Mathematical Modelling and Analysis
Volume 9 Number 3, 2004, pages 179–192

APPLICATIONS OF INSPECTION GAMES

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Received July 27, 2004; revised September 9, 2004

Abstract. An inspection game is a mathematical model of a non-cooperative situation where an inspector verifies that another party, called inspectee, adheres to legal rules. The inspector wishes to deter illegal activity on the part of the inspectee and, should illegal activity nevertheless take place, detect it with the highest possible probability and as soon as possible. The inspectee may have some incentive to violate his commitments and violation, if observed, will incur punishment. Therefore if he chooses illegal behaviour, the inspectee will wish to avoid detection with the highest possible probability.

Three examples of applications are presented. The first one deals with random controls in public transportation systems. The second one describes the problem of verification of arms control and disarmament in a very general way. The third one deals with inspections over time which are important in the context of non-proliferation verification.

Key words: Extensive form game, interim inspection, Nash equilibrium, normal form game, public transportation, verification of arms control and disarmament agreements

1. Introduction

An inspection game is a mathematical model of a situation where an inspection authority, called inspector, verifies that another party, called inspectee, adheres to certain legal rules [4]. This legal behaviour may be defined, for example, by an arms control treaty, and the inspectee has a potential interest in violating these rules. Typically, the inspector's resources are limited so that verification can only be partial.

A mathematical analysis should help in designing an optimal inspection scheme, where it must be assumed that an illegal action is executed strategically. This defines a game theoretic problem, usually with two players, inspector and inspectee. In some cases, several inspectees are considered as individual players. Game theory is not only adequate to describe an inspection situation, it also produces results which may be used in practical applications. But what does that mean? Theoreticians and practitioners have, as we know, very different views about applications. Instead of discussing this question in an abstract manner, in this paper three cases will be presented which illustrate three different kinds of applications. For that purpose, we define operational, conceptual and structural models.

The first one which deals with random controls of passengers using public transportation systems gives a concrete advise what effort the inspector should spend in order to achieve his objectives. The second one which describes the problem of the verification of arms control and disarmament in a very general way provides insight into the nature of the inspection problem. The third one which deals with inspections over time which are important in the context of non-proliferation verification shows how sensibly the best inspection strategies depend on assumptions about the information the inspectee gains, or does not gain, in the course of the game. Whereas these three examples by no means do exhaust the wealth of models developed in the past forty years - the first serious attempts were made in the early 1960s where game theoretic studies of arms control inspections were commissioned by the US ACDA [10] – they should at least give an idea of what inspection models can achieve and furthermore that each inspection problem has its own characteristics which require new models and appropriate solution techniques.

Since in this paper the emphasis is put on the modelling aspect and furthermore on the use of the models for practical applications, proofs are only sketched and results are not presented in form of theorems; both proofs and theorems can be found in the references.

2. Operational Model: Passenger Ticket Control

In its edition of July 8th, 1997, the daily Süddeutsche Zeitung reports about the complaints by the city treasurer of Munich regarding the passenger ticket control applied within the area of the Munich Transport and Fares Tariff association (Münchner Verkehrs- und Tarifverbund, MVV). The deployment of inspectors was not worthwhile since they make up for only about half of what they cost themselves by charging the extra fares (fines) i.e., the employment of them was not profitable. It is obvious that there must be an optimum high incidence of controlling: if there was only one inspector many passengers would go without paying, i.e., this one single inspector would collect a lot of fines which is certainly not in line with the interest of the MVV, although the inspector would pay off. If on the other hand all passengers were checked, all of them would pay for the fares. This would please the MVV, however, the numerous and expensive ticket inspectors would not take in any fines at all. Where does thus the optimum regulation for the entire MVV system lie?

Since the behaviour of passengers, of whom one can assume strategical conduct i.e. reflections regarding payment or non-payment (morale aspects should be disregarded in this incident), must be taken into account when the assessment of the optimum frequency of controls by the MVV is made, a decision theoretical, precisely a game theoretical analysis of the problem is required.

The ”game” is conducted by the inspector, representing the MVV on one part, the frequency of controls being his strategic variable, and the passenger on the other side who decides between the alternatives of paying (legal behaviour) or not paying the fare (illegal behaviour).

Let f be the normal fare, b the fine and e the costs of controls per passenger. We assume $e < b$. Then the payoffs to the two „players" (inspector, passenger) in the four possible situations (outcomes) are

$$\begin{aligned} &(f-e,-f) &&\text{for control and legal behaviour,}\\ &(f,-f) &&\text{for no control and legal behaviour,}\\ &(b-e,-b) &&\text{for control and illegal behaviour,}\\ &(0,0) &&\text{for no control and illegal behaviour.} \end{aligned} \tag{2.1}$$

We consider the normal form indicated in Table 1 showing a two-person-game between the MVV being represented by the inspector as first player and the passenger as second. In this diagram the pure strategies of the first player (control/no control) are depicted as rows and the second player's as columns (legal/illegal behaviour); in the individual squares the payments to the first player resulting from the respective combination of strategies are put down on the left bottom and those to the second player on the top right.

Table 1. Normal form of the two person game between the inspector representing the MVV and the passenger. The arrows indicate the preference directions of the two players.

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	inspector \ passenger	legal behaviour (q)	illegal behaviour (1-q)	
↓	control (p)	-f f-e	-b b-e	↑
	no control (1-p)	-f f	0 0	

⟶

Taking this formulation of the problem we ignore the costs on the part of the MVV for maintenance of the business since these do not influence the decisions of both players immediately and also for the same reason we ignore the ideal or material gain the passenger has from his trip.

According to John Nash [8], one of the Economics Nobel prize winners of 1994, we understand by a solution of this game a pair of equilibrium strategies implying the quality that if one of the two players deviates unilaterally from his equilibrium strategy he cannot improve his payment. In so doing we ignore the difficult problem of the existence of multiple equilibria since in our cases they do not occur.

Since according to Table 1 the preference directions of the two players, i. e., the incentive to deviate from a chosen strategy go cyclical, there is no equilibrium in pure strategies. The inspector will thus control with probability p, and the passenger will behave legally with probability q. The expected payments to the two players are given in this case by

$$\begin{aligned} E_1(p,q) &= (f-e)\,p\,q + (b-e)\,p\,(1-q) + f\,(1-p)\,q\,,\\ E_2(p,q) &= -f\,p\,q - b\,p\,(1-q) - f\,(1-p)\,q\,. \end{aligned} \tag{2.2}$$

If we designate *the mixed equilibrium strategies* of the two players as p^*, and q^*, and the equilibrium payments as $E_i^* = E_i(p^*, q^*), i = 1, 2$, the equilibrium conditions according to John Nash are

$$E_1^*(p,q) \geqslant E_1(p,q^*) \quad \text{for all} \quad p \in [0,1]\,,$$
$$E_2^*(p,q) \geqslant E_2(p^*,q) \quad \text{for all} \quad q \in [0,1]\,. \tag{2.3}$$

In our case, the equilibrium strategies can be determined so that the adversary is indifferent as regards to the choice of his own strategy, see e.g. Morris [7]. As a result, the equilibrium strategies and – payments are given as follows [1]:

$$p^* = \frac{f}{b}, \qquad E_1^* = f\,\left(1 - \frac{e}{b}\right)\,, \tag{2.4}$$

$$q^* = 1 - \frac{e}{b}, \quad E_2^* = -f\,. \tag{2.5}$$

Thus in the equilibrium the passenger with a positive probability $1 - q^*$ behaves illegally, in the mean, however, he pays the same price he would pay if he always behaved legally. The reduced price achieved by dodging the fare is compensated by the obligatory fine.

The mean value of the control expenditure by the MVV per passenger is $e\,p$, while the profit from the fines is $b\,p\,(1-q)$. Thus the difference is

$$e\,p - b\,p\,(1-q) = \big(e - b\,(1-q)\big)\,p \gtreqless 0 \quad \text{for} \quad \frac{e}{b} \gtreqless 1 - q\,.$$

If the passenger chooses his equilibrium strategy q^* given by (2.5), the following holds

$$\big(e - b\,(1-q^*)\big)\,p = 0 \tag{2.6}$$

for any control probability p, i.e., *the investment of control is just being compensated by the amount of fines taken in.* It must be noted that these considerations only include the parameters e and b, but not the fare f of the trip.

The optimum control probability p^* satisfies the condition $p^*\,b = f$, which can be understood intuitively: if the passenger behaves legally he has to pay $-f$, whereas his expected payment in case of illegal behaviour is $-b\,p^*$. Thus the optimum control probability renders the passenger indifferent as regards to the strategy to be chosen by him.

Thus, in conclusion, this game theoretical model gives an advise how frequently passengers should be controlled if the fare and the fine are fixed. It should be mentioned that the actual figures for Munich approximately satisfy (2.4). One may speculate why then, according to the City Treasurer's complaints, the equilibrium condition (2.6) is not satisfied. Certainly one reason is that the inspections are nor purely random: passengers who systematically do not buy tickets frequently recognize the inspectors already before they can do their job. Another reason are the considerable deviations of passengers in frequency, hour of the day and dwelling time from the average which is not adequately taken into account by the inspectors.

A stratification of inspection procedures which would be required here has been analysed in the context of arms control, see [2]. There it turns out that the inspection efforts in the different strata have to be the higher, the more profitable illegal actions are, and in turn that the illegal actions are concentrated there as well. It will be pointed out in the last section, however, that one has to be very careful in predicting results of yet unsolved problems.

Finally, it should be mentioned that there is also not intentional illegal behaviour – e.g., passengers use monthly tickets but forget to take them with them – which can be modelled as well [1]; the analysis shows, however that it does not change the results in a significant way.

3. Conceptual Models: Arms Control and Disarmament Verification

As a second case, let us consider an international arms control and disarmament agreement, for example the Treaty for the Non-Proliferation of Nuclear Weapons, or the Chemical Weapons Convention. A State who signs this agreement is obliged not to act illegally in that sense that he does not do anything that is forbidden by the agreement, for example to acquire nuclear or chemical weapons.

Let us assume furthermore that, together with the agreement, a verification system is established which means that an international authority verifies with the help of well-defined measures - measurements, on-site inspections and others - that the inspected State adheres to the provisions of the agreement. For the Non-Proliferation Treaty, for example, the International Atomic Energy Agency (IAEA) plays that role. The purpose of the verification is to deter the State from illegal behaviour or, should he behave illegally, to detect this with as high a probability and as quickly as possible.

On the other hand, the inspected State may have some incentive to violate his commitments – otherwise the situation is pointless, we will come back to this issue, – and violation, if observed, will incur punishment of the State. Therefore, if he chooses illegal behaviour, the inspected State will wish to avoid detection with the highest possible probability.

In the following we will describe this conflict situation between the verification authority (in short inspector) and the State (in short inspectee) with the help of a non-cooperative two-person game. Let the payoffs to the inspector as the first player and to the inspectee as the second player be given by

$$\begin{array}{ll} (0,0) & \text{for legal behaviour of the inspectee,} \\ (-a,-b) & \text{for detected illegal behaviour of the inspectee,} \\ (-c,d) & \text{for undetected illegal behaviour of the inspectee.} \end{array} \tag{3.1}$$

Note that inspection costs are not taken into account explicitly. We assume

$$0 < a < c, \quad 0 < b, \quad 0 < d\,, \tag{3.2}$$

the first inequality expresses the fact that the highest priority of the inspector is to deter the inspectee from illegal behaviour.

In keeping with common notation, let us call $1-\beta$ be the probability to detect illegal behaviour. Then the expected payoffs to the two players are

$$(0,0) \quad \text{for legal behaviour of the inspectee,} \tag{3.3}$$

$$(-a\,(1-\beta) - c\,\beta,\ -b\,(1-\beta) + d\,\beta) \quad \text{for illegal behaviour of the inspectee.}$$

Furthermore we assume that the inspector, in a concrete situation, decides either to verify or not, and the inspectee, in turn, to behave legally or not. The normal form of this two-by-two-game is given by Table 2.

Table 2. Normal form of the two person game between a State (inspectee) and the verification authority (inspector). False alarms are not possible. The arrows indicate the preference directions of the two players if (3.4) is fulfilled.

	inspector \ passenger	legal behaviour	illegal behaviour ($\longleftarrow$ above, $\longrightarrow$ below)	
$\updownarrow$	Verification	0 (inspectee) 0 (inspector)	$-b(1-\beta)+d\beta$ (inspectee) $-a(1-\beta)-c\beta$ (inspector)	$\uparrow$
	No verification	0 (inspectee) 0 (inspector)	d (inspectee) $-c$ (inspector)	

As a solution of this game we consider again the Nash equilibrium. Using the method of incentive directions, we see immediately that legal behaviour is the only equilibrium strategy of the inspectee if

$$0 > -b\,(1-\beta) + d\,\beta\,,$$

or, equivalently, if

$$\beta < \frac{1}{1+\frac{d}{b}}\,. \tag{3.4}$$

Thus, as a result we see that the inspectee will be induced to legal behaviour if the non-detection probability is smaller than some threshold, which is the lower, the larger the ratio between gain in case of undetected illegal behaviour and the sanctions b in case of detected illegal behaviour is. Otherwise he will behave illegally. Alternatively one may say that the inspectee will behave legally if either the probability of no detection or the ratio $\dfrac{d}{b}$ is small enough.

Now let us consider a more complicated problem: Let us assume that false alarms may happen with probability α. Let the payoffs to the two players in case of a false alarm be $-e < 0$ and $-f < 0$. We assume

$$0 < e < a < c, \quad 0 < f < b, \quad 0 < d\,.$$

Then the normal form of the verifi cation game is given by Table 3.

We see immediately that legal behaviour is not an equilibrium strategy. However, for

$$-f\alpha > -b\,(1-\beta) + d\,\beta\,,$$

or equivalently, for

Table 3. Normal form of the two person game between a State (inspectee) and the verification authority (inspector). False alarms occur with probability α. The arrows indicate the preference directions of the two players if (3.5) is fulfilled.

	inspector \ passenger	legal behaviour (q)	illegal behaviour (1-q)	
		⟵		
↓	Verification	$-f\alpha$ / $-e\alpha$	$-b(1-\beta)+d\beta$ / $-\alpha(1-\beta)-c\beta$	↑
	No verification	0 / 0	d / $-c$	
			⟶	

$$\beta < \frac{-f}{b+d}\,\alpha + \frac{b}{b+d} \tag{3.5}$$

there exists a Nash equilibrium in mixed strategies: The inspectee will act illegally with probability q^* as given by

$$\frac{1}{q^*} = 1 + \frac{c-a}{e}\,\frac{1-\beta}{\alpha}\,. \tag{3.6}$$

Since, as mentioned initially, and contrary to the previous example, where morale problems were not dominating, the purpose of the treaty is that the State fulfi lls the provisions of the treaty, the question arises if there is any possibility to induce the inspectee to legal behaviour.

Let us assume that α is a strategic variable of the inspector. Of course, β depends on α. For unbiased test procedures we have

$$\alpha + \beta < 1\,. \tag{3.7}$$

Let us assume in addition

$$\beta = 1 \quad \text{for } \alpha = 0\,, \quad \text{and} \quad \beta = 0 \quad \text{for } \alpha = 1\,, \tag{3.8}$$

and furthermore,

$$\frac{d\beta}{d\alpha} < 0, \quad \frac{d^2\beta}{d\alpha^2} < 0. \tag{3.9}$$

Then the problem of choosing an appropriate value of α can be represented graphically as given in Figure 1*a*.

We see that for $\alpha = \alpha_0$ and $\beta = \beta_0 = \beta(\alpha_0)$ as defi ned in the fi gure the inspectee is indifferent between legal and illegal behaviour.

Now let us change the rules of the game [2]. Instead of the inspector's two alternatives considered so far, namely verifying with a fi xed false alarm probability or not verifying, we now assume that the inspector always verifi es, and that his set of strategies consists in the possible choices of the false alarm probability. In addition, he will announce his strategy in a credible way. The extensive form of this so-called inspector leadership game is given by Figure 1*b* .

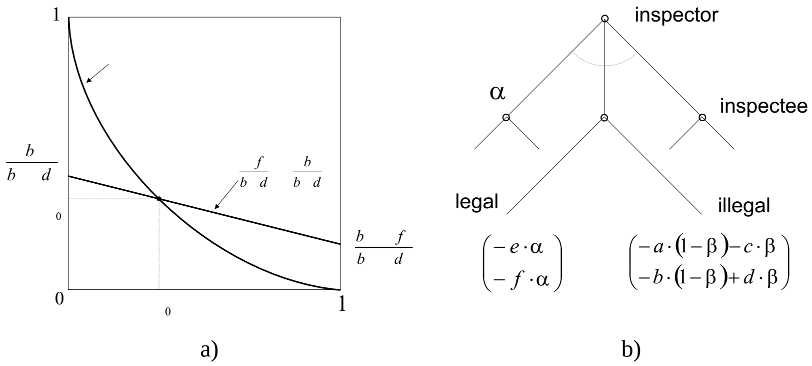

Figure 1. a) graphical representation of the value of which makes the inspectee indifferent between legal and illegal behaviour, b) extensive form of the leadership game between the verification authority (inspector) and the State (inspectee).

According to the backward induction procedure the inspectee will

$$\begin{cases} \text{behave legally}, & \text{if } -f\,\alpha > -b\,(1-\beta)+d\,\beta\,, \\ \text{be indifferent}, & \text{if } -f\,\alpha > -b\,(1-\beta)+d\,\beta\,, \\ \text{behave illegally}, & \text{if } -f\,\alpha > -b\,(1-\beta)+d\,\beta\,. \end{cases} \tag{3.10}$$

or, equivalently, with α_0 as defi ned in Figure 1*a*,

$$\begin{cases} \text{behave legally}, & \text{if } \alpha > \alpha_0\,, \\ \text{be indifferent}, & \text{if } \alpha = \alpha_0\,, \\ \text{behave illegally}, & \text{if } \alpha < \alpha_0\,. \end{cases} \tag{3.11}$$

For this best reply of the inspectee, the payoff to the inspector is given as represented graphically in Figure 2.

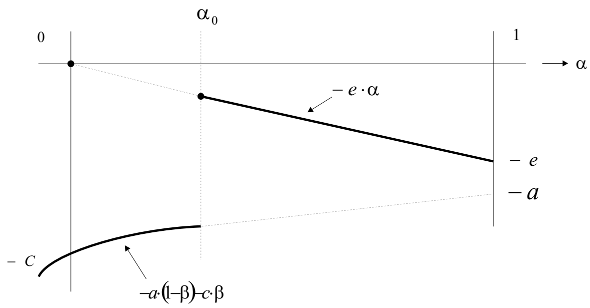

Figure 2. Payoff to the inspector for the best reply of the inspectee.

One sees immediately that the payoff of the inspector has its maximum at α_0. Now, surprisingly enough it can be shown that $\alpha = \alpha_0$ and

$$\begin{cases} \text{legal behaviour}, & \text{for } \alpha \geq \alpha_0 \,, \\ \text{illegal behaviour}, & \text{for } \alpha < \alpha_0 \end{cases} \tag{3.12}$$

are Nash equilibrium strategies. This means in effect that the inspector chooses $\alpha = \alpha_0$ and that consequently, the inspectee acts legally – even though at that point he is indifferent between legal and illegal behaviour.

In discussions on the usefulness of verification in general, arms control and disarmament officials, administrators and political scientists have frequently criticized, and still do so, that game theorists or more generally, analysts working quantitatively, always assume that the State might behave illegally even though he has ratified the agreement under consideration. In the beginning we mentioned that without this assumption the situation would be pointless. Now we can be more precise: in order to show that appropriate verification on one hand and legal behaviour of the State on the other are equilibrium strategies we have to study deviations – quite in the spirit of Nash's equilibrium concept.

4. Structural Models: Interim Inspections

Finally, as a third case, we consider a single inspected object, for example a nuclear or chemical facility subject to verification in the framework of an international treaty, and a reference period of one time unit (e.g. one calendar year). In order to separate the timeliness aspect of routine inspection from the overall goal of detecting illegal activity, we assume that a thorough and unambiguous inspection takes place at the end of the reference period which will detect an illegal activity with certainty once it has occurred.

In addition there are a number of less intensive and strategically placed „interim" inspections which are intended to reduce the time to detection below the length of the reference period. An interim inspection will detect a preceding or coincident illegal activity, but with some lower probability. Again in keeping with common notation, we call this probability $1 - \beta$, where β is the probability of an error of the second kind, or non-detection probability.

Associated with each interim inspection which is not preceded by an illegal action is a corresponding probability of an error, the false alarm probability α. Moreover, again only an unbiased inspection procedure is considered.

We assume that, by agreement, k interim inspections will occur within the reference period. For convenience we label the inspections backwards in time. Also we label the beginning of the reference time t_{k+1} and the end t_0, so we have

$$0 = t_{k+1} < t_k < \ldots < t_1 < t_0 = 1. \tag{4.1}$$

The utilities of the protagonists (inspector, inspectee) are taken to be as follows:

$(0, 0)$	for legal behaviour over the reference time, and no false alarm,
$(-le, -lf)$	for legal behaviour over the reference time, and l false alarms, $l = 1, \ldots, k$
$(-\alpha\Delta t, d\Delta t - b)$	for detection of illegal activity after elapsed time $\Delta t \geq 0$,

where

$$0 < e < a, \quad 0 < f < b < d.$$

Thus the utilities are normalized to zero for legal behaviour without false alarms, and the loss (profi t) to the inspector (inspectee) grows proportionally with the time elapsed to detection of an illegal action. A false alarm is resolved unambiguously with time independent costs $-e$ to the inspector and $-f$ to the inspectee, whereupon the game continues. The quantity b is the cost to the inspectee of immediate detection. Note that, if $b > d$, the inspectee will behave legally even if there are no interim inspections at all. Since interim inspections introduce false alarm costs for both parties, there would be no point in performing them.

The extensive form of the inspection game for one single observable interim inspection is represented graphically in Figure 3. Without going through the analysis which uses similar techniques as sketched before, we just present the results [3]:

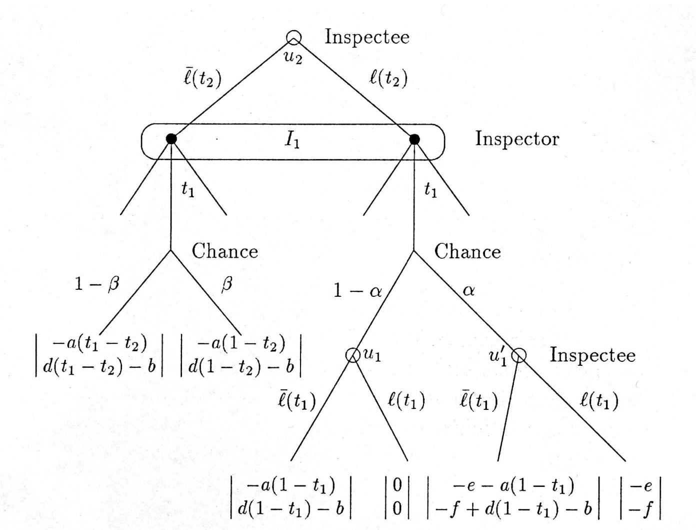

Figure 3. Extensive form of the two person game between the inspector and the inspectee for one interim inspection.

Let $\{t_1 : \; 0 < t_1 < 1\}$ be the set of pure strategies of the inspector, and the probabilities g_2, g_1 to start illegal actions at time moments $t = 0$, t_1 be the mixed behavioral strategies of the inspectee. Let V_2 and W_2 be the payoffs to the two players (here we use a notation different from that of the second section in order to remain consistent with the notation in the published literature). Taking into account that the inspectee will behave legally if his payoff is larger than in case he behaves illegally, and vice versa, the equilibrium strategies and payoffs are given as follows.

Under the assumption

$$\frac{b}{d} < \frac{1}{2-\beta}\left(1+\frac{f\,\alpha}{d}\right) \tag{4.2}$$

in equilibrium the inspectee acts illegally, with payoffs and strategies given by

$$
\begin{aligned}
V_2^* &= -a\,A_2 - e\,\alpha\,B_2\,,\\
W_2^* &= d\,A_2 - f\,\alpha\,B_2 - b\,,\\
t_1^* &= (1-\beta)\,A_2 - \frac{f\,\alpha}{d}\,\big((1-\beta)\,B_2 + \beta\big)\,,\\
g_2^* &= A_2, \quad g_1^* = 1,
\end{aligned}
$$

where A_2 and B_2 are given by

$$A_2 = \frac{1}{2-\beta}, \quad B_2 = \frac{1-\beta}{2-\beta}. \tag{4.3}$$

Under the assumption (we exclude equality being practically not important)

$$\frac{b}{d} > \frac{1}{2-\beta}\left(1 + \frac{f\alpha}{d}\right) \tag{4.4}$$

in equilibrium the inspectee acts legally, with the following payoffs to the two antagonists:

$$V_2^* = -ea, \quad W_2^* = -f\alpha\,.$$

The equilibrium strategy of the inspector is not unique; it is given by what M. Kilgour [6] called the cone of deterrence:

$$1 - \frac{b}{d} \leqslant t_1^* \leqslant \frac{1}{1-\beta}\left(\frac{b}{d} - \frac{f\alpha}{d} - \beta\right)\,. \tag{4.5}$$

It can be shown that the equilibrium strategy of the inspector in case of illegal behaviour of the inspectee is an element of the cone of deterrence (4.5). Thus, the inspector is on the safe side if he always uses the former one.

It is also possible to generalize this solution to more than one interim inspection however, the analysis gets rather involved since non-trivial information sets have to be taken into account and furthermore, since unrealistic solutions may occur where some interim inspections may have to be conducted right at the beginning of the reference time [3]. Nevertheless are the realistic solutions for more than one interim inspection of the same structure as that given by (4.3), with more complicated expressions for A and B; whereas it is not easy to fi nd them, it is straightforward to prove their validity via complete induction.

We will not delve into these intricacies. Instead, we consider an inspection problem which differs from the previous one only by the fact that now the interim inspections are unobservable or – formally the same – that prior commitment on the part of the inspectee is assumed. That means that now we consider a simultaneous rather than a sequential game.

This game has been analysed by H. Diamond [5] for an arbitrary number of interim inspections, however, without taking into account false alarms. The analysis of the game for one interim inspection and the possibility of false alarms is due to Sohrweide [9] and again we just present the main results.

For one interim inspection an equilibrium strategy for the inspector is to choose his single interim inspection time t_1 on an interval $0 < t_1 \leqslant \kappa < 1$ according to the distribution function

$$F^*(t_1) = -\frac{1}{1-\beta} \ln \left(1 - \frac{t_1}{1 - \frac{f}{d}\frac{\alpha}{1-\beta}}\right), \tag{4.6}$$

where κ is given by

$$\kappa = \left(1 - \frac{1}{e^{1-\beta}}\right)\left(1 - \frac{1}{d}\frac{\alpha}{1-\beta}\right). \tag{4.7}$$

Under the assumption

$$\frac{b}{d} > 1 - \kappa \tag{4.8}$$

the inspectee behaves illegally; he randomizes similarly, however, his distribution function $Q^*(t)$, as given by

$$Q^*(t) = \frac{1 - \kappa + \frac{e}{a}\frac{\alpha}{1-\beta}}{1 + \frac{e}{a}\frac{\alpha}{1-\beta}} \quad \text{for } 0 \leqslant t \leqslant \kappa, \tag{4.9}$$

has an atom at $t = 0$:

$$Q^*(0) = 1 - \frac{\kappa}{1 + \frac{e}{a}\frac{\alpha}{1-\beta}} > 0\,.$$

Thus, both players necessarily play mixed strategies in equilibrium, with payoffs

$$\begin{aligned} V_2^* &= -a\left[\beta\,(1-\kappa) - (1 - \kappa + \frac{e}{a}\frac{\alpha}{1-\beta})\ln\left(1 - \frac{\kappa}{1 - t + \frac{e}{a}\frac{\alpha}{1-\beta}}\right)\right], \\ W_2^* &= d\,(1-\kappa) - f - b\,. \end{aligned} \tag{4.10}$$

If (4.8) is not fulfilled (we exclude equality being practically not important) the inspectee behaves legally, with payoffs being the same as in the previous model.

Whereas the inspectee's payoff W_2^* as given by (4.10) can be understood easily – it is just his payoff in case he starts his illegal action at time $t = \kappa$ – this is not so easy in case of the inspector's payoff unless we have $e = -f$ which can hardly be justified.

It turns out, not surprisingly, that the unobservability places the inspectee at a disadvantage: his payoff in case of illegal behaviour is smaller than that for an observable inspection, and the limit for $\frac{b}{d}$ to induce the inspectee to legal behaviour is lower.

At first sight it is very surprising that for one well specified inspection problem different assumptions about the information the inspectee gains during the course of the game or does not gain, lead to totally different results: In the first case the inspector plays in equilibrium a pure, in the second case a mixed strategy. This is the general lesson to be drawn from this very concrete inspection problem: Even if one has studied so many different problems, one hardly will be able to predict the outcome of a new or even only modified one.

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Kontrolės lošimų taikymai

R. Avenhaus

Kontrolės lošimas yra matematinis modelis tam tikrų nekooperacinių situacijų, kai inspektorius kontroliuoja kitą pusę, skatindamas korektišką elgesį. Inspektorius turi atbaidyti uždraustus veiksmus su kuo galima didesne tikimybe ir kuo greičiau. Tai reiškia, kad už nustatytų taisyklių pažeidimą mokama tam tikra bauda ir šie pažeidimai aptinkami su maksimalia tikimybe. Straipsnyje nagrinėjami trys šio modelio taikymo pavyzdžiai: atsitiktinė visuomeninio transporto keleivių kontrolė, ginklų kontrolės modelis ir paplitimo ribojimo per tam tikrą laiką modelis.