

REDUCTION OF PLANE THERMOELASTICITY PROBLEM IN INHOMOGENEOUS STRIP TO INTEGRAL VOLTERRA TYPE EQUATION

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Abstract. We have developed a method for analytical solving of the plane thermoelasticity problem in terms of stresses for a strip, which is infinite with respect to width. To derive the governing equations, we have used a method of direct integration of differential equilibrium and compatibility equations. Reducing the governing equations to the integral Volterra type equation of the second kind, we have solved it in Fourier transforms by applying a method of simple iteration.

Key words: inhomogeneous strip, plane thermoelasticity problem

1. Introduction

Recently, the demands of engineering caused the development of new directions in mechanics, thermoelasticity theory of inhomogeneous solids inclusive. It is well known that all the materials are inhomogeneous to certain extent [10]. Of special interest for theoretical and practical research are the solids with continuous dependence of their elastic properties on coordinate. As an example, we mention the functionally graded materials [9], whose elastic properties can be formed technologically, composites, etc. [10].

The main methods for constructing the analytical solutions of thermoelasticity problems for inhomogeneous solids are described in [5, 7, 10]. The essential difficulty one faces while using them consists in solving the differential equations with variable coefficients. In most cases, certain approximations are used, e.g., replacement of an inhomogeneous solid by a set of conjuncted homogeneous solids [17].

See [3, 4, 6] for application of such an approach to a plane elasticity problem in a strip inhomogeneous with respect to width.

Plevako showed [11] that representation of the continuously inhomogeneous material (elastic cylinder inhomogeneous in radial direction) by the soldered homogeneous layers gives a very slow convergence to an exact solution if the number of layers increases. So, he proposed to consider the inhomogeneous cylinders having such elastic characteristics that enable easy construction of the solution. Then the elastic characteristics can be approximated by continuous polylines instead of piecewise-constant functions, improving the approximation towards an exact solution.

Despite of many approaches to solution of the thermoelasticity problems for inhomogeneous solids, there exists a strong need in analytical methods. Those methods would enable finding solutions in the form of a functional dependence on the loadings thus being efficient for different kinds of inhomogeneity, loadings, and shapes. It is known, that such solutions are most convenient for solving inverse problems of thermomechanics and the problems of optimal control of thermo-stressed state [13].

The paper deals with construction of an analytical solution of the plane thermoelasticity problem in terms of stresses for a strip inhomogeneous in its cross-section. To solve the problem, we use a method of direct integration of equilibrium and compatibility equations proposed by Vihak [14]. Such an approach enables easy application of the method for solving the problems for inhomogeneous solids, since the equilibrium equations, which are integrated directly, are independent of the mathematical model of physical relations between stresses and strains. The method has been already applied to some one-dimensional problems [15, 16].

2. Statement of the Problem

We consider a plane quasi-static thermoelasticity problem in the strip

$$D = \{ (x, y) : (x, y) \in (-\infty, \infty) \times [-1, 1] \}$$

for the case of inhomogeneous isotropic material. The problem is governed by the equilibrium equations [1, 8]

$$\begin{cases} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + F_x = 0, & (x, y) \in D, \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + F_y = 0, \end{cases} \quad (2.1)$$

compatibility equation in terms of strains

$$\frac{\partial^2 e_x}{\partial y^2} + \frac{\partial^2 e_y}{\partial x^2} = \frac{\partial^2 e_{xy}}{\partial x \partial y}, \quad (2.2)$$

and the physical relations for plane strain ($e_z = 0$)

$$\begin{aligned} E e_x &= \sigma_x - \nu(\sigma_y + \sigma_z) + \alpha E T, & E e_y &= \sigma_y - \nu(\sigma_x + \sigma_z) + \alpha E T, \\ \sigma_z &= \nu(\sigma_x + \sigma_y) - \alpha E T, & G e_{xy} &= \sigma_{xy}, & 2G &= E/(1 + \nu). \end{aligned} \quad (2.3)$$

We prescribe the tractions at the boundary

$$\begin{aligned}\sigma_y|_{y=1} &= -p_1(x), & \sigma_y|_{y=-1} &= -p_2(x), \\ \sigma_{xy}|_{y=1} &= q_1(x), & \sigma_{xy}|_{y=-1} &= q_2(x)\end{aligned}\quad (2.4)$$

and assume that stresses are tending to zero as $|x| \rightarrow \infty$. Here $\sigma_j, \sigma_{xy}; e_j, e_{xy}$, ($j = x, y, z$) are the stress and strain tensor components, respectively; $x = x^*/b$, $y = y^*/b$; x^*, y^* are the Cartesian coordinates ($y^* \in [-b, b]$); E, G, ν, α denote the Young's modulus, shear modulus, Poisson's ratio, and the coefficient of linear thermal expansion, which are the functions of the y -coordinate; F_x, F_y are the body forces in the dimension of stress, and T denotes a prescribed temperature field. We assume that the force and thermal loadings depend on time parametrically, so, we skip the t -variable for shortening of notation.

3. Reduction of the Problem to Governing Equations

Following [14], we reduce the set of equations (2.1)–(2.4) to two governing equations for the normal stress σ_y and total stress $\sigma = \sigma_x + \sigma_y$ (we call them the key stresses). To derive the first governing equation, we represent (2.2) in terms of stresses. For that, we eliminate σ_z from (2.3) and express σ_x in terms of the key stresses, to obtain

$$\begin{aligned}2Ge_x &= (1 - \nu)\sigma - \sigma_y + 2\alpha G(1 + \nu)T, \\ 2Ge_y &= -\nu\sigma + \sigma_y + 2\alpha G(1 + \nu)T.\end{aligned}$$

Using the obtained expressions for strains in terms of stresses, the fourth relation (2.3), and the equilibrium equations (2.1), we represent (2.2) in the form

$$\begin{aligned}\frac{\partial^2}{\partial y^2} \left(\frac{1 - \nu}{2G} \sigma + \alpha(1 + \nu)T \right) + \frac{1 - \nu}{2G} \frac{\partial^2 \sigma}{\partial x^2} + \alpha(1 + \nu) \frac{\partial^2 T}{\partial x^2} \\ = \frac{\sigma_y}{2} \frac{d^2}{dy^2} \left(\frac{1}{G} \right) - F_y \frac{d}{dy} \left(\frac{1}{G} \right) - \frac{1}{2G} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \right).\end{aligned}\quad (3.1)$$

Further, we use the relation

$$\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial F_x}{\partial x} = \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial F_y}{\partial y},$$

following from (2.1) by elimination of shear stress. Addition $\partial^2 \sigma_y / \partial x^2$ to both sides of the latter equation yields

$$\Delta \sigma_y = \frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial F_x}{\partial x} - \frac{\partial F_y}{\partial y}, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.\quad (3.2)$$

We complement (3.1) – (3.2) by two boundary conditions (2.4) for σ_y and those for the derivatives

$$\frac{\partial \sigma_y}{\partial y} \Big|_{y=1} = -\frac{dq_1}{dx} - F_y(x, 1), \quad \frac{\partial \sigma_y}{\partial y} \Big|_{y=-1} = -\frac{dq_2}{dx} - F_y(x, -1).\quad (3.3)$$

They follow from (2.4)_{3,4} by satisfying (2.1) at strip's sides $y = \pm 1$.

After determining the key stresses, the stress σ_x is calculated by the formula $\sigma_x = \sigma - \sigma_y$. Finally, the shear stress is determined by integration of the equilibrium equations:

$$4\sigma_{xy} = q_1 + q_2 - \int_{-1}^1 \left(\frac{\partial \sigma_x}{\partial x} + F_x \right) \text{sign}(y - \xi) d\xi - \int_{-\infty}^{\infty} \left(\frac{\partial \sigma_y}{\partial y} + F_y \right) \text{sign}(x - \eta) d\eta. \quad (3.4)$$

4. Solution of the Governing Equations

To calculate the key stresses, we apply the integral Fourier transform [2] by x to (3.1), (3.2), (2.4)_{3,4}, and (3.3), to obtain the following problem in Fourier space:

$$\frac{d^2 \bar{\sigma}_y}{dy^2} - s^2 \bar{\sigma}_y = -s^2 \bar{\sigma} + is \bar{F}_x - \frac{d\bar{F}_y}{dy}, \quad (4.1)$$

$$\begin{aligned} \frac{d^2}{dy^2} \left(\frac{1-\nu}{2G} \bar{\sigma} + \alpha(1+\nu) \bar{T} \right) - s^2 \left(\frac{1-\nu}{2G} \bar{\sigma} + \alpha(1+\nu) \bar{T} \right) \\ = \frac{\bar{\sigma}_y}{2} \frac{d^2}{dy^2} \left(\frac{1}{G} \right) - \bar{F}_y \frac{d}{dy} \left(\frac{1}{G} \right) - \frac{1}{2G} \left(is \bar{F}_x + \frac{d\bar{F}_y}{dy} \right), \end{aligned}$$

$$\bar{\sigma}_y|_{y=1} = -\bar{p}_1, \quad \bar{\sigma}_y|_{y=-1} = -\bar{p}_2, \quad (4.2)$$

$$\left. \frac{d\bar{\sigma}_y}{dy} \right|_{y=1} = -is\bar{q}_1 - \bar{F}_y(1), \quad \left. \frac{d\bar{\sigma}_y}{dy} \right|_{y=-1} = -is\bar{q}_2 - \bar{F}_y(-1).$$

Here s denotes a parameter of the integral transform, $i = \sqrt{-1}$.

By solving (4.1) – (4.2), we arrive at the expression for $\bar{\sigma}_y$

$$\begin{aligned} \bar{\sigma}_y = -\bar{p}_2 \cosh(s(1+y)) - \left(i\bar{q}_2 + \frac{1}{s} \bar{F}_y(-1) \right) \sinh(s(1+y)) \\ + \int_{-1}^y \left(i\bar{F}_x(\xi) - \frac{1}{s} \frac{d\bar{F}_y(\xi)}{d\xi} - s\bar{\sigma}(\xi) \right) \sinh(s(y-\xi)) d\xi \end{aligned} \quad (4.3)$$

and two integral conditions

$$\begin{aligned}
\int_{-1}^1 \bar{\sigma} \sinh(s\xi) d\xi &= (\bar{q}_1 + \bar{q}_2) \frac{i \sinh(s)}{s} - (\bar{p}_1 - \bar{p}_2) \frac{\cosh(s)}{s} \\
&+ \frac{\sinh(s)}{s^2} (\bar{F}_y(1) + \bar{F}_y(-1)) + \frac{1}{s} \int_{-1}^1 \left(i \bar{F}_x - \frac{1}{s} \frac{d\bar{F}_y}{d\xi} \right) \sinh(s\xi) d\xi, \\
\int_{-1}^1 \bar{\sigma} \cosh(s\xi) d\xi &= (\bar{q}_1 - \bar{q}_2) \frac{i \cosh(s)}{s} - (\bar{p}_1 + \bar{p}_2) \frac{\sinh(s)}{s} \\
&+ \frac{\cosh(s)}{s^2} (\bar{F}_y(1) - \bar{F}_y(-1)) + \frac{1}{s} \int_{-1}^1 \left(i \bar{F}_x - \frac{1}{s} \frac{d\bar{F}_y}{d\xi} \right) \cosh(s\xi) d\xi.
\end{aligned} \tag{4.4}$$

The second equation (4.1), accompanied by (4.3), yields

$$\begin{aligned}
\bar{\sigma} &= \frac{2G}{1-\nu} \{ A \cosh(sy) + B \sinh(sy) + P_2 \bar{p}_2 + Q_2 \bar{q}_2 + \Phi - \alpha(1+\nu) \bar{T} \\
&- \frac{1}{2} \int_{-1}^y \frac{d^2}{d\xi^2} \left(\frac{1}{G(\xi)} \right) \sinh(s(y-\xi)) \int_{-1}^{\xi} \bar{\sigma}(\eta) \sinh(s(\xi-\eta)) d\eta d\xi, \tag{4.5}
\end{aligned}$$

where

$$\begin{aligned}
\Phi &= \frac{1}{2s} \int_{-1}^y \frac{d^2}{d\xi^2} \left(\frac{1}{G(\xi)} \right) \sinh(s(y-\xi)) \int_{-1}^{\xi} \left(i \bar{F}_x(\eta) - \frac{1}{s} \frac{d\bar{F}_y(\eta)}{d\eta} \right) \\
&\quad \times \sinh(s(\xi-\eta)) d\eta d\xi - \frac{1}{s} \int_{-1}^y \left[\bar{F}_y(\xi) \frac{d}{d\xi} \left(\frac{1}{G(\xi)} \right) \right. \\
&\quad \left. + \frac{1}{2G(\xi)} \left(i s \bar{F}_x(\xi) + \frac{d\bar{F}_y(\xi)}{d\xi} \right) \right] \sinh(s(y-\xi)) d\xi + \frac{Q_2}{is} \bar{F}_y(-1), \\
P_2 &= -\frac{1}{2s} \int_{-1}^y \frac{d^2}{d\xi^2} \left(\frac{1}{G(\xi)} \right) \cosh(s(1+\xi)) \sinh(s(y-\xi)) d\xi, \\
Q_2 &= -\frac{i}{2s} \int_{-1}^y \frac{d^2}{d\xi^2} \left(\frac{1}{G(\xi)} \right) \sinh(s(1+\xi)) \sinh(s(y-\xi)) d\xi.
\end{aligned}$$

The constants A and B are determined by (4.4).

Change of order of integration in the integral of (4.5) yields the integral Volterra type equation of the second kind for the transform of total stress:

$$\bar{\sigma} = \frac{2G}{1-\nu} \left(A \cosh(sy) + B \sinh(sy) + P_2 \bar{p}_2 + Q_2 \bar{q}_2 + \Phi \right. \\ \left. - \alpha(1+\nu) \bar{T} - \frac{1}{2} \int_{-1}^y \bar{\sigma}(\eta) K(\xi, \eta, y) d\eta \right), \quad (4.6)$$

where

$$K(\xi, \eta, y) = \int_{\eta}^y \frac{d^2}{d\xi^2} \left(\frac{1}{G(\xi)} \right) \sinh(s(y-\xi)) \sinh(s(\xi-\eta)) d\xi.$$

Following [15, 16, 18], we solve (4.6) by a method of simple iteration [12]:

$$\bar{\sigma}_n = \frac{2G}{1-\nu} \left(A_n \cosh(sy) + B_n \sinh(sy) + P_2 \bar{p}_2 + Q_2 \bar{q}_2 + \Phi \right. \\ \left. - \alpha(1+\nu) \bar{T} - \frac{1}{2} \int_{-1}^y \bar{\sigma}_{n-1}(\eta) K(\xi, \eta, y) d\eta \right).$$

To compute the constants A_1 and B_1 , it is assumed that $\bar{\sigma}_0 = 0$, and (4.4) is used.

After having found $\bar{\sigma}$, we determine $\bar{\sigma}_y$ by (4.3). Applying the inverse Fourier transform [2], we calculate the normal stresses σ , σ_y and, after that, the shear stress σ_{xy} by means of (3.4).

Note that if $\frac{1}{G}$ is linear in y , equation (4.6) has an exact solution already at $n = 1$:

$$\bar{\sigma} = \frac{2G}{1-\nu} \left\{ A \cosh(sy) + B \sinh(sy) + H - \alpha(1+\nu) \bar{T} \right\}, \quad (4.7)$$

$$\bar{\sigma}_y = -\bar{p}_2 \cosh(s(1+y)) - \left(i\bar{q}_2 + \frac{1}{s} \bar{F}_y(-1) \right) \sinh(s(1+y)) - 2As \\ \times \int_{-1}^y \frac{G(\xi) \cosh(s\xi) \sinh(s(y-\xi))}{1-\nu(\xi)} d\xi - 2Bs \int_{-1}^y \frac{G(\xi) \sinh(s\xi) \sinh(s(y-\xi))}{1-\nu(\xi)} d\xi \\ + s \int_{-1}^y \frac{\alpha(\xi) E(\xi)}{1-\nu(\xi)} \bar{T}(\xi) \sinh(s(y-\xi)) d\xi - 2s \int_{-1}^y \frac{G(\xi) H(\xi)}{1-\nu(\xi)} \sinh(s(y-\xi)) d\xi,$$

where

$$A = \frac{I_2}{I_2 I_3 - I_1^2} \Psi_1 - \frac{I_1}{I_2 I_3 - I_1^2} \Psi_2, \quad B = \frac{I_3}{I_2 I_3 - I_1^2} \Psi_2 - \frac{I_1}{I_2 I_3 - I_1^2} \Psi_1,$$

$$I_1 = \int_{-1}^1 \frac{G(\xi)}{1-\nu(\xi)} \sinh(s\xi) \cosh(s\xi) d\xi, \quad I_2 = \int_{-1}^1 \frac{G(\xi)}{1-\nu(\xi)} \sinh^2(s\xi) d\xi,$$

$$I_3 = \int_{-1}^1 \frac{G(\xi)}{1-\nu(\xi)} \cosh^2(s\xi) d\xi,$$

$$\Psi_1 = \frac{1}{2} \int_{-1}^1 \bar{\sigma} \cosh(s\xi) d\xi - H_c + \frac{1}{2} \Theta_c, \quad \Psi_2 = \frac{1}{2} \int_{-1}^1 \bar{\sigma} \sinh(s\xi) d\xi - H_s + \frac{1}{2} \Theta_s,$$

$$H_c = \int_{-1}^1 \frac{G(\xi)H(\xi)}{1-\nu(\xi)} \cosh(s\xi) d\xi, \quad \Theta_c = \int_{-1}^1 \frac{\alpha(\xi)E(\xi)\bar{T}(\xi)}{1-\nu(\xi)} \cosh(s\xi) d\xi,$$

$$H_s = \int_{-1}^1 \frac{G(\xi)H(\xi)}{1-\nu(\xi)} \sinh(s\xi) d\xi, \quad \Theta_s = \int_{-1}^1 \frac{\alpha(\xi)E(\xi)\bar{T}(\xi)}{1-\nu(\xi)} \sinh(s\xi) d\xi,$$

$$H(y) = -\frac{1}{s} \int_{-1}^y \left[\bar{F}_y(\xi) \frac{d}{d\xi} \left(\frac{1}{G(\xi)} \right) + \frac{1}{2G(\xi)} \left(is\bar{F}_x(\xi) + \frac{d\bar{F}_y(\xi)}{d\xi} \right) \right] \times \sinh(s(y-\xi)) d\xi,$$

and the integral expressions of $\bar{\sigma}$ are determined by (4.4).

Finally, if $E, G, \nu = \text{const}$, then (4.7) provides us with the same expressions for σ_y and σ that have been found while solving the analogous problem for homogeneous material [14].

In the case of plane stress [1], [8], the governing equation (3.1) takes the form

$$\begin{aligned} \frac{\partial^2}{\partial y^2} \left(\frac{1}{E} \sigma + \alpha T \right) + \frac{1}{E} \frac{\partial^2 \sigma}{\partial x^2} + \alpha \frac{\partial^2 T}{\partial x^2} \\ = \frac{\sigma_y}{2} \frac{d^2}{dy^2} \left(\frac{1}{G} \right) - F_y \frac{d}{dy} \left(\frac{1}{G} \right) - \frac{1}{2G} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \right), \end{aligned}$$

and (4.6) reduces to

$$\begin{aligned} \bar{\sigma} = E \left(A \cosh(sy) + B \sinh(sy) + P_2 \bar{p}_2 + Q_2 \bar{q}_2 + \bar{\Phi} \right. \\ \left. - \alpha(1+\nu)\bar{T} - \frac{1}{2} \int_{-1}^y \bar{\sigma}(\eta) K(\xi, \eta, y) d\eta \right). \end{aligned}$$

5. Numerical Results

Consider an inhomogeneous strip D loaded by the tractions

$$p_1 = p_2 = \frac{\exp(-x^2)}{2}, \quad q_1 = q_2 = 0 \quad \text{at } F_x = F_y = T \equiv 0.$$

Let $G = G_0 = \text{const}$, $\nu = 1 - \frac{2}{3-ay}$ ($a = \text{const}$). By (4.7), the Fourier transforms of σ , σ_y are

$$\bar{\sigma} = \frac{2}{1-\nu} \left(A \cosh(sy) + B \sinh(sy) \right),$$

$$\bar{\sigma}_y = -\bar{p}_2 \cosh(s(1+y)) - i\bar{q}_2 \sinh(s(1+y))$$

$$- 2sA \int_{-1}^y \frac{\cosh(s\xi) \sinh(s(y-\xi))}{1-\nu} d\xi - 2sB \int_{-1}^y \frac{\sinh(s\xi) \sinh(s(y-\xi))}{1-\nu} d\xi,$$

where $D = I_1^2 - I_2 I_3$,

$$A = \frac{1}{2sD} \left[(i(\bar{q}_1 + \bar{q}_2) \sinh(s) - (\bar{p}_1 - \bar{p}_2) \cosh(s)) I_1 \right. \\ \left. + (i(\bar{q}_2 - \bar{q}_1) \cosh(s) + (\bar{p}_1 + \bar{p}_2) \sinh(s)) I_2 \right],$$

$$B = \frac{1}{2sD} \left[(i(\bar{q}_1 - \bar{q}_2) \cosh(s) - (\bar{p}_1 + \bar{p}_2) \sinh(s)) I_1 \right. \\ \left. - (i(\bar{q}_1 + \bar{q}_2) \sinh(s) - (\bar{p}_1 - \bar{p}_2) \cosh(s)) I_3 \right],$$

$$I_1 = \int_{-1}^1 \frac{\sinh(s\xi) \cosh(s\xi)}{1-\nu(\xi)} d\xi, \quad I_2 = \int_{-1}^1 \frac{\sinh^2(s\xi)}{1-\nu(\xi)} d\xi, \quad I_3 = \int_{-1}^1 \frac{\cosh^2(s\xi)}{1-\nu(\xi)} d\xi.$$

We see that $\bar{\sigma}$, $\bar{\sigma}_y$ are independent of G_0 , depending on the Poisson's ratio only, which is varying in y -coordinate.

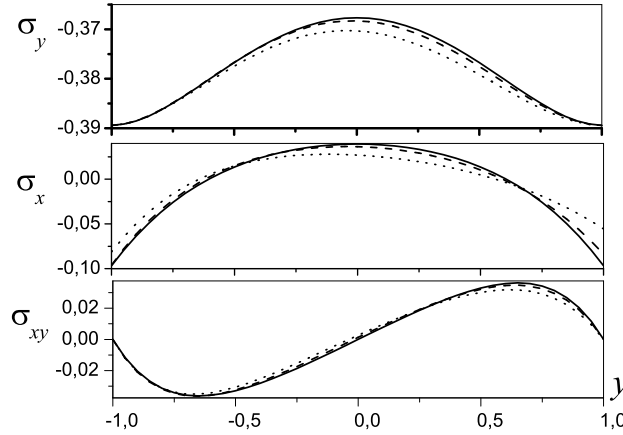


Figure 1. y -dependence of dimensionless stresses in D at $x = 0.5$ (solid line – $a = 0$, dashed line – $a = 0.5$, dotted line – $a = 1.0$).

Figure 1 demonstrates the y -distribution of dimensionless stresses in a strip D for different values of the parameter a . The solid curves correspond to the case of

a homogeneous material ($a = 0$, $\nu = 0.33$). Then the stresses σ_y and σ_x are even functions of planar coordinates, while σ_{xy} is an odd function. The dashed and dotted lines reflect the influence of material's inhomogeneity on stress distribution. Due to dependence of the Poisson's ratio on the y -coordinate, the law of paired normal stresses is violated. Moreover, the peaks of stresses are shifted in the direction of greater Poisson's ratio.

So, the coordinate dependence of the elastic characteristics of material has an enormous effect on the distribution of stresses.

6. Conclusions

The paper develops an approach to solving the plane thermoelasticity problem in terms of stresses for an inhomogeneous strip. The approach is based on the method of direct integration of differential equilibrium equations, which are independent of the mathematical model of relations between strains and stresses. Due to derived relations between stress tensor components, we can simplify calculation of the stressed state in an inhomogeneous strip considerably, if compared to solving such a problem in terms of displacements. In particular, we reduce the order of the governing differential equations with variable coefficients, derived on the basis of the compatibility and equilibrium equations.

The solution we have constructed enables calculation of the stressed state in a strip inhomogeneous with respect to width. It can be also applied for solving the corresponding inverse thermoelasticity problems as well as optimization problems.

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Nehomogeninio strypo termoelastiškumo uždavinio suvedimas į Volterra tipo integralinę lygtį

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Straipsnyje vystomas analizinio sprendinio metodas nehomogeninio strypo termoelastiškumo uždaviniui strypo įtempimams rasti, kai strypo ilgis yra begalinis pločio atžvilgiu. Pagrindinės lygtys išvedamos naudojant diferencialines pusiausvyros ir suderinamumo lygtis ir tiesioginį integravimą. Suvedus pagrindines lygtis į antrojo tipo Volterra integralinę lygtį, naudojant Furje transformaciją, ji sprendžiama paprastosios iteracijos metodu.