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TRAJECTORY PLANNING ALGORITHM FOR MERGING CONTROL OF HETEROGENEOUS VEHICULAR PLATOON ON CURVE ROAD

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Abstract. The main goal of this study is to propose a trajectory planning algorithm for the merging control of heterogeneous vehicular platoons. Merging control is essential for the application of vehicular platoons, by which vehicles can be coordinated to form a platoon in a lane. While most previous researches on merging control only considered the operation on straight roads and ramps. Few studies have investigated the merging operation on curve roads, which may hinder the application of platoons on general traffic environment. In this study, a trajectory planning algorithm is proposed for the merging control of heterogeneous vehicular platoons on curve roads with constant radius. The proposed algorithm consists of two stages for the operation of merging: the first stage is to align the vehicles in each lane to form a structure with a certain clearance; the second stage is to conduct a lane changing manoeuvre for each merging vehicle to form a platoon in a lane. In the proposed algorithm, the dynamic limits of speed and acceleration are considered. The distance of each vehicle can be guaranteed to avoid undesired collisions. Two simulations are carefully conducted for the merging control of heterogeneous vehicular platoons on a curve road to demonstrate the effectiveness of the proposed algorithm. The results of simulations indicate that the proposed algorithm is capable of the merging control of platoons on a curve road.

Keywords: heterogeneous vehicular platoon, merging control, trajectory planning, curve road, vehicular communication.

Introduction

With the development of vehicular network and communication, the technology of vehicular platoon has drawn considerable attention in both academic and industry circles. Various studies have investigated different aspects of vehicular platoons (Li *et al.* 2015). One of the basic requirements for a platoon is that it should be able to merge new vehicles and form a new platoon. Thus, more vehicles can be coordinated simultaneously to reduce energy consumption (Yu *et al.* 2016) and improve traffic safety (Amoozadeh *et al.* 2015).

Most previous studies focused on the merging control on straight roads and ramps. According to the shape of the studied road, the previous algorithms on merging control can be classified into three groups. The first group focused on the merging control on a straight road (Goli, Eskandarian 2014; Jeon, Choi 2001; Kazerooni, Ploeg 2015). Jeon and Choi (2001) used the linear feedback control to form ideal gaps between vehicles for the merging control of one merging vehicle. Based on the multi-agent algorithm, Kazerooni and Ploeg (2015) designed an interaction protocol for cooperative merging control of multiple merging vehicles. Goli and Eskandarian (2014) proposed an algorithm consisting of proportional-derivative and sliding mode controllers for merging operation in a platoon. The second group considered the merging control on a ramp (Cao *et al.* 2013; Rai *et al.* 2015; Sakaguchi *et al.* 1997). Sakaguchi *et al.* (1997) introduced a merging algorithm on a ramp based on the pole assignment technology and virtual vehicles. Cao *et al.* (2013) suggested a scheme of merging control on a ramp based the model predictive control algorithm. Rai *et al.* (2015) considered the artificial potential field for the design of the controller of merging operation. The third group investigated the merging control on both straight roads and ramps (Awal *et al.* 2013; Lam, Katupitiya 2013; Lu, Hedrick 2000; Lu *et al.* 2004; Uno *et al.* 1999; Wu, Chen 2008). Wu and Chen (2008) proposed the concept of a virtual platoon for the design of a merging control algorithm. Uno *et al.* (1999) discussed the principle of inter-vehicle communication for merging control. Lu *et al.* (2004) designed a vehicle controller for merging manoeuvre using the back-stepping and sliding mode algorithms. Lam and Katupitiya (2013) introduced a

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multi-agent event-based approach for the merging control on highways. Lu and Hedrick (2000) provided an adaptive close-loop algorithm to solve the problem of merging control. Awal *et al.* (2013) suggested an optimal merging strategy considering the average merging time and fuel consumption. These researches show very encouraging results in the field of merging control on straight roads and ramps. However, few previous researches considered the merging control of vehicular platoons on a curve road. As a merging operation is a conventional demand for a platoon, it is essential to study the merging control algorithm in this scope.

In this paper, the merging control on a curve road with constant radius is considered. A trajectory planning algorithm is proposed for the merging control of a heterogeneous platoon. The dynamic limits and collision avoidance are taken into account for the design of the proposed algorithm. The proposed algorithm consists of two stages to perform a merging operation. The first stage is to align the vehicles in each lane to form a structure with a certain clearance. The trajectory planning scheme based on optimization is designed in this stage to handle the constraints of dynamics. In the second stage, a scheme for the planning of lane changing trajectory on a curve road is proposed. Based on the planned trajectories the vehicles in adjacent lanes can merge into the lane of the desired platoon.

The main contribution of this paper is that a trajectory planning algorithm for merging control is proposed, this algorithm is probably the first attempt for the merging control of heterogeneous platoons on a curve road with constant radius. Based on the proposed algorithm, the study of heterogeneous platoons is extended to curve roads, which facilitates the application of vehicular platoons on more general traffic conditions.

The remainder of the paper is organized as follows: the problem of the merging control of heterogeneous platoons is demonstrated in Section 1; the trajectory planning algorithm for merging control is proposed in Section 2; in Section 3, numerical simulations are conducted to demonstrate the effectiveness of the proposed algorithm; finally, the conclusions and future research directions are presented.

1. Problem statement

Consider a platoon consisting of several vehicles in a lane. The merging control of the platoon is to merge some vehicles in the adjacent lanes into the lane of the platoon. The trajectory planning for merging control is to find the trajectories of these vehicles to merge into a new platoon. Several requirements should be considered for merging control. First, the collisions among these vehicles should be avoided, which is critical to prevent traffic accident. Second, the dynamic limits of these vehicles should be satisfied to avoid an impractical trajectory.

Several assumptions are made to limit the scope of this paper:

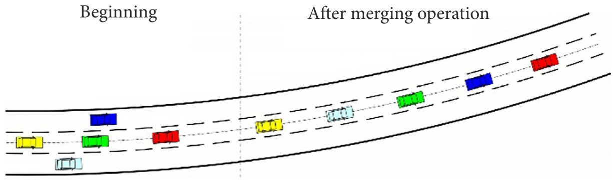

Figure 1. Example of merging control
(source: created by the authors)

- »» the vehicles in the same lane are of the same speed in the beginning of merging operation, which can be achieved by cruise control systems;
- »» the considered road is of constant radius, which means the central paths of the lanes are concentric circles;
- »» the considered vehicles are equipped with motor drive system and can follow the planned trajectories within the dynamic limits.

To facilitate the discussion, the lane of the platoon is called the main lane. The centre path of the main lane is called the main path. The lane of the merging vehicles is called the merging lane. The centre path of the merging lane is called the merging path. The radius of main path is called the main radius.

An example of merging control of a vehicular platoon is shown in Figure 1. The road consists of three lanes. As shown in the left of this figure, a platoon with three vehicles is moving on the middle lane from left to right. Two vehicles are moving on the two adjacent lanes, respectively. It is desired to merge the two vehicles into the platoon to form a new platoon as shown in the right of Figure 1. The speeds and clearances of these vehicles after merging operation should be of the desired values.

2. Trajectory planning algorithm

In this section, the trajectory planning algorithm is proposed for merging control on a curve road. In the beginning, several definitions are made to facilitate the discussion of the proposed algorithm. Next, the principle of the proposed algorithm is shown. Last the details of the proposed trajectory planning algorithm are presented.

2.1. Definitions

For a vehicle, the distance from the Centre of Gravity (CG) to the front and rear is defined as the front length and the rear length, respectively. In this paper, the road with constant radius is considered. Thus the centre paths of all lanes are concentric circles with the same centre. The centre is called road centre. The line that links the CG of a vehicle with the road centre is defined as the radial line of the vehicle. Note that, the radial line will be changing with the moving of the vehicle. The length of the radius line of a vehicle is defined as the path radius of the vehicle. The angle of the radial line to the horizontal is defined as the central angle of the vehicle. The speed component that is perpendicular to the path radius is defined as the circumferential speed of the vehicle. The circumferential angular speed of a vehicle is defined as:

$$\omega_c = \frac{v_c}{R_v}, \quad (1)$$

where: ω_c is the circumferential angular speed; v_c is the circumferential speed; R_v is the path radius.

If two vehicles are with the same circular path, then the segment of circular path between the CGs of the two vehicles is defined as the circular segment of the two vehicles. The length of the circular segment is defined as the circular distance between the two vehicles.

An example to demonstrate some of these definitions discussed above is shown in Figure 2. In this figure, there are two vehicles (labelled by A and B) moving along a circular path from left to right. p_c is the path centre; R_A and R_B are the path radius of vehicles A and B, respectively; c_{AB} denotes the circular segment of the two vehicles; ϑ_A and ϑ_B are the central angles of vehicles A and B, respectively.

The position of a vehicle can be expressed as:

$$\begin{cases} X = x_0 + R_v \cdot \cos(\vartheta_v); \\ Y = y_0 + R_v \cdot \sin(\vartheta_v). \end{cases} \quad (2)$$

where: (x_0, y_0), (X, Y) are the coordinates of the path centre and the vehicle, respectively; R_V, ϑ_v are the path radius and central angle of the vehicle, respectively. It can be found in Equation (2) that the position of a vehicle can be determined by its path radius and central angle. Thus, the trajectory planning of the vehicles on merging manoeuvre can be achieved by planning the path radiuses and central angles of these vehicles.

The point that projected from the CG of a vehicle to the main path along the radius line is defined as the projection point of the vehicle. The speed of the projection point is defined as the projection speed of the vehicle. The segment on the main path between the projection points of two vehicles is defined as the projection segment of the two vehicles. The length of the projection segment is defined as the projection distance of the two vehicles. The projection clearance d_c is defined as:

$$d_c = m_d - l_{r,F} - l_{f,R}, \quad (3)$$

where: m_d is the projection distance of the two vehicles; $l_{r,F}$, $l_{f,R}$ are the rear length and front length of the front and back vehicles, respectively. If all vehicles are of the same projection distance and the projection speeds of these vehicle are the speed of the desired platoon, then these vehicles are called synchronous.

An example to demonstrate these definitions discussed above is shown in Figure 3. There are two vehicles (labelled by A and C) on the main path and a vehicle (labelled by B) on the merging path. The projection point of vehicle B is p_B. As vehicles A and C are on the main path, the projection points of the two vehicles are their CGs. m_{AB} is the projection distance of vehicles A and B; m_{BC} is the projection distance of vehicles B and C. Provided that these vehicles can follow their paths accurately.

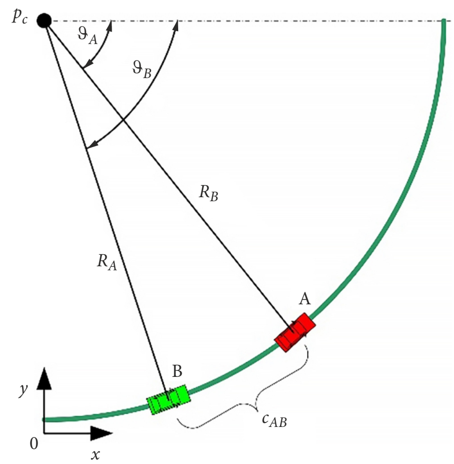

Figure 2. Example of two vehicles on curve road with constant radius (source: created by the authors)

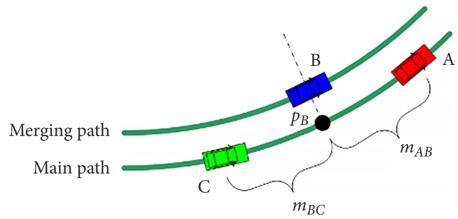

Figure 3. Example of synchronous vehicles (source: created by the authors)

If Equations (4) and (5) are satisfied, then these vehicles are synchronous:

$$\begin{cases} m_{AB} = c_d + l_{r,A} + l_{f,B}; \\ m_{BC} = c_d + l_{r,B} + l_{f,C}; \end{cases} \quad (4)$$

$$\begin{cases} v_{A,p} = v_d; \\ v_{B,p} = v_d; \\ v_{C,p} = v_d, \end{cases} \quad (5)$$

where: c_d, v_d are the clearance and speed of the desired platoon, respectively; $l_{f,B}$, $l_{r,B}$ are the front and rear lengths of vehicle B, respectively; $l_{r,A}$ is the rear length of vehicle A; $l_{f,C}$ is the front length of vehicle C; $v_{A,p}$, $v_{B,p}$, $v_{C,p}$ are the projection speeds of vehicles A, B and C, respectively. As vehicles A and C are on the main lane, the projection speeds of these vehicles are equal to their longitudinal speeds. The projection speed of vehicle B is $v_{B,p} = v_b \cdot \frac{R}{R_B}$ (where: v_b, R_B are the longitudinal speed and path radius of vehicle B, respectively; R is the main radius).

The concept of synchronization is introduced to facilitate the analysis of merging control. When several vehicles are synchronous, a platoon can be formed by performing lane changing manoeuvres for the merging vehicles. Provided that all vehicles are of the same circumferential angular speed, collisions will not occur during the lane changing manoeuvres. Thus the lane changing trajectories of the merging vehicles can be designed without considering collision avoidance, which reduces the complexity of the planning of lane changing trajectories.

2.2. Principle of algorithm

The considered merging operation is divided into two stages to simplify the algorithm for merging control as shown in Figure 4. The first stage is the operation of synchronization between the time of t_0 and t_s with the time span of t_{ax}, during which the vehicles are coordinated to become synchronous. In this stage, each vehicle follows its lane without lane changing and its path radius will not change. The second stage is the operation of lane changing between the time of t_s and t_e with the time span of t_{lc}, during which these vehicles are coordinated to form a platoon in the main lane. In this stage, the circumferential angular speed of each vehicle is constant, and the path radius of each vehicle will be changing during the operation of lane changing.

Consider the merging control problem shown in Figure 1, which can be achieved by two stages as discussed above. The beginning of merging operation is shown in Figure 5a. The result of synchronization is illustrated in Figure 5b, in which a safe clearance for each merging vehicle is formed in the main lane, and each merging vehicle is aligned with a safe clearance of the platoon. The result of lane changing is plotted in Figure 5c, which shows that the platoon is formed after the operation of lane changing.

The division of merging operation is made to simplify the trajectory planning of merging control. In the first stage, only the collisions of the vehicles in the same lane must be considered. In the second stage, provided constant circumferential angular speed, the collisions caused by lane changing manoeuvres will not occur. Without the division, the collisions among the vehicles in the same lane and adjacent lanes must be considered simultaneously, thereby complicating the problem of merging operation.

An example of a vehicle trajectory during merging operation is shown in Figure 6, in which a vehicle is merged into the main path from the merging path. R_m is the radius of merging path. R means the main radius. According to the division of merging operation, in the stage of synchronization operation, the path radius of the vehicle is constant. In the stage of lane changing operation, the path radius of the vehicle changes from R_m to R. In fact, the trajectory of each vehicle during the merging operation can be divided according to the division of merging operation. In the first stage, the vehicle trajectory can be planned with varying circumferential angular speed and constant path radius. In the second stage, the vehicle trajectory can be planned with varying path radius and constant circumferential angular speed.

2.3. Trajectory planning for the operation of synchronization

As there is no lane changing during the operation of synchronization, the collision among the vehicles in adjacent lanes will not occur. Thus, the trajectories of vehicles in each lane can be planned individually. In this subsection, the trajectory planning algorithm for the vehicles in the same lane is designed. The synchronization operation can be achieved by the proposed algorithm in this subsection for each lane.

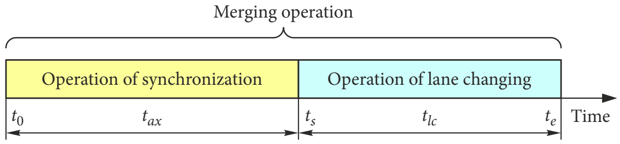

Figure 4. Procedure of merging operation
(source: created by the authors)

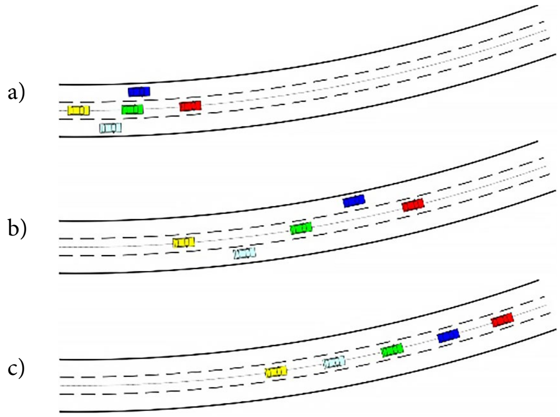

Figure 5. Merging operation (source: created by the authors):
a – beginning of merging operation; b – result of synchronization; c – result of lane changing

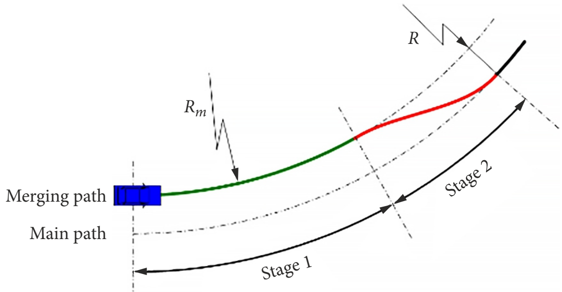

Figure 6. Division of vehicle trajectory
(source: created by the authors)

For the lane with only one vehicle, there is no need to consider collisions in this lane. The trajectory of the vehicle in the lane can be planned independently. For the lane with more than one vehicle, the collisions among these vehicles should be considered. To simplify the calculation, the trajectories of all vehicles are planned successively from front to back, in which the back vehicle is responsible to maintain a safe clearance with the front vehicle.

Consider the merging operation shown in Figure 1. The synchronization operation of these vehicles in the main lane is shown in Figure 7. It is desired to coordinate these vehicles to the ideal positions and speeds of the synchronous vehicles. Following the sequence of planning discussed above, the trajectory of the front vehicle is planned first, followed by the middle one. Last the trajectory of the rear one is planned. In the following parts of this subsection, the trajectory planning algorithm for one vehicle is designed. The trajectories of these vehicles can be obtained using the algorithm successively.

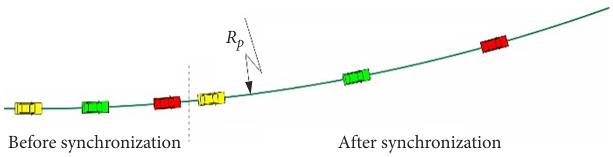

Figure 7. Example of synchronization operation in a lane (source: created by the authors)

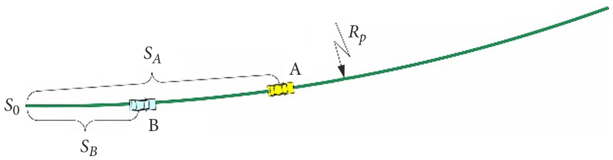

Figure 8. Path position (source: created by the authors)

2.3.1. Concept of path position

The concept of the circular position is proposed for the trajectory planning of a vehicle in a circular lane. The arc between the CG of a vehicle and the start point of a circular path is defined as the route of the vehicle. The length of the route of the vehicle is defined as the path position of the vehicle.

These concepts are demonstrated in Figure 8. In this figure, there is a path with constant radius starting from s_0. The radius of path is R_p. Two vehicles are moving along the path from left to right. s_A and s_B are the path positions of vehicles A and B, respectively. The clearance c_{AB} between vehicles A and B can be calculated as:

$$c_{AB} = s_A - s_B - \left(l_{r,A} + l_{f,B}\right), \quad (6)$$

where: $l_{r,A}$, $l_{f,B}$ are the rear length and front length of vehicles A and B, respectively.

As shown in Equation (6), the clearance between the two vehicles can be calculated according to the path positions of these vehicles to facilitate the discussion of collision avoidance among the vehicles in the same lane.

2.3.2. Modelling of vehicle

The longitudinal dynamic equation of a vehicle can be written by:

$$\begin{cases} \dot{s} = v; \\ \dot{v} = a, \end{cases} \quad (7)$$

where: s is the path position; v is the longitudinal speed; a means the longitudinal acceleration.

To simplify the problem of trajectory planning, the time span in t_{ax} is divided into n intervals with the same length:

$$\mathbf{t} = \left[t_0 \ \ t_1 \ \ \dots \ \ t_n\right]^T, \quad (8)$$

where: $n = 10$, $t_i = t_0 + i \cdot t_\Delta$, $t_\Delta = \dfrac{t_{ax}}{n}$ is the length of time in each interval; $t_n = t_s$ is the end time of synchronization.

The accelerations of the vehicle in these intervals are used for trajectory planning. It is assumed that the acceleration in each interval is constant:

$$\mathbf{a} = \left[a_1 \ \ a_2 \ \ \dots \ \ a_n\right]^T, \quad (9)$$

where: a_i means the planned acceleration in the i th interval.

The speeds and path positions at the time of t_i can be calculated as:

$$\mathbf{v} = \mathbf{t_v} \cdot \mathbf{a} + \mathbf{v_0}, \quad (10)$$

and

$$\mathbf{s} = \mathbf{t_s} \cdot \mathbf{a} + \mathbf{s_v} + \mathbf{s_0}, \quad (11)$$

where:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}; \ \mathbf{t_v} = \begin{bmatrix} t_\Delta & 0 & \dots & 0 \\ t_\Delta & t_\Delta & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ t_\Delta & t_\Delta & \dots & t_\Delta \end{bmatrix}; \ \mathbf{v_0} = \begin{bmatrix} v_0 \\ v_0 \\ \vdots \\ v_0 \end{bmatrix}; \ \mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix};$$

$$\mathbf{t_s} = \begin{bmatrix} \frac{1}{2} \cdot t_\Delta^2 & 0 & \dots & 0 \\ \frac{3}{2} \cdot t_\Delta^2 & \frac{1}{2} \cdot t_\Delta^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{2 \cdot n - 1}{2} \cdot t_\Delta^2 & \frac{2 \cdot n - 3}{2} \cdot t_\Delta^2 & \dots & \frac{1}{2} \cdot t_\Delta^2 \end{bmatrix};$$

$$\mathbf{s_v} = \begin{bmatrix} t_\Delta \cdot v_0 \\ 2 \cdot t_\Delta \cdot v_0 \\ \vdots \\ n \cdot t_\Delta \cdot v_0 \end{bmatrix}; \ \mathbf{s_0} = \begin{bmatrix} s_0 \\ s_0 \\ \vdots \\ s_0 \end{bmatrix};$$

v_i, s_i are the speed and path position at the time of t_i, respectively; v_0, s_0 are the speed and path position at t_0.

2.3.3. Constraints and objective function of trajectory planning

The constraint of acceleration can be expressed as:

$$\mathbf{a_{min}} \le \mathbf{E} \cdot \mathbf{a} \le \mathbf{a_{max}}, \quad (12)$$

where:

$\mathbf{a_{min}} = \left[a_{\min} \ \ a_{\min} \ \ \dots \ \ a_{\min}\right]^T$; $\mathbf{a_{max}} = \left[a_{\max} \ \ a_{\max} \ \ \dots \ \ a_{\max}\right]^T$; and $\mathbf{E}$ is a n dimensional identity matrix; $a_{\max} = \min\left(a_{\max,veh}, a_\mu\right)$, $a_{\min} = \max\left(a_{\min,veh}, -a_\mu\right)$ are the upper and lower bounds of planned acceleration, respectively; $a_{\max,veh}$, $a_{\min,veh}$ are the upper and lower bounds of the acceleration limited by the power of the vehicles, respectively; $a_\mu = f_\mu \cdot \mu \cdot g$ is the maximum acceleration limited by the friction; $f_\mu < 1$ is a constant safe factor to limit the acceleration; μ is the friction coefficient; g denotes the acceleration of gravity.

On a curve road, the centripetal acceleration of a vehicle should be lower than the maximum acceleration limited by friction:

$$a_c < f_v \cdot \mu \cdot g, \quad (13)$$

where: $a_c = \dfrac{v^2}{R}$ is the centripetal acceleration of the vehicle; $f_v < 1$ is the constant factor; R means the path radius. Thus the maximum speed limited by the friction can be calculated as: $v_\mu = \sqrt{f_v \cdot \mu \cdot g \cdot R}$.

The constraint of speed is:

$$\mathbf{v_{min}} \le \mathbf{v} \le \mathbf{v_{max}}, \tag{14}$$

where:
$\mathbf{v_{min}} = \left[v_{\min} \;\; v_{\min} \;\; \cdots \;\; v_{\min} \right]^T$; $\mathbf{v_{max}} = \left[v_{\max} \;\; v_{\max} \;\; \cdots \;\; v_{\max} \right]^T$; $v_{\max} = \min\left(v_{\max,veh}, v_{\mu}\right)$, $v_{\min} = \max\left(v_{\min,veh}, -v_{\mu}\right)$ are the upper and lower bounds of planned speed, respectively; $v_{\max,veh}$, $v_{\min,veh}$ are the upper and lower bounds limited by the performance of vehicle during merging control, respectively.

The performance of collision avoidance should be ensured. Thus one has:

$$\mathbf{s} \le \mathbf{s_f} - \mathbf{d_0}, \tag{15}$$

where: $\mathbf{d_0} = \left[d_0 \;\; d_0 \;\; \cdots \;\; d_0 \right]^T$; $\mathbf{s_f} = \left[s_{f,1} \;\; s_{f,2} \;\; \cdots \;\; s_{f,n} \right]^T$; $d_0 = f_{safe} \cdot \left(l_f + l_{r,f}\right)$ is the minimum safe clearance between the planned vehicle and its frontal vehicle; l_f is the front length of the planned vehicle; $l_{r,f}$ means the rear length of the frontal vehicle; $f_{safe} > 1$ is the safe factor for the calculation of d_0; $s_{f,i}$ is the path position of the front vehicle at t_i. Note that, if there is no vehicle in front of the planned vehicle, then the constraint of collision avoidance described in Equation (15) can be omitted.

The constraint of the errors of position and speed at t_n is:

$$\begin{cases} -v_{tol} \le v_n - v_d \le v_{tol}; \\ -s_{tol} \le s_n - s_d \le s_{tol}, \end{cases} \tag{16}$$

where: s_d, v_d are the desired path position and speed at t_n, respectively; s_{tol}, v_{tol} are the tolerances of position error and speed error, respectively. This constraint is used to limit the errors of position and speed at t_n.

The objective function is defined as:

$$f(\mathbf{a}) = w_s \cdot \left(s_n - s_d\right)^2 + w_v \cdot \left(v_n - v_d\right)^2 + w_a \cdot \sum_{i=1}^{n} \left(a_i\right)^2, \tag{17}$$

where: w_s = 100, w_v = 100 are the weight of position and speed errors at t_n, respectively; w_a = 1 is the weight of vehicle acceleration. The first two items are defined to reduce the errors of position and speed at t_s. The third item is used to reduce the longitudinal acceleration and energy consumption.

2.3.4. Construction of the optimal problem

To facilitate the trajectory planning, a convex quadratic programming is constructed. Substituting Equation (10) into Equation (14), the constraint of the planned speed can be rewritten as:

$$\mathbf{v_{min}} - \mathbf{v_0} \le \mathbf{t_v} \cdot \mathbf{a} \le \mathbf{v_{max}} - \mathbf{v_0}. \tag{18}$$

Substituting Equation (11) into Equation (15), the constraint of collision avoidance can be rewritten as:

$$\mathbf{t_s} \cdot \mathbf{a} \le \mathbf{s_f} - \left(\mathbf{d_0} + \mathbf{s_v} + \mathbf{s_0}\right). \tag{19}$$

According to Equations (10) and (11), the speed and position at the time of t_n are:

$$\begin{cases} v_n = \mathbf{t_{\Delta,v}} \cdot \mathbf{a} + v_0; \\ s_n = \mathbf{t_{\Delta,s}} \cdot \mathbf{a} + n \cdot t_\Delta \cdot v_0 + s_0, \end{cases} \tag{20}$$

where:

$$\mathbf{t_{\Delta,v}} = \left[t_\Delta \;\; t_\Delta \;\; \cdots \;\; t_\Delta \right];$$

$$\mathbf{t_{\Delta,s}} = \left[\frac{2 \cdot n - 1}{2} \cdot t_\Delta^2 \;\; \frac{2 \cdot n - 3}{2} \cdot t_\Delta^2 \;\; \cdots \;\; \frac{1}{2} \cdot t_\Delta^2 \right].$$

Substituting Equation (20) into Equation (16), the constraint of speed and position at t_n can be rewritten as:

$$\begin{cases} -v_{tol} + v_d - v_0 \le \mathbf{t_{\Delta,v}} \cdot \mathbf{a} \le v_{tol} + v_d - v_0; \\ -s_{tol} + s_d - \left(n \cdot t_\Delta v_0 + s_0\right) \le \mathbf{t_{\Delta,s}} \cdot \mathbf{a} \le s_{tol} + s_d - \left(n \cdot t_\Delta v_0 + s_0\right), \end{cases} \tag{21}$$

According to Equations (12), (18), (19) and (21), the constraints of acceleration, speed and position of the planned trajectory can be summarized as:

$$\mathbf{A} \cdot \mathbf{a} \le \mathbf{B}, \tag{22}$$

where:

$$\mathbf{A} = \left[\mathbf{A_1} \;\; \mathbf{A_2} \;\; \mathbf{A_3} \;\; \mathbf{A_4} \;\; \mathbf{A_5} \;\; \mathbf{A_6} \;\; \mathbf{A_7} \;\; \mathbf{A_8} \;\; \mathbf{A_9} \right]^T;$$

$$\mathbf{B} = \left[\mathbf{B_1} \;\; \mathbf{B_2} \;\; \mathbf{B_3} \;\; \mathbf{B_4} \;\; \mathbf{B_5} \;\; \mathbf{B_6} \;\; \mathbf{B_7} \;\; \mathbf{B_8} \;\; \mathbf{B_9} \right]^T;$$

$$\mathbf{A_1}^T = \mathbf{E};$$

$$\mathbf{A_2}^T = -\mathbf{E};$$

$$\mathbf{A_3}^T = \mathbf{t_v};$$

$$\mathbf{A_4}^T = -\mathbf{t_v};$$

$$\mathbf{A_5}^T = \mathbf{t_s};$$

$$\mathbf{A_6}^T = \mathbf{t_{\Delta,v}};$$

$$\mathbf{A_7}^T = -\mathbf{t_{\Delta,v}};$$

$$\mathbf{A_8}^T = \mathbf{t_{\Delta,s}};$$

$$\mathbf{A_9}^T = -\mathbf{t_{\Delta,s}};$$

$$\mathbf{B_1}^T = \mathbf{a_{max}};$$

$$\mathbf{B_2}^T = -\mathbf{a_{min}};$$

$$\mathbf{B_3}^T = \mathbf{v_{max}} - \mathbf{v_0};$$

$$\mathbf{B_4}^T = -\left(\mathbf{v_{min}} - \mathbf{v_0}\right);$$

$$\mathbf{B_5}^T = \mathbf{s_f} - \left(\mathbf{d_0} + \mathbf{s_v} + \mathbf{s_0}\right);$$

$$\mathbf{B_6}^T = v_{tol} + v_d - v_0;$$

$$\mathbf{B_7}^T = v_{tol} - v_d + v_0;$$

$$\mathbf{B_8}^T = s_{tol} + s_d - \left(n \cdot t_\Delta \cdot v_0 + s_0\right);$$

$$\mathbf{B_9}^T = s_{tol} - s_d + \left(n \cdot t_\Delta \cdot v_0 + s_0\right).$$

Substituting Equation (20) into Equation (17), the objective function can be rewritten as:

$$f(\mathbf{a}) = \frac{1}{2} \cdot \mathbf{a}^T \cdot \mathbf{H} \cdot \mathbf{a} + \mathbf{g} \cdot \mathbf{a} + c, \tag{23}$$

where:

$\mathbf{H} = 2 \cdot \left(w_s \cdot \mathbf{t_{\Delta,s}}^T \cdot \mathbf{t_{\Delta,s}} + w_v \cdot \mathbf{t_{\Delta,v}}^T \cdot \mathbf{t_{\Delta,v}} + w_a \cdot \mathbf{E} \right);$

$\mathbf{g} = 2 \cdot w_s \cdot \mathbf{t_{\Delta,s}} \cdot c_s + 2 \cdot w_v \cdot \mathbf{t_{\Delta,v}} \cdot c_v,$

$c_s = n \cdot t_\Delta \cdot v_0 + s_0 - s_d,$

$c_v = v_0 - v_d,$

$c = w_s \cdot c_s^2 + w_v \cdot c_v^2.$

As c is constant, the objective function in Equation (23) can be further written as:

$$F(\mathbf{a}) = \frac{1}{2} \cdot \mathbf{a}^T \cdot \mathbf{H} \cdot \mathbf{a} + \mathbf{g} \cdot \mathbf{a}. \tag{24}$$

The optimal problem can be constructed as:

$$\min\ F(\mathbf{a}) = \frac{1}{2} \cdot \mathbf{a}^T \cdot \mathbf{H} \cdot \mathbf{a} + \mathbf{g} \cdot \mathbf{a}, \tag{25}$$

subjected to Equation (22). In this study the time of synchronization operation is t_{ax} = 15 s. Thus one has $t_\Delta = \frac{t_{ax}}{n} = 1.5$ s. Note that w_s, w_v and w_a are constant. Thus $\mathbf{H}$ is a constant matrix. Substituting the values of t_Δ, w_s, w_v and w_a into $\mathbf{H}$, it is easy to deduce that $\mathbf{H}$ is a positive definite matrix. Note that the constraints shown in Equation (22) are linear. Thus the optimal problem described above is a convex quadratic programming problem. The active-set method is one of the effective methods to solve the problem described above (Ferreau *et al.* 2014). The principle of active method is that the inequality constraints are divided into an active group and an inactive group in each iteration. The active inequality constraints are viewed as equality constraints, and the inactive inequality constraints are temporarily ignored. The division of inequality constraints will be updated according to the solution in each iteration. A detailed discussion of the active-set method can be found in Ferreau *et al.* (2014).

2.3.5. Calculation of the vehicle trajectory

When the optimal problem is solved, the planned trajectory can be expressed as:

$$s(t) = \begin{cases} s_0 + v_0 \cdot t + \frac{1}{2} \cdot a_1 \cdot (t - t_0)^2, & t \in [t_0, t_1); \\ s_1 + v_1 \cdot t + \frac{1}{2} \cdot a_2 \cdot (t - t_1)^2, & t \in [t_0, t_1); \\ \vdots & \vdots \\ s_{n-1} + v_n \cdot t + \frac{1}{2} \cdot a_n \cdot (t - t_{n-1})^2, & t \in [t_{n-1}, t_n]. \end{cases} \tag{26}$$

The central angle of the vehicle at the time of t can be calculated as:

$$\vartheta_v(t) = \vartheta_0 + \frac{s(t)}{R_v}, \tag{27}$$

where: ϑ_0 is the central angle of the vehicle at t_0; R_v means the path radius of the vehicle, which is constant during the operation of synchronization.

The vehicle trajectory in the general Cartesian coordinate system can be obtained according to Equation (2).

2.4. Trajectory planning for the operation of lane changing

In the stage of lane changing, the circumferential angular speed is constant. As these vehicles are synchronous, the performance of collision avoidance can be ensured during the lane changing manoeuvre, as discussed in Section 2.2. Thus, the respective trajectories of all vehicles can be planned. The trajectory of lane changing on a curve road is extended from the polynomial based lane changing trajectory. The fifth order polynomial for lane changing can be expressed as (Papadimitriou, Tomizuka 2003):

$$y(t) = \begin{cases} y_s, & t < t_s; \\ eqs, & t_s \le t \le t_e; \\ y_e, & t_e < t, \end{cases} \tag{28}$$

where: $eqs = q_1 \cdot t^5 + q_2 \cdot t^4 + q_3 \cdot t^3 + q_4 \cdot t^2 + q_5 \cdot t + q_6$; y is the lateral displacement of the vehicle; y_s, y_e are the start and end lateral positions of lane changing manoeuvre, respectively; t_s, t_e are the start and end time of lane changing, respectively; $q_{i\,(i=1,2,\ldots,6)}$ are constant parameters to determine the planned trajectory.

The parameters in Equation (28) can be calculated by:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} t_s^5 & t_s^4 & t_s^3 & t_s^2 & t_s & 1 \\ 5 \cdot t_s^5 & 4 \cdot t_s^4 & 3 \cdot t_s^3 & 2 \cdot t_s^2 & t_s & 1 \\ 20 \cdot t_s^5 & 12 \cdot t_s^4 & 6 \cdot t_s^3 & 2 \cdot t_s^2 & t_s & 1 \\ t_e^5 & t_e^4 & t_e^3 & t_e^2 & t_e & 1 \\ 5 \cdot t_e^5 & 4 \cdot t_e^4 & 3 \cdot t_e^3 & 2 \cdot t_e^2 & t_e & 1 \\ 20 \cdot t_e^5 & 12 \cdot t_e^4 & 6 \cdot t_e^3 & 2 \cdot t_e^2 & t_e & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \\ \ddot{y}(t_s) \\ y(t_e) \\ \dot{y}(t_e) \\ \ddot{y}(t_e) \end{bmatrix}, \tag{29}$$

where:

$y(t_s) = 0;$

$\dot{y}(t_s) = 0;$

$\ddot{y}(t_s) = 0;$

$y(t_e) = R_m - R;$

$\dot{y}(t_e) = 0;$

$\ddot{y}(t_e) = 0.$

For the lane changing on a curve road with constant radius, the lateral displacement is extended to represent the changing of path radius. The path radius of the planned vehicle at the time of t can be expressed as:

$$R_v(t) = R_m - y(t). \tag{30}$$

As the circumferential angular speed is constant during the lane changing operation, the central angle of the vehicle at the time of t can be expressed as:

$$\vartheta_v(t) = \vartheta_s + \omega \cdot (t - t_s). \tag{31}$$

where: ϑ_s is the central angle of vehicle at the time of t_s; $\omega = \frac{v_d}{R}$ is the circumferential angular speed; v_d is the desired longitudinal speed of the platoon. The vehicle trajectory in the general Cartesian coordinate system can be obtained according to Equation (2).

3. Numerical simulation

To verify the effectiveness of the proposed algorithm, two simulations are conducted using MATLAB and qpOASES (Ferreau *et al.* 2014) on a notebook computer (ASUS F450, Intel i5-3230M, 2.6 GHz). Using qpOASES, the optimal problem shown in Equation (25) can be solved within 1 ms. The memory used for optimization is less than 2 MB. Thus the proposed algorithm can be used for practical application.

3.1. Definitions for simulation

To facilitate the discussion of simulation, several definitions are made. $a_{\max}$ is the maximum resultant acceleration of a vehicle during simulation. Provided $a_{\max} < \mu \cdot g$, a vehicle will not drift because of the saturation of friction. The safe distance of a vehicle is defined as the minimum distance of the vehicle to others, and can be obtained by calculating the distance between the sample points among the rectangular boundaries of all vehicles.

3.2. Simulation A

In this simulation, there is a platoon with three vehicles (vehicles 1, 2 and 4) in the main lane as shown in Figure 9. It is desired to control vehicle 3 to merge into the platoon.

The vehicle parameters used in simulation are listed in Table 1. The time spans of the synchronization operation and lane changing are 15 s and 10 s, respectively. The main radius and the friction are 1200 m and 0.85, respectively. The desired clearance and speed of the platoon are 20 m and 27.7 m/s (100 km/h), respectively.

The planned paths and the maximum resultant accelerations are shown in Figure 10. The planned longitudinal accelerations and speeds are shown in Figure 11. The camera shots of these vehicles are demonstrated in Figure 12. Note that in the figure of camera shots, the path after each vehicle is plotted to show the trajectory more clearly.

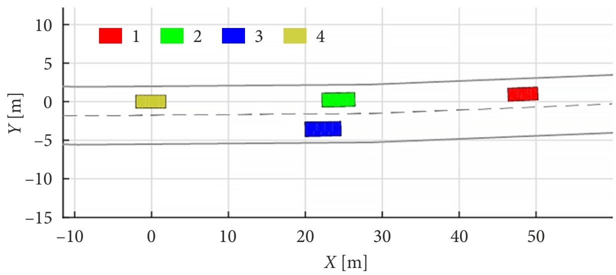

Figure 9. Positions of the vehicles at t_0
(source: created by the authors)

It can be seen from Figure 10a and Figure 12 that vehicle 3 is merged into the main lane after the merging operation. Figure 11 indicates that the planned accelerations converge to zero and the speeds of these vehicles reach the desired speed.

Figure 11 shows that all planned longitudinal accelerations and speeds are within the predefined limits, indicating that the planned trajectories are practical for actual application. Figure 10b indicates that the maximum resultant acceleration of each vehicle is lower than 1.5 m/s^2,

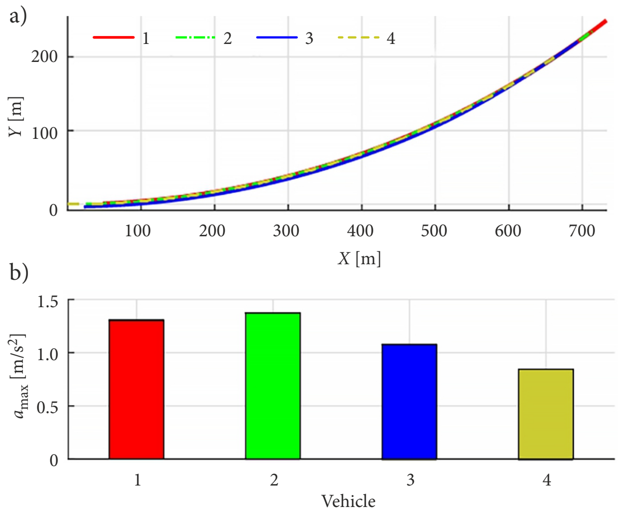

Figure 10. Planned paths and maximum resultant accelerations
(source: created by the authors):
a – planned paths; b – maximum resultant accelerations

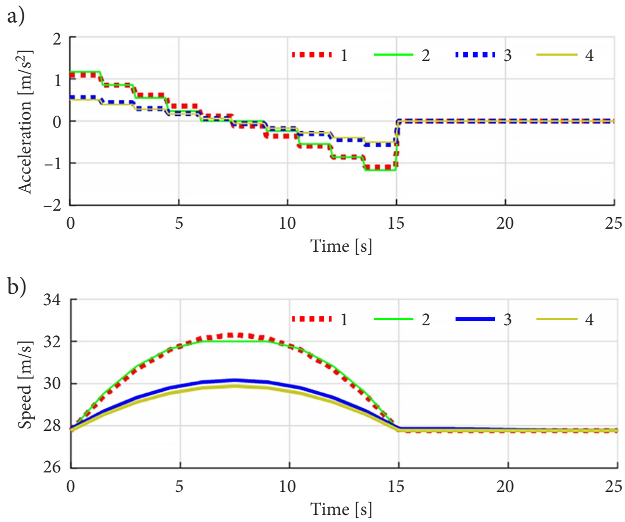

Figure 11. Planned longitudinal accelerations and speeds
(source: created by the authors):
a – planned longitudinal accelerations; b – planned speeds

Table 1. Vehicle parameters

Vehicle	$v_{\max,veh}$ [m/s]	$v_{\min,veh}$ [m/s]	$a_{\max,veh}$ [m/s^2]	$a_{\min,veh}$ [m/s^2]	l_f [m]	l_r [m]
1	35	0	2.4	–3	1.8	2.0
2	32	0	2.0	–3	2.0	2.2
3	30	0	1.6	–3	2.2	2.4
4	35	0	2.4	–3	1.8	2.0

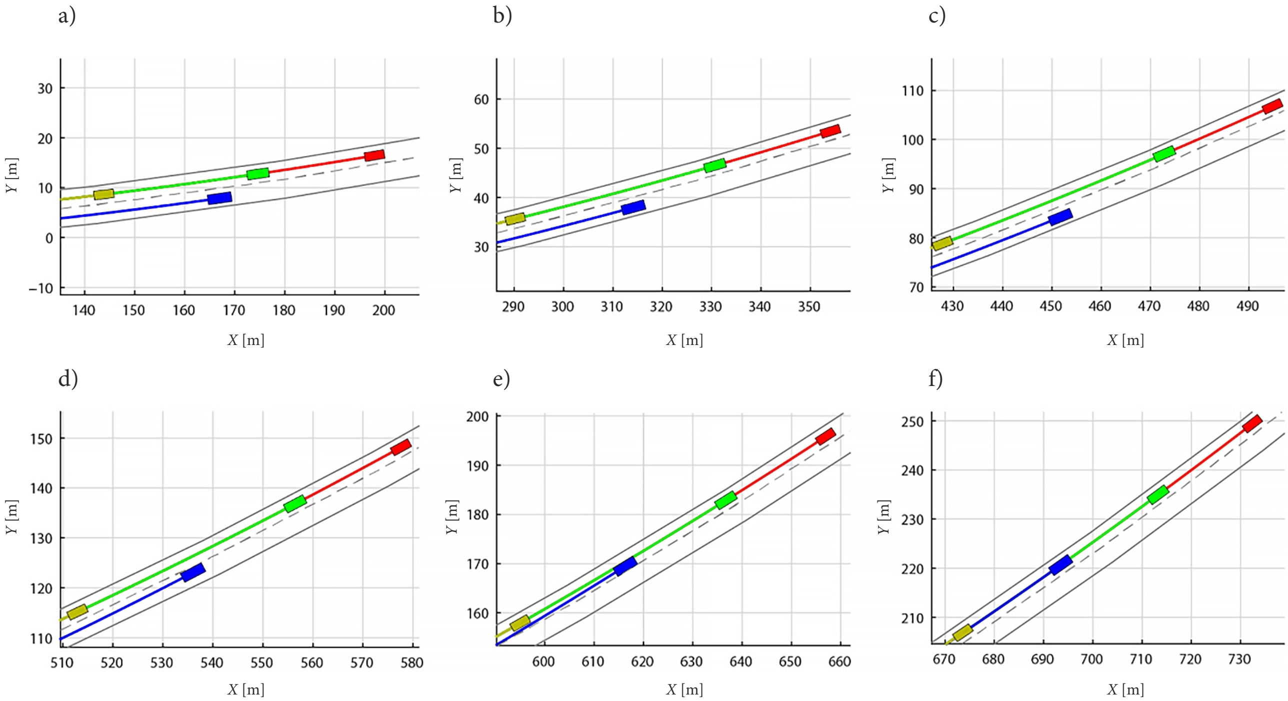

Figure 12. Camera shots of merging operation (source: created by the authors):
a – $t = t_0 + \frac{1}{3} \cdot t_{ax}$; b – $t = t_0 + \frac{2}{3} \cdot t_{ax}$; c – $t = t\ \ + t_{ax}$; d – $t = t_s + \frac{1}{3} \cdot t_{lc}$; e – $t = t_s + \frac{2}{3} \cdot t_{lc}$; f – $t = t_s + t_{lc}$

which is lower than $\mu \cdot g$. Thus the slips of these vehicles will not occur. The projection clearances and safe distances of all vehicles are demonstrated in Figure 13. Figure 13a shows that the projection clearances between vehicles converge to the desired value. Figure 13b shows that the safe distance of each vehicle is above zero, i.e., that the collision avoidance among these vehicles can be guaranteed.

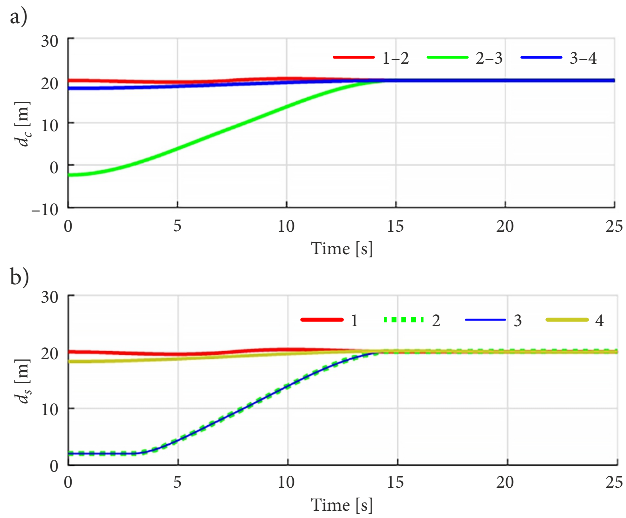

Figure 13. Projection clearances and safe distances (source: created by the authors):
a – projection clearances; b – safe distances

3.3. Simulation B

In this simulation, a platoon consisting of three vehicles (vehicles 1, 3 and 6) is considered, as indicated in Figure 14. Three merging vehicles are on the right and left lanes of the platoon.

The desired clearance and speed of platoon are 20 m and 15 m/s, respectively. The vehicle parameters used in simulation are listed in Table 2. The time spans of synchronization and lane changing are 15 s and 10 s, respectively. The main radius and the friction are 1000 m and 0.3, respectively.

The planned paths and maximum resultant accelerations are shown in Figure 15. The planned longitudinal accelerations and speeds are plotted in Figure 16. Several camera shots are shown in Figure 17, revealing that these merging vehicles are successfully merged into the platoon.

Figure 16 shows that all planned accelerations and speeds are within the predefined constraint. Thus, the planned trajectories are practical for the vehicles with different dynamics limits. The projection clearances and safe distances are plotted in Figure 18. The plotted projection clearances indicate that the platoon is formed with desired clearance. The safe distances of these vehicles imply that

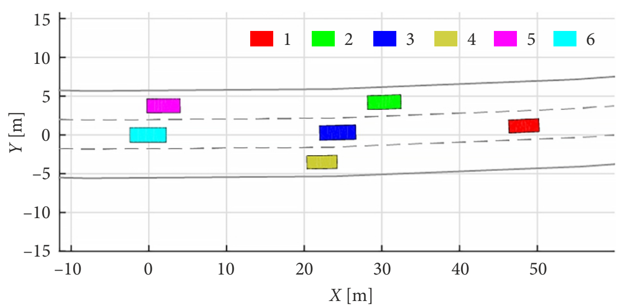

Figure 14. Positions of vehicles at t_0
(source: created by the authors)

all vehicles can maintain adequate clearance with the other vehicles to avoid undesired collisions. Figure 15b indicates that the maximum resultant acceleration of each vehicle is lower than 2 m/s^2, which is lower than $\mu \cdot g$. Thus the drifts of these vehicles will not occur.

Compared with the previous simulation, this simulation consists of more merging vehicles on both sides of the platoon. The successful result of merging control shows that the proposed algorithm not only is suitable for the simple merging operation of one merging vehicle, but can also be utilized for the merging operation with more vehicles on both sides.

Table 2. Vehicle parameters

Vehicle	$v_{\max,veh}$ [m/s]	$v_{\min,veh}$ [m/s]	$a_{\max,veh}$ [m/s^2]	$a_{\min,veh}$ [m/s^2]	l_f [m]	l_r [m]
1	35	0	2.4	–3	1.8	2.0
2	32	0	2.0	–3	2.0	2.2
3	30	0	1.6	–3	2.2	2.4
4	35	0	2.4	–3	1.8	2.0
5	32	0	2.0	–3	2.0	2.2
6	30	0	1.6	–3	2.2	2.4

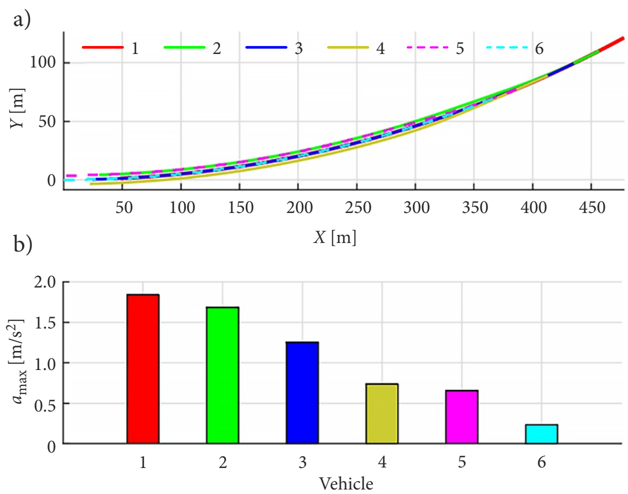

Figure 15. Planned paths and maximum resultant accelerations (source: created by the authors):
a – planned paths; b – maximum resultant accelerations

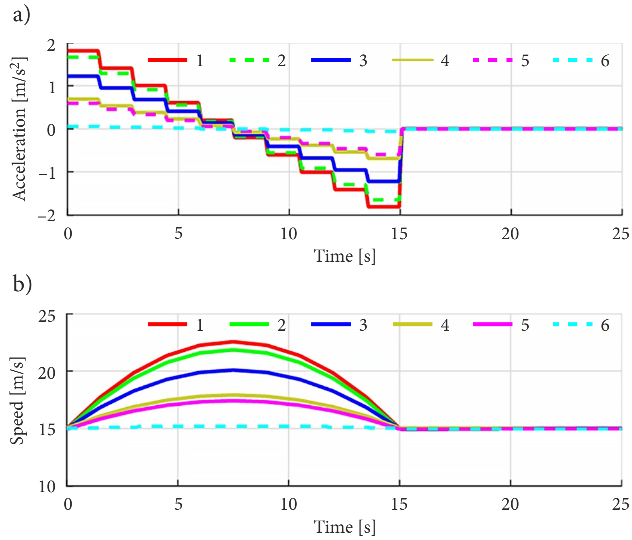

Figure 16. Planned longitudinal accelerations and speeds (source: created by the authors):
a – planned longitudinal accelerations; b – planned speeds

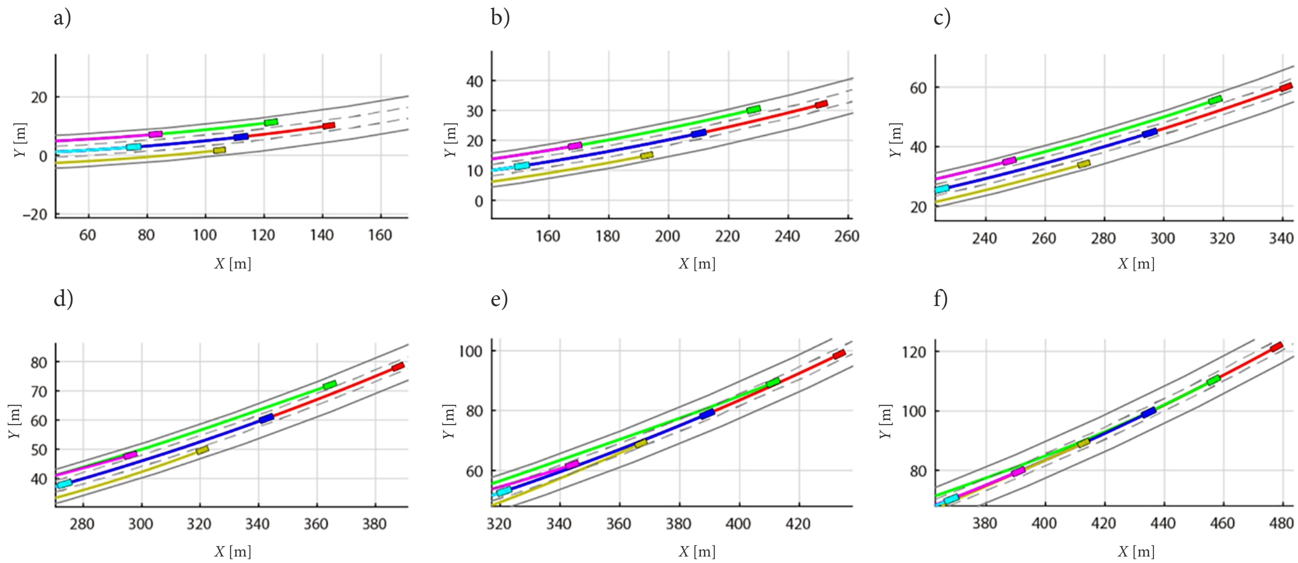

Figure 17. Camera shots of merging control (source: created by the authors):
a – $t = t_0 + \frac{1}{3} \cdot t_{ax}$; b – $t = t_0 + \frac{2}{3} \cdot t_{ax}$; c – $t = t_0 + t_{ax}$; d – $t = t_s + \frac{1}{3} \cdot t_{lc}$; e – $t = t_s + \frac{2}{3} \cdot t_{lc}$; f – $t = t_s + t_{lc}$

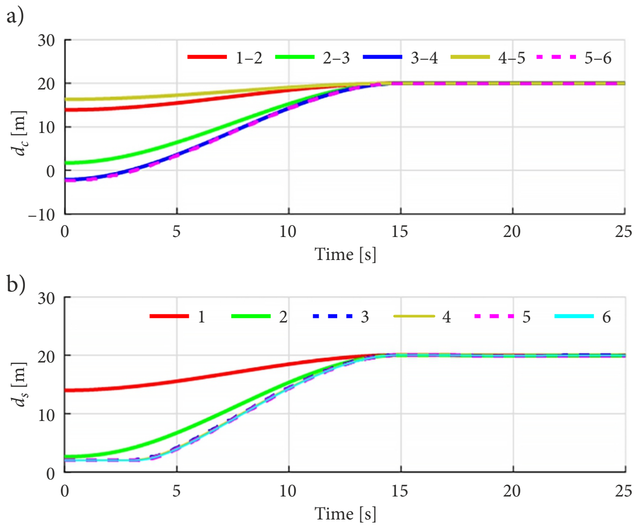

Figure 18. Projection clearances and safe distances (source: created by the authors): a – projection clearances; b – safe distances

Conclusions

This paper considers the trajectory planning for merging control of platoons on a curve road. The heterogeneous of dimensions and dynamics limits are taken into consideration to obtain practicable trajectories. Both of the dynamics constraints and collision avoidance can be guaranteed.

The performance of the proposed algorithm is validated using simulations. In practical term, the vehicles on road are always heterogeneous. In addition, curve roads are commonly observed in the traffic system. Therefore, it is essential to study the merging control for heterogeneous vehicles on curve roads.

The main advantage of the proposed algorithm is that it can be used for the merging operation of heterogeneous platoon on a curve road with constant radius, which is seldom discussed in previous studies.

Further research will be undertaken to achieve merging control on roads with varying curvature.

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Disclosure statement

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