COMPARISON OF NUMERICAL METHODS FOR THE PROBLEM ARISING IN THE GYROTRON THEORY

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Abstract. There are considered some aspects for numerical solving of problem with Robin's boundary conditions arising in the gyrotron theory. The single mode case is carefully investigated. The obtained observations make possible to offer the suitable strategy for the numerical solving of the problem for general system of nonstationary gyrotron oscillations.
Key words: finite-difference schemes, gyrotrons, method of lines, Robin's boundary conditions

1. Introduction and Formulation of the Mathematical Model

In recent papers [1, 2, 3] we have presented the experience obtained by numerical solving of partial cases of certain problem for time-dependent gyrotron equations. Now we are considering a more general situation of investigated previously problems. Using the obtained as well as previous results we propose a suitable strategy for solving the problem for full system of nonstationary gyrotron oscillations.

A competition between the amplitudes of nonstationary gyrotron oscillations f_s and the transverse momentum of electrons p in different modes ($s = 1, \dots, m$) is described by the corresponding nonlinear system of complex partial differential equations (see, [6])

$$
\begin{cases}
\dfrac{\partial p}{\partial x} + i(|p|^2 - 1)p = i\sum_{s=1}^{m} f_s \exp(i\Delta_s x + \Psi_s), \\[2mm]
\dfrac{\partial^2 f_s}{\partial x^2} - i\dfrac{\partial f_s}{\partial t} + \delta_s f_s = \dfrac{I_s}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} p\exp[-i(\Delta_s x + \Psi_s)]\,d\theta_0\,d\varphi.
\end{cases}
$$

Here i is the imaginary unit, $x \in [0, L]$ and $t \geq 0$ are the normalized axial and time coordinates, for every number of mode s correspondingly Δ_s is the frequency mis-

match, $\delta_s = \delta_s(x)$ describes variation of critical frequencies, I_s is the dimensionless current, $\Psi_s = a_s t + m_s \varphi$ is the phase of the mode, a_s, m_s are the physical constants, $0 \leq \varphi < 2\pi$. The initial conditions for momentum

$$p|_{x=0} = \exp(i\theta_0)$$

depend on parameter $0 \leq \theta_0 < 2\pi$.

In order to study properties of numerical methods we investigate the single mode case having also in mind special restrictions on the variable p. If

$$|p|^2 = 1 + C, \quad C = const \leq 0, \quad m = 1, \quad f_1 = f$$

the nonlinear system of gyrotron equations for determination of function $f(t, x)$ reduces to the linearized complex integro-differential equation:

$$i\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2} + \delta f - iIe^{-i\Delta_* x} \int_0^x f(t, \xi)e^{i\Delta_* \xi}\, d\xi, \tag{1.1}$$

where $\Delta_* = \Delta + C$. We formulate boundary conditions of Robin's type for the function $f(t, x)$ in the following form

$$f(t, 0) = 0, \quad \frac{\partial f(t, L)}{\partial x} = -i\gamma f(t, L) \tag{1.2}$$

with some positive constant γ. The initial condition is presented in the form

$$f(0, x) = \phi(x), \tag{1.3}$$

where $\phi(x)$ is a given complex function.

Using the notation

$$y(t, x) = \int_0^x f(t, \xi)e^{i\Delta_* \xi}\, d\xi$$

and the transformation $\bar{f} = e^{i\Delta_* x} f$, the integro-differential equation (1.1) can be written in the form of two partial differential equations.

$$\begin{cases} \dfrac{\partial \bar{f}}{\partial t} = -i\left(\dfrac{\partial^2 \bar{f}}{\partial x^2} - 2i\Delta_* \dfrac{\partial \bar{f}}{\partial x} + (\delta - \Delta_*^2)\bar{f}\right) - Iy(t, x), \\ \dfrac{\partial y}{\partial x} = \bar{f}(t, x), \end{cases} \tag{1.4}$$

and conditions (1.2), (1.3) are formulated as follows

$$\begin{cases} \bar{f}(t, 0) = 0, \quad \dfrac{\partial \bar{f}(t, L)}{\partial x} = -i(\gamma - \Delta_*)\bar{f}(t, L), \\ \bar{f}(0, x) = e^{i\Delta_* x}\phi(x). \end{cases} \tag{1.5}$$

In section 2 the case $\delta = const$ is considered. We reduce the main problem to the boundary-value problem for the ordinary differential equations considering the quasistationary solutions. In sections 3-5 we apply the method of lines and finite-difference schemes for the more general case. Finally, in section 6 we present results of numerical experiments. The main conclusions of our development are gathered in section 7.

2. The Quasistationary Solution of the Differential Problem

We represent the quasistationary solution of the problem (1.4), (1.5) in the following form

$$\bar{f}(t,x) = g(x)\exp(i\alpha t), \quad y(t,x) = w(x)\exp(i\alpha t), \tag{2.1}$$

where α is a complex number $\alpha = \alpha_1 + i\alpha_2$, α_2 is a temporal damping factor: if $\alpha_2 > 0$, the solution of (1.4) decreases in time, if $\alpha_2 < 0$, the solution increases, and for $\alpha_2 = 0$ the solution is oscillating in time.

Substituting functions (2.1) into equation (1.4) and boundary conditions (1.5), we obtain the boundary-value problem of Sturm-Liouville type for the system of two ordinary differential equations

$$\begin{cases} g''(x) - 2i\Delta_* g'(x) + \lambda g(x) - Iiw(x) = 0 \\ w'(x) = g(x) \\ g(0) = w(0) = 0, \quad g'(L) = -i(\gamma - \Delta_*)g(L), \end{cases} \tag{2.2}$$

where $\lambda = \alpha + \delta - \Delta_*^2$ is a complex eigenvalue.

If $\delta = const$, then applying a standard scheme in order to solve this linear boundary-value problem we obtain a transcendental complex equation for the calculation of eigenvalues

$$\mu_3 \kappa_3 (\mu_2 - \mu_1) + \mu_2 \kappa_2 (\mu_1 - \mu_3) + \mu_1 \kappa_1 (\mu_3 - \mu_2) = 0, \tag{2.3}$$

where $\kappa_j = \big(\mu_j - i(\Delta_* - \gamma)\big)\exp(\mu_j L)$, μ_j, $j = 1,2,3$ are the three roots of the following complex cubic equation:

$$\mu^3 - 2i\Delta_* \mu^2 + \lambda\mu - Ii = 0.$$

The roots of equation (2.3) $\lambda^{(k)}$, $k = 1,2,\ldots$ are sorted in increasing order of their real parts.

The corresponding complex eigenfunctions $g^{(k)}(x)$, $w^{(k)}(x)$ are obtained from problem (2.2) for $\lambda = \lambda^{(k)}$ and the quasistationary solution of the problem (1.4), (1.5) for every eigenvalue $\alpha^{(k)} = \lambda^{(k)} - \delta + \Delta_*^2$ is given in the form

$$f^{(k)}(t,x) = \exp(-i\Delta_* x)g^{(k)}(x)\exp(i\alpha^{(k)}t).$$

Particularly, if $\alpha_2^{(k)} = \mathrm{Im}\,(\lambda^{(k)}) > 0$ than these solutions asymptotically tend to zero, if t tends to infinity.

3. Discrete Grids and Approximation of Derivatives

For approximation of derivatives we consider two types of discrete grids.

1) A uniform grid is defined by grid points

$$x_j = jh, \quad j = 0,\ldots,n, \quad h = \frac{L}{n}, \tag{3.1}$$

where h is the space step.

2) A nonuniform grid, grid points of which are the roots of the Chebyshev polynomials of the second kind [2]

$$x_j = 0.5L\big(1 - \cos(\pi j/n)\big), \quad j = 0, \dots, n. \tag{3.2}$$

Using grid points (3.1) or (3.2) we can approximate partial derivatives $\dfrac{\partial}{\partial x}$, $\dfrac{\partial^2}{\partial x^2}$ in equations (1.4) for a fixed time moment t with matrixes D, D^2 of derivatives in the form

$$f'_h = Df_h, \ \ f''_h = D^2 f_h, \ \ y'_h = Dy_h, \ \ \bar{f}'_h = D\bar{f}_h, \ \ \bar{f}''_h = D^2\bar{f}_h,$$

where

$$f_h = (f_0, f_1, \dots, f_n), \ \ f'_h = (f'_0, f'_1, \dots, f'_n), \ \ f''_h = (f''_0, f''_1, \dots, f''_n),$$

$$y_h = (y_0, y_1, \dots, y_n), \ \ y'_h = (y'_0, y'_1, \dots, y'_n), \ \ x_h = (x_0, x_1, \dots, x_n),$$

are the vector–columns of the corresponding values of the grid and grid functions

$$f_j = f(t, x_j), \ \ f'_j = \frac{\partial f(t, x_j)}{\partial x}, \ \ f''_j = \frac{\partial^2 f(t, x_j)}{\partial x^2}.$$

It follows from the Lagrange interpolation formula (see, [5]), that the elements of matrix D are given in the form

$$D_{j,k} = \frac{dl_k(x_j)}{dx}, \quad j, k = 0, \dots, n,$$

where

$$l_k(x) = \frac{\omega(x)}{\omega'(x_k)(x - x_k)}, \quad \omega = \prod_{k=0}^{n}(x - x_k)$$

are the elementary Lagrange multipliers.

It must be mentioned that for the nonuniform grid (3.2) if the grid points are chosen as the roots of the Chebyshev polynomials of the second kind the interpolation error is small [5].

Next we define a uniform temporal grid:

$$t_m = m\tau, \quad m = 1, 2, \dots, M,$$

where a corresponding time–step τ is used. We substitute the continuous function f on this grid by the discrete values $f_j^m \approx f(t_m, x_j)$. The corresponding derivatives are approximated by finite-differences

$$\frac{\partial^2 f(t_m, x_j)}{\partial x^2} \approx \frac{f_{j+1}^m - 2f_j^m + f_{j-1}^m}{h^2}, \quad \frac{\partial f(t_m, x_j)}{\partial t} \approx \frac{f_j^{m+1} - f_j^m}{\tau},$$

$$\frac{\partial f(t_m, x_j)}{\partial x} \approx \frac{f_{j+1}^m - f_{j-1}^m}{2h}, \quad \frac{\partial f(t_m, L)}{\partial x} \approx \frac{f_n^m - f_{n-1}^m}{h}, \quad j = 1, \dots, n - 1.$$

4. Method of Lines for Solving Differential Problems

Considering only the spatial discretization of problem (1.4), (1.5), when the variable x is discretized and the variable t is continuous, and using the matrixes D, D^2 of derivatives in the grids (3.1) or (3.2) we obtain for grid functions $f_h = f_h(t)$, $y_h = y_h(t)$ the system of $(n-1)$ complex ordinary differential equations (the boundary conditions (1.2) are taken into account during construction of matrixes D, D^2)

$$\frac{df_h}{dt} = Bf_h - IE^- y_h, \quad Dy_h = E^+ f_h, \tag{4.1}$$

where $B = -i(D^2 + \delta E)$, $E^{\pm} = diag(\exp(\pm i\Delta_* x_h))$ are quadratic $(n-1) \times (n-1)$ matrixes, $E^{\pm}$, E are the corresponding diagonal matrixes and the unit matrix. The initial conditions are defined' in the form

$$f_h(0) = \phi_h,$$

where

$$\phi_h = (\phi_1, \ldots, \phi_{n-1}), \quad \phi_j = \phi(x_j), \quad j = 1, \ldots, n-1.$$

Excluding the vector y_h in (4.1) we derive the initial-value problem:

$$\frac{df_h}{dt} = Gf_h, \quad f_h(0) = \phi_h, \tag{4.2}$$

where $G = B - IE^-(D^{-1}E^+)$.

The solution $f_h(t)$ of the problem (4.2) can be obtained in two forms:

1) by using matrix-exponent function

$$f_h(t) = \exp(Gt)\phi_h,$$

2) by using the spectral decomposition of matrix $G = RD_0R^{-1}$, where D_0 is the diagonal matrix of eigenvalues, R is the eigenvectors matrix with corresponding eigenvectors as columns of the matrix

$$f_h(t) = R\exp(D_0 t)(R^{-1}\phi_h). \tag{4.3}$$

Using matrixes D, D^2 and the approximation of integrals $y(t, x_j)$ we obtain for solving the integro-differential equation (1.1) the system (4.2), where the elements of matrix G are given in the following form

$$\begin{cases} G_{k,j} = B_{k,j} - I\exp(-i\Delta_* x_k)\big(h_j B_j \exp(i\Delta_* x_{j-1}) \\ \qquad\qquad + h_{j+1}A_j \exp(i\Delta_* x_j)\big), \quad j = 1, \ldots, k-1, \\ G_{k,k} = B_{k,k} - Ih_k \exp(-i\Delta_* h_k)B_k, \end{cases} \tag{4.4}$$

where $B_{k,j}$ are elements of matrix B;

$$B_j = \exp(i\Delta_* h_j)\big((\Delta_* h_j)^{-2} + (i\Delta_* h_j)^{-1}\big) - (\Delta_* h_j)^{-2},$$

$$A_j = (\Delta_* h_j)^{-2} - (i\Delta_* h_j)^{-1} - \exp(i\Delta_* h_j)(\Delta_* h_j)^{-2}$$

are coefficients of the quadrature formula, $h_j = x_j - x_{j-1}$, $k = 1, \ldots, n$.

5. Discrete Approximations in Time and Finite-Difference Schemes

Using the uniform time–grid $t_m = m\tau$, $m = 0, 1, 2, \ldots, M$ we can consider following approximation of the system (4.1):

$$\begin{cases} \dfrac{f_h^{m+1} - f_h^m}{\tau} = B(\sigma f_h^{m+1} + (1-\sigma)f_h^m) - IE^-(\sigma y_h^{m+1} + (1-\sigma)y_h^m), \\ Dy_h^m = E^+ f_h^m, \quad f_h^0 = \phi_h, \end{cases}$$

where $\sigma \in [0, 1]$ is the weight coefficient of the scheme,

$$f_h^m = f_h(t_m), \quad m = 0, 1, \ldots, M.$$

By eliminating vector y_h^m from the system we obtain the two level finite–difference scheme

$$f_h^{m+1} = \bar{G} f_h^m, \tag{5.1}$$

where the transition matrix $\bar{G}$ has the form

$$\bar{G} = \left(\tau^{-1}E - \sigma(B - IE^-D^{-1}E^+)\right)^{-1}(\tau^{-1}E + (1-\sigma)(B - IE^-D^{-1}E^+)).$$

It follows from (5.1) that $f_h^m = (\bar{G})^m \phi_h$.

The discrete approximation in time for the integro-differential equations (1.1) is given by

$$\frac{f_h^{m+1} - f_h^m}{\tau} = G(\sigma f_h^{m+1} + (1-\sigma)f_h^m), \tag{5.2}$$

where $f_h^0 = \phi_h$, $m = 0, 1, \ldots, M$ and the elements of matrix G are written in the form (4.4), $\sigma \in [0, 1]$.

In this case we can rewrite (5.2) in the form (5.1), where

$$\bar{G} = (\tau^{-1}E - \sigma G)^{-1}(\tau^{-1}E + (1-\sigma)G). \tag{5.3}$$

For the finite-difference approximation of integro-differential equation (1.1) using the uniform grid (3.1) we apply the method of lines in the form (4.2) with matrix G written in three-diagonal and left-triangle forms. The nonzero elements of this matrix are defined as

$$\begin{cases} G_{k,k} = i(2h^{-2} - \delta) - Ih\exp(-i\Delta_* h)B_1, \ G_{k,k+1} = -ih^{-2}, \\ G_{k,j} = -\big(ih^{-2}\delta_{j,k-1} + Ih(B_1\exp(-i\Delta_*(k-j+1)h) \\ \qquad\qquad + A_1\exp(-i\Delta_*(k-j)h))\big), \end{cases} \tag{5.4}$$

where $\delta_{j,k-1}$ is the Kronecker delta function, A_1, B_1 are coefficients of the quadrature formula with $h_j = h$, $j = 1, \ldots, k-1$, $k = 1, \ldots, n-1$.

Boundary conditions (1.2) are approximated in the form

$$\frac{f_n - f_{n-1}}{h} = -i\gamma f_n,$$

thus the element $G_{n-1,n-1}$ of matrix G must be supplemented with additional term $-ih^{-2}/(1 + ih\gamma)$.

For finite-difference approximations in space coordinate by using the uniform grid (3.1) and time we can consider a two level finite-difference scheme (5.2), where elements of matrix G are given in the form (5.4). The transition matrix $\bar{G}$ is defined by (5.3).

In the case of $I = 0$ the stability condition of this scheme is $\sigma \geq 0.5$ (see, [3]). This condition is valid for $I \neq 0$.

A similar finite-difference scheme for solving differential equations (1.1) can be obtained from (4.1), where matrixes D, B are defined using the finite differences in space. Difference equations approximate the differential equation with the second order both in space and time, if $\sigma = 1/2$, and with the first order in time, if $\sigma \neq 1/2$ (see, [4]). Boundary conditions (1.2) are approximated only with the first order in space. Therefore, problem (1.1) - (1.3) is approximated only with the first order of accuracy.

6. Numerical Experiments with Different Methods

All numerical results are obtained by means of computer programs MATLAB and MAPLE for the following parameters:

$$\gamma = 2, \quad L = 15, \quad \delta = 0, \quad \delta = \delta(x) = \frac{1}{900}\sinh(x - 7.5), \quad C = 0,$$

$$I = 0, \quad I = 0.01, \quad \Delta = \pm 10, \quad \phi = \sin(\pi x/L).$$

The first 15 eigenvalues $\bar{\lambda}^{(k)}$, $k = 1, \ldots, 15$ of matrix $G(n = 25)$ in (4.2) are presented in Tab. 1 ($\Delta = 10$).

For the nonuniform grid (3.2) with $n = 25$, $\delta = 0$ the first 15 eigenvalues $\bar{\lambda}^{(k)}$ of matrix G of the differential equations (4.2) are equal to $i(\lambda^{(k)} + \Delta_*^2)$ and numerical approximations have the accuracy of 4 decimal digits, where $\lambda^{(k)}$ are the roots of equation (2.3). The maximal value $M_1 = \text{Im}(\bar{\lambda}^{(n)})$ of eigenvalues of matrix G increase, but $M_0 = -\text{Re}(\bar{\lambda}^{(n)})$ decrease if the number of grid points n is enlarged. As an example, if

$$I = 1, \quad \Delta_* = 2, \quad \delta = 0,$$

then for the nonuniform grid (3.2)

$$M_1 = 331.13, \quad M_0 = 0.050, \quad (n = 25),$$
$$M_1 = 5271.6, \quad M_0 = 0.0001, \quad (n = 50).$$

These values depend little on Δ_* and I. Therefore, function $\exp\left((-M_0 + iM_1)t\right)$ slowly decreases and rapidly oscillates in time. The calculations show that

$$\text{Re}(\bar{\lambda}^{(k)}) < 0, \quad k = 1, \ldots, n$$

Table 1. The first 15 eigenvalues $\bar{\lambda}^{(k)}$ for matrix G.

k	$\delta = I = 0$	$\delta(x), I = 0$	$\delta = 0, I = 0.01$	$\delta(x), I = 0.01$
1	-.0029+.0437i	-.0147-.0387i	-.0039+.0336i	-.0145-.0287i
2	-.0118+.1748i	-.0350-.1678i	-.0129+.1689i	-.0364-.1615i
3	-.0272+.3933i	-.0565+.3912i	-.0305+.3856i	-.0597-.3832i
4	-.0497+.6989i	-.0853+.7024i	-.0556+.6926i	-.0916+.6957i
5	-.0808+1.091i	-.1249+1.099i	-.0898+1.088i	-.1343+1.095i
6	-.1230+1.570i	-.1809+1.580i	-.1310+1.571i	-.1891+1.581i
7	-.1807+2.133i	-.2664+2.140i	-.1849+2.137i	-.2707+2.145i
8	-.2636+2.773i	-.4141+2.736i	-.2632+2.776i	-.4141+2.740i
9	-.3863+3.458i	-.4775+3.253i	-.3847+3.457i	-.4754+3.252i
10	-.4749+4.116i	-.3912+3.970i	-.4779+4.112i	-.3935+3.966i
11	-.4268+4.886i	-.3487+4.839i	-.4324+4.888i	-.3547+4.840i
12	-.3780+5.822i	-.3266+5.802i	-.3802+5.826i	-.3289+5.806i
13	-.3497+6.866i	-.3132+6.854i	-.3487+6.866i	-.3122+6.855i
14	-.3319+8.002i	-.3041+7.994i	-.3339+8.000i	-.3060+7.992i
15	-.3197+9.228i	-.2976+9.223i	-.3230+9.229i	-.3009+9.224i

for all parameters and methods. At first the values $|\mathrm{Re}\,(\bar{\lambda}^{(k)})|$ increase, if k enlarged, then those reach the maximum and then asymptotically tend to zero.

The algorithm of method of lines can be improved, if the last eigenvalues with large M_1 are eliminated. If $n = 100$, $\delta = 0$, then the first $n_1 = 60$ eigenvalues of matrix G coincide with the roots of equation (2.3), but the last $n_2 = 40$ eigenvalues are not close to the roots of equation (2.3). Fig. 1 ($I = 0.01$, $\delta = 0$, $\Delta = 10$) shows the first 80 eigenvalues $\alpha^{(k)} = -i\bar{\lambda}^{(k)}$, $k = 1,\ldots,80$ with 20 "parasitic" eigenvalues.
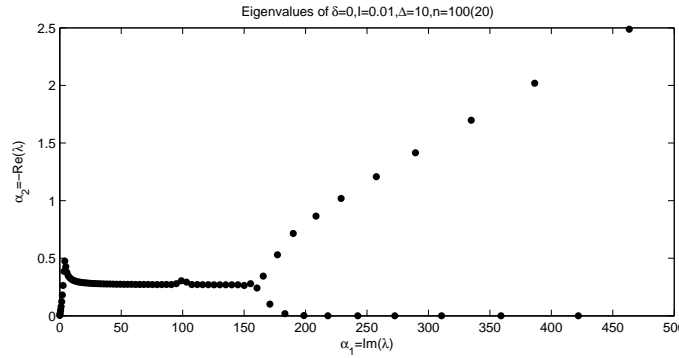
Figure 1. Eigenvalues $\alpha^{(k)}$ of $\delta = 0$, $I = 0.01$, $\Delta = 10$, $n = 100(20)$.

Therefore, we can use the spectral decomposition method (4.3) for solving the system of ODEs (4.2)

$$f_h(t) = R_* \exp(D_0^* t)(R^{-1}\phi_h)_*, \tag{6.1}$$

or

$$f_h(t) = R_* \exp(D_0^* t)(R_*' R_*)^{-1} R_*' \phi_h, \tag{6.2}$$

where R_* is the matrix R without the last n_2 columns, R_*' is the transpose matrix of R_*, D_0^* is the quadratic matrix of D_0 without the last n_2 columns and rows, and $(R^{-1}\phi)_*$ is the vector-column $(R^{-1}\phi)$ without the last n_2 rows. The expressions (6.1), (6.2) are in the form $f_h(t) = R_* \exp(D_0^* t) w_h^*$, where w_h^* is the vector-column w_h without the last n_2 rows, but w_h, w_h^* are the following solutions of algebraic systems equations:

1) the system with full matrix R (order n) – $w_h = R^{-1}\phi_h$ in (6.1),

2) the system with matrix $R_*' R_*$ (order n_1) from the least-squares approximation or Gauss transformation – $w_h^* = (R_*' R_*)^{-1} R_*' \phi_h$ in (6.2).
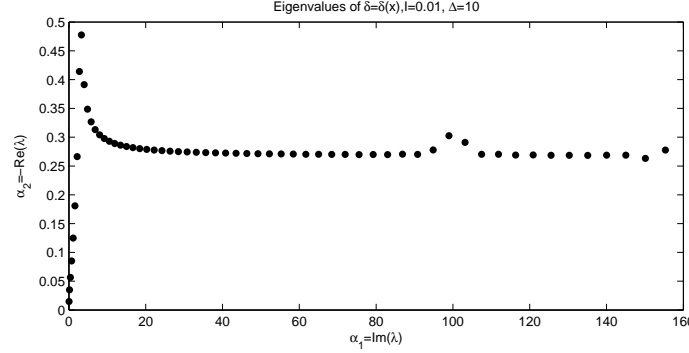

Figure 2. Eigenvalues $\alpha^{(k)}$ of $\delta(x)$, $I = 0.01$, $\Delta = 10$, $n = 100(40)$.

Fig. 2, shows the first 60 eigenvalues $\alpha^{(k)}$, $k = 1, \ldots, 60$, where $n = 100$; $\delta = \delta(x)$ and $I = 0.01$, $\Delta = 10$. Here the values $|\text{Re}\,(\bar{\lambda}^{(k)})|$ tend to value 0.270, if the number k is enlarged.

For the uniform grid (3.1) using the finite-difference schemes gives us a possibility to compute accurately only the first 3-5 eigenvalues, if $n \geq 100$ (see Fig. 3 with $n = 100$, $I = 0.01$, $\delta = 0$, $\Delta = 10$, here the exact value of α_2 tends asymptotically to 0.2675 when α_1 tends to infinity, but the corresponding value obtained by the finite-difference method tends to zero).

Next we present results of numerical experiments, when problem (1.1) – (1.3) is solved with different methods.

Results of application of the method of lines ($n = 100$, $n_1 = 60$) for values $|f(t, L)|$, $t \in [0, 400]$ are presented in Fig. 4 ($\delta = 0$, $I = 0.01$, $\Delta = 10$) and Fig. 5 ($\delta = \delta(x)$, $I = 0.01$, $\Delta = 10$). If $\delta = 0$ then the solution tends to zero very slowly.

Tab.2 shows that the calculation with $n_2 = 0.4n$, using expressions (6.1), (6.2) are more stable in comparison with $n_2 = 0$ from (6.1).
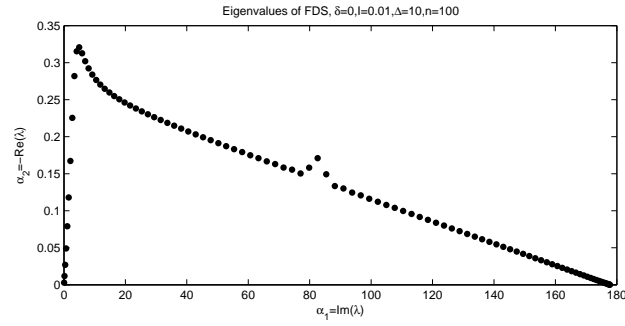
Figure 3. Eigenvalues $\alpha^{(k)}$ of $\delta = 0, I = 0.01, \Delta = 10, n = 100$ for finite-difference scheme.
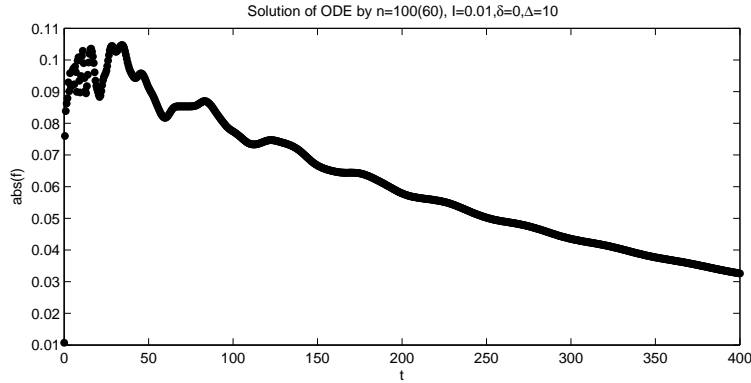

Figure 4. Solution $|f(t, 15)|$ by $n = 100$, $I = 0.01$, $\delta = 0$, $\Delta = 10$.

Table 2. The dependence of value $F_n = |f(1, L)|$ from n for $I = 0.01$, $\delta = \delta(x)$, $\Delta = 10$.

n	(6.1), $n_2 = 0.4n$	(6.2), $n_2 = 0.4n$	(6.1), $n_2 = 0$
5	0.0809	0.0842	0.0776
10	0.0918	0.0990	0.1008
20	0.0983	0.1018	0.1049
40	0.1001	0.1003	0.1003
60	0.1001	0.1002	0.0996
80	0.1003	0.1003	0.1003
100	0.1003	0.1003	0.1011

7. Conclusions

It follows from the numerical experiments that for the two level approximation the time step τ must be small. For the finite-difference scheme the number of grid points is increased $(n \geq 200)$. Therefore this requires large computer time and it is more suitable to use the nonuniform grid and the algorithm based on the method of lines.
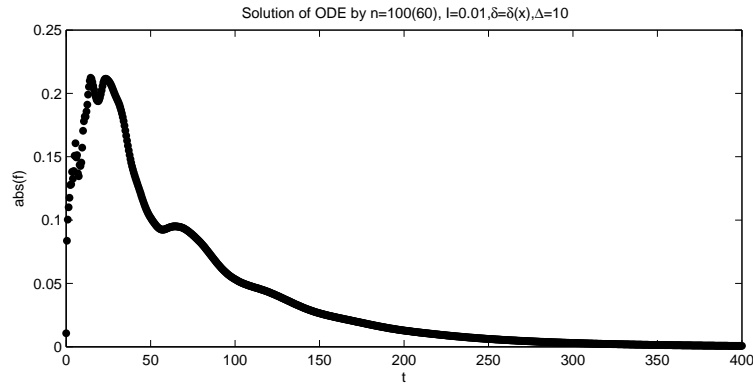
Figure 5. Solution $|f(t, 15)|$ by $n = 100$, $I = 0.01$, $\delta = \delta(x)$, $\Delta = 10$.

The finite-difference schemes (5.2), (5.4) are stable for $\sigma \geq 0.5$, but they are not sufficiently accurate. Using the matrix of derivatives on the uniform grid it is possible to make calculations only for small n ($n \leq 25$), otherwise the calculations are unstable.

References

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Skaitinių metodų girotrono teorijos uždaviniuose palyginimas

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Straipsnyje nagrinėjami girotrono teorijos uždavinių su Robino kraštine sąlyga kai kurie skaitinio sprendimo metodų aspektai. Atidžiai nagrinėjamas vienos modos atvejis ir gauti pastebėjimai leidžia sudaryti tinkamą skaitinio šio uždavinio sprendimo strategiją bendrajai girotrono lygčių sistemai, aprašančiai jo nestacionarius virpesius.