{cases} \tag{3.3}$$

where $\bar{L} \equiv \bar{L}_1 = (b, \infty)$ for partially immersed plate and $\bar{L} \equiv \bar{L}_2 = (0, a) \cup (b, \infty)$ for submerged plate and $f(y)$ is an unknown function having square root singularity at $y = b$ for $\bar{L} = \bar{L}_1$ and at $y = a, b$ for $\bar{L} = \bar{L}_2$.

Then, (2.1) and (3.1)–(3.3) provides

$$A_- = -A_+ = 2i \left[\int_L g(u)e^{-Ku} du + \int_{\bar{L}} f(u)e^{-Ku} du \right],$$

$$A(k) = -kB(k) = \frac{2}{\pi} \frac{1}{k(k^2 + K^2)} \left[\int_L g(u)S(k, u)du + \int_{\bar{L}} f(u)S(k, u)du \right].$$

Now the continuity of pressure at $x = 0, 0 < y < \infty$ gives

$$\phi(+0, y) = \phi(-0, y), \quad 0 < y < \infty.$$

This provides the integral equation for $f(y) \equiv f_1(y)$ for partially immersed plate given by

$$\int_{L_1} f_1(y)M_1(y, u)du = G_1(y), \quad y \in \bar{L}_1, \tag{3.4}$$

where

$$M_1(y, u) = 2ie^{-K(y+u)} - \frac{1}{\pi} \ln \left| \frac{y-u}{y+u} \right| - \frac{2}{\pi} e^{-K(y+u)} \int_{-\infty}^{K(y+u)} \frac{e^v}{v} dv, \quad y, u \in \bar{L}_1$$

and $\forall y \in \bar{L}_1$

$$G_1(y) = \int_{L_1} g(u) \left[\frac{1}{\pi} \ln \left| \frac{y-u}{y+u} \right| + \frac{2}{\pi} e^{-K(y+u)} \int_{-\infty}^{K(y+u)} \frac{e^v}{v} dv - 2ie^{-K(y+u)} \right] du,$$

while

$$f_1(y) = O((y-b)^{-\frac{1}{2}}) \quad \text{as } y \rightarrow b+0.$$

The integral equation for $f(y) \equiv f_2(y)$ for submerged plate is given by

$$\int_{\bar{L}_2} f_2(y) M_2(y, u) du = G_2(y), \quad y \in \bar{L}_2, \quad (3.5)$$

where

$$M_2(y, u) = 2ie^{-K(y+u)} - \frac{1}{\pi} \ln \left| \frac{y-u}{y+u} \right| - \frac{2}{\pi} e^{-K(y+u)} \int_{-\infty}^{K(y+u)} \frac{e^v}{v} dv, \quad y, u \in \bar{L}_2$$

and

$$G_2(y) = \int_{L_2} g(u) \left[\frac{1}{\pi} \ln \left| \frac{y-u}{y+u} \right| + \frac{2}{\pi} e^{-K(y+u)} \int_{-\infty}^{K(y+u)} \frac{e^v}{v} dv - 2ie^{-K(y+u)} \right] du$$

$y \in \bar{L}_2$,

while

$$f_2(y) = \begin{cases} O((a-y)^{-\frac{1}{2}}) & \text{as } y \rightarrow a-0, \\ O((y-b)^{-\frac{1}{2}}) & \text{as } y \rightarrow b+0. \end{cases}$$

It may be noted that the integral equations (3.4) and (3.5) are weakly singular integral equations. These integral equations were solved in the literature in closed form using somewhat complicated procedure. However, the complicated procedure for each barrier configuration is avoided here and the integral equations are solved numerically using Galerkin approximation involving simple polynomials multiplied by appropriate weight functions whose form is dictated by the edge condition. Very accurate numerical estimates for the amplitude of the radiated waves are obtained from the approximate solution of the integral equations.

4 Numerical solutions of the integral equations

Numerical solutions of the integral equations (3.4) and (3.5) are now obtained by using Galerkin approximation in terms of simple polynomials multiplied by weight functions whose forms are dictated by the appropriate edge conditions. Thus we expand $f_1(y)$ and $f_2(y)$ as

$$f_1(y) = \left(\frac{b}{b-y} \right)^{\frac{1}{2}} e^{-Ky} \sum_{n=0}^N a_n \left(\frac{y}{b} \right)^n \quad \text{for } b < y < \infty \quad (4.1)$$

and

$$f_2(y) = \begin{cases} \left(\frac{a}{a-y}\right)^{\frac{1}{2}} \sum_{n=0}^N b_n \left(\frac{y}{a}\right)^n & \text{for } 0 < y < a, \\ \left(\frac{b}{y-b}\right)^{\frac{1}{2}} e^{-Ky} \sum_{n=0}^N c_n \left(\frac{y}{b}\right)^n & \text{for } b < y < \infty, \end{cases} \tag{4.2}$$

where N is an integer to be chosen suitably.

Substituting the expression of $f_1(y)$ given by (4.1), $f_2(y)$ given by (4.2) in place of $f(y)$ and $g(y)$ given by (2.2) in (3.3) we obtain A_- and A_+ as given by,

$$A_- = -A_+ = \begin{cases} 2i \left[-\int_0^b i\omega\theta_0(c-u)e^{-Ku} du + \sum_{n=0}^N a_n \int_b^\infty \left(\frac{b}{b-u}\right)^{\frac{1}{2}} \left(\frac{u}{b}\right)^n e^{-2Ku} du \right] \\ \text{for partially immersed plate,} \\ 2i \left[-\int_a^b i\omega\theta_0(c-u)e^{-Ku} du + \sum_{n=0}^N b_n \int_0^a \left(\frac{a}{a-u}\right)^{\frac{1}{2}} \left(\frac{u}{a}\right)^n e^{-Ku} du \right. \\ \left. + \sum_{n=0}^N c_n \int_b^\infty \left(\frac{b}{u-b}\right)^{\frac{1}{2}} \left(\frac{u}{b}\right)^n e^{-2Ku} du \right] & \text{for submerged plate.} \end{cases}$$

To find the unknown constants a_n ($n = 0, 1, 2, \dots, N$), we put $y = \frac{b}{y_i}$ ($i = 0, 1, 2, \dots, N$), ($0 < \frac{b}{y_i} < 1$) in the relation (3.4) to obtain the linear system

$$\sum_{n=0}^N a_n A_{in} = G\left(\frac{b}{y_i}\right), \quad i = 0, 1, 2, \dots, N, \tag{4.3}$$

$$A_{in} = \int_b^\infty \left(\frac{b}{b-u}\right)^{\frac{1}{2}} \left(\frac{u}{b}\right)^n e^{-Ku} M_1\left(\frac{b}{y_i}, u\right) du, \quad i = 0, 1, 2, \dots, N.$$

Suitable transformations like $u = \frac{b}{w}$ can be made to change the range of the integration to a finite one. The collocation points $\frac{b}{y_i}$ are to be chosen suitably. Here we have chosen

$$b/y_i = 1 - i/N, \quad i = 0, 1, 2, \dots, N.$$

Actually $i = 0$ or N have not been used. For $i = 0$ or N a point little bit away from end points (towards right for $i = 0$ and towards left for $i = N$) have been chosen during the numerical computation to avoid singularity.

To find the unknown constants b_n and c_n ($n = 0, 1, 2, \dots, N$) we put

$$y = \begin{cases} y_i \quad (i = 0, 1, 2, \dots, N) & \text{for } 0 < y_i < a, \\ b/y_i \quad (i = 0, 1, 2, \dots, N) & \text{for } 0 < \frac{b}{y_i} < 1 \end{cases}$$

to obtain the combined linear system

$$\sum_{n=0}^N b_n B_{in} + \sum_{n=0}^N c_n C_{in} = G(y_i), \quad i = 0, 1, 2, \dots, N, \tag{4.4}$$

$$\sum_{n=0}^N b_n B_{in} + \sum_{n=0}^N c_n C_{in} = G\left(\frac{b}{y_i}\right), \quad i = 0, 1, 2, \dots, N, \tag{4.5}$$

where

$$B_{in} = \int_0^a \left(\frac{a}{a-u} \right)^{\frac{1}{2}} \left(\frac{u}{a} \right)^n M_2(y_i, u) du, \quad i = 0, 1, 2, \dots, N,$$

$$C_{in} = \int_b^\infty \left(\frac{b}{u-b} \right)^{\frac{1}{2}} \left(\frac{u}{b} \right)^n e^{-Ku} M_2\left(\frac{b}{y_i}, u\right) du, \quad i = 0, 1, 2, \dots, N.$$

To find C_{in} ($i = 0, 1, 2, \dots, N$), suitable transformations like $u = \frac{b}{w}$ can be made to change the range of the integration to a finite one. The collocation points are to be chosen as

$$y_i = ia/N \quad \text{for } 0 < y_i < a, \quad i = 0, 1, 2, \dots, N,$$

$$b/y_i = 1 - i/N \quad \text{for } 0 < b/y_i < 1, \quad i = 0, 1, 2, \dots, N.$$

Here also, $i = 0$ or N have not been used. For $i = 0$ or N a point little bit away from end points (towards right for $i = 0$ and towards left for $i = N$) have been chosen during the numerical computation to avoid singularity.

Now, the linear systems (4.3), (4.4) and (4.5) can be solved by any standard method to obtain the constants a_n , b_n and c_n ($n = 0, 1, 2, \dots, N$). Hence the approximate solution of the integral equation (3.4) and (3.5) are obtained.

Special cases

In the case of swaying of a partially immersed and submerged plate given by Evans [6] and Mandal [19], that is when $\theta_0 \rightarrow 0$ and $c \rightarrow \infty$ in such a way that $\theta_0 c = \Gamma$ is a finite real constant called the amplitude of sway (cf. Evans [6]), then the expression of $g(y)$ given by (2.2) becomes

$$g^*(y) = -i\omega\Gamma$$

and the amplitude of the radiated wave of a swaying plate is then obtained.

5 Numerical results

5.1 Convergence of the numerical results

To study the convergence of the numerical results on $N(N+1$ is the number of terms in the polynomial approximation), numerical estimates for $|A^*|$ ($= \left| \frac{A}{\omega\theta_0} \right|$) as in [18], $|A| = |A_+| = |A_-|$) is tabulated in Tables 1 and 2 for the partially immersed and submerged plates respectively for $N = 1, 2, \dots, 5$ and different values of wave number Kb and other parameters mentioned therein. Table 1 shows the number of terms of the simple polynomial in the Galerkin approximation given by $N = 1, 2, \dots, 5$. The value of the amplitude converges up to three or four decimal places for the partially immersed plate as computed from the exact expression given by Ursell [31] for $c/b = 0.4$. Similarly, Table 2 shows the number of terms of the simple polynomial in the Galerkin approximation given by $N = 1, 2, \dots, 5$. The value of the amplitude converges up to three or four decimal places for the partially immersed plate as computed from the exact expression given by Mandal [19] for $a/b = 0.2$ and $c/b = 0.5$. Thus for all numerical computations $N = 4$ is chosen throughout.

Table 1. Values of $|A^*|$ against Kb for different values of N for a partially immersed plate with $c/b = 0.4$.

Kb	Ursell (1947)	$ A^* $ ($N = 1$)	$ A^* $ ($N = 2$)	$ A^* $ ($N = 3$)	$ A^* $ ($N = 4$)	$ A^* $ ($N = 5$)
0.30	0.0044968	0.0042668	0.004328	0.0043568	0.0044712	0.0044774
0.45	0.0005737	0.0004967	0.0004973	0.0005343	0.00055745	0.0005634
0.60	0.0113109	0.0110232	0.0112766	0.0113365	0.0113023	0.0113219
0.75	0.0242648	0.0249897	0.0246533	0.0243655	0.0242751	0.0242688
0.90	0.035554	0.0354981	0.0355198	0.0355378	0.0355117	0.0355294
1.05	0.0442971	0.0443568	0.0443414	0.04424351	0.0442543	0.0442658
1.20	0.0511054	0.0511126	0.0511047	0.0511086	0.0511091	0.0511035
1.35	0.0566166	0.0567652	0.056734	0.0566128	0.0566354	0.0566187
1.50	0.0612313	0.0611874	0.0612645	0.0612726	0.0612944	0.0612388

Table 2. Values of $|A^*|$ against Kb for different N for a submerged plate with $a/b = 0.2$ and $c/b = 0.5$.

Kb	Mandal (1991)	$ A^* $ ($N = 1$)	$ A^* $ ($N = 2$)	$ A^* $ ($N = 3$)	$ A^* $ ($N = 4$)	$ A^* $ ($N = 5$)
0.31	0.0209182	0.0204345	0.0207587	0.0209654	0.0209764	0.0209134
0.51	0.0392564	0.0391747	0.0391158	0.0391941	0.0392747	0.0392843
0.71	0.0471074	0.0473679	0.0471288	0.0471105	0.0471064	0.0471087
0.91	0.0491133	0.0496542	0.0492761	0.0491545	0.0491087	0.0491241
1.11	0.0477695	0.0478254	0.0476429	0.0477635	0.0477583	0.0477847
1.31	0.0446953	0.0445788	0.0447277	0.0446105	0.0446235	0.0446876
1.51	0.0400838	0.0401852	0.0401208	0.0400965	0.0400557	0.0400896
1.71	0.0322326	0.0322175	0.0322658	0.0322759	0.0322484	0.0322398
1.91	0.0217085	0.0217652	0.0217275	0.0217118	0.0217077	0.0217064

5.2 Graphical representation of the numerical results

In Figure 2, we have plotted $|A^*|$ for the rolling motion of a partially immersed plate for $c/b = 0.4$ against Kb . The curve totally coincides with the curve drawn from the explicit solutions of the amplitude at infinity given by Ursell [31]. In Figure 3, we have plotted $|A^*|$ for the rolling motion of the submerged plate for $c/b = 0.5$ and $a/b = 0.2$ against Kb . This curve totally coincides with the curve drawn from the explicit expression given by Mandal [19]. These two figures provide a check on the correctness of the method employed here.

In Figure 4, $|A^*|$ is depicted against Kb for a submerged plate taking $a/b = 0.0001, 0.01, 0.05, 0.1$ and $c/b = 0.5$. When $a/b = 0.0001$, the value of $|A^*|$ is almost zero when Kb is zero and then increases with the wave number Kb . For the other values of $a/b = 0.01, 0.05, 0.1$, the amplitude increases at first and then decreases nearly to zero for some value of Kb and then again increases with Kb for larger values of a/b .

In Figure 5, $|A^*|$ is depicted against Kb for a submerged plate for different values of $a/b = 0.1, 0.15, 0.2, 0.25$ and $c/b = 0.5$. Here, the value of the amplitude starts from zero and rapidly increases until it reaches a peak and then

decreases upto zero at some point and again increases as Kb increases further, thus showing a oscillatory behaviour.

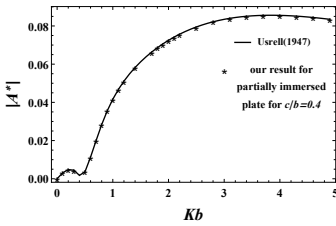


Figure 2. $|A^*|$ vs Kb for partially immersed plate for $c/b = 0.4$.

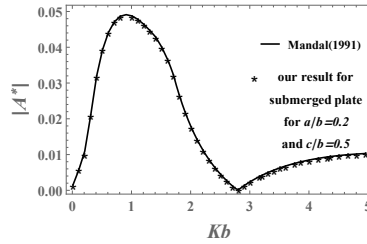


Figure 3. $|A^*|$ vs Kb for submerged plate for $c/b = 0.5$ and $a/b = 0.2$.

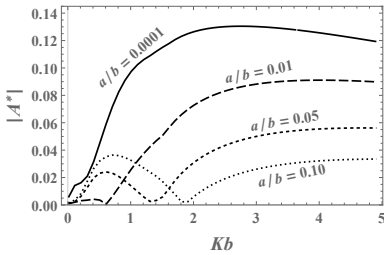


Figure 4. $|A^*|$ vs Kb for $a/b = 0.0001$ to $a/b = 0.1$ and $c/b = 0.5$.

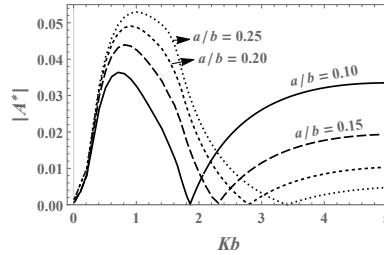


Figure 5. $|A^*|$ vs Kb for $a/b = 0.1$ to $a/b = 0.25$ and $c/b = 0.5$.

In Figure 6, $|A^*|$ for a submerged plate is plotted against Kb for $a/b = 0.25, 0.35, 0.45$ and $c/b = 0.5$. From this figure it is observed that, as a/b increases $|A^*|$ first increases upto a peak value as a function of the wave number Kb and then decreases asymptotically to zero as Kb further increases.

Figure 7 depicts $|A^*|$ for a submerged plate when the point $(0, c)$ lies above the plate i.e. the plate undergoes a small rolling oscillations about a point which lies above it. We choose $a/b = 0.6, 0.7, 0.8, 0.95$ and $c/b = 0.5$. Here also the same nature of the amplitude can be observed as before but the peak value of $|A^*|$ is smaller when larger values of a/b are considered. Here also the value of the amplitude asymptotically tends to zero for higher values of Kb .

The difference of change of $|A^*|$ can also be observed for the different values of the oscillatory point c/b keeping a/b fixed. In Figure 8, $|A^*|$ is depicted against Kb for $c/b = 0.5, 0.6, 0.7$ and $a/b = 0.2$. Here the oscillatory behaviour of the amplitude is observed and in Figure 9, $|A^*|$ is depicted for the same set of values of c/b with fixed $a/b = 0.45$. Different peak points for $|A^*|$ can be observed for different values of c/b and for all cases $|A^*|$ becomes small as Kb increases further.

Figure 10 depicts $|A^*|$ for the case of swaying of a partially immersed plate. The curve for $|A^*|$ almost coincides with the same drawn from the expression

given by Evans [6] considering the value of the constant $\Gamma = \frac{1}{\omega\pi}$. It is observed that $|A^*|$ is zero when the value of Kb is zero and then $|A^*|$ increases while Kb increases and then decreases after reaching a peak value as Kb further increases. The coincidence of the numerical values of the amplitude of a swaying partially immersed plate with Evans [6] gives another check on the correctness of the method employed here.

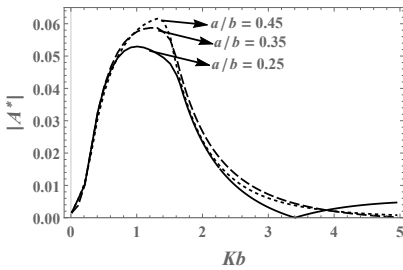


Figure 6. $|A^*|$ vs Kb for $a/b = 0.25$ to $a/b = 0.45$ and $c/b = 0.5$.

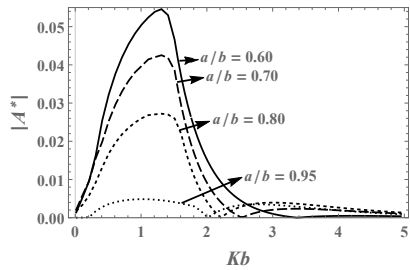


Figure 7. $|A^*|$ vs Kb for $a/b = 0.6$ to $a/b = 0.95$ and $c/b = 0.5$.

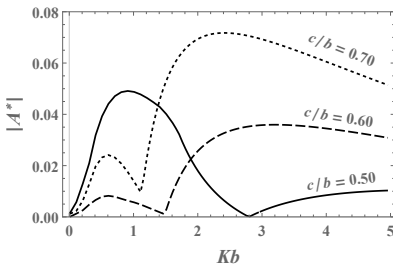


Figure 8. $|A^*|$ vs Kb for $a/b = 0.2$ for different values of c/b .

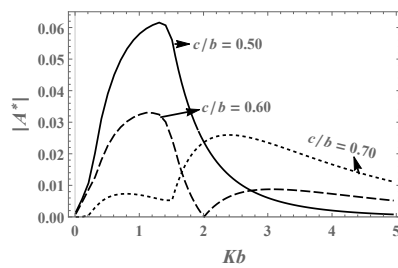


Figure 9. $|A^*|$ vs Kb for $a/b = 0.45$ for different values of c/b .

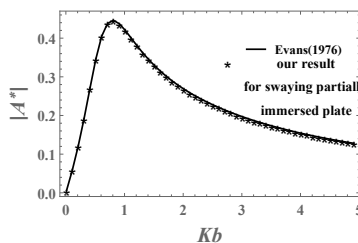


Figure 10. $|A^*|$ vs Kb coincides with Evans [6] for a swaying partially immersed plate.

The amplitude of a swaying submerged plate can similarly be observed in Figures 11 and 12 for $a/b = 0.0001, 0.05, 0.2, 0.4$ and for $a/b = 0.5, 0.6, 0.7, 0.9$ respectively. Here also the amplitude becomes higher at first and then begin to decrease along with the increase of Kb and tends to zero for further increase

of Kb . It is also observed that for $a/b = 0.9$ i.e. when the size of the plate is very much small the amplitude becomes almost zero, which is obvious.

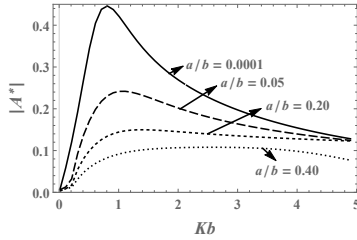


Figure 11. $|A^*|$ vs Kb for $a/b = 0.0001$ to $a/b = 0.4$ for swaying plate.

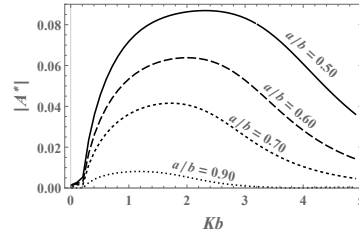


Figure 12. $|A|$ vs Kb for $a/b = 0.5$ to $a/b = 0.9$ for swaying plate.

6 Conclusions

Here the problems of surface water wave generated by rolling oscillations of a thin vertical plate partially immersed as well as submerged in deep water are reinvestigated. The integral equation for the horizontal component of velocity along the plate is solved numerically by Galerkin technique wherein the unknown functions are expanded in terms of simple polynomials multiplied by a weight function whose form is dictated by the edge condition of the plate. Very accurate numerical estimates for the amplitude of the radiated waves are obtained and are depicted in a number of figures against the wave number. The effect of change of the size of the submerged plate has been studied for rolling as well as swaying of the plates.

It may be noted that, in some water wave problems involving thin plates the original integral equations to which these are reduced are logarithmically singular (weakly singular). Then after some mathematical operations involving differentiation (once or twice) these are reduced to strongly singular (Cauchy singular or hypersingular) integral equations. Fortunately, in some cases the strongly singular integral equations (usually Cauchy singular) can be solved in closed forms. However, this may not succeed always, for example for the case of thin curved plates. In those cases, one has to employ some appropriate numerical methods. As most of these methods involve expansion involving some special functions (which are basically polynomials), the polynomial approximation employed here is also applicable. That Cauchy singular and hypersingular integral equations can be solved numerically by using polynomial approximations can be seen in the papers by Mandal and Bera ([21, 22]).

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