A NEW FUZZY ADDITIVE RATIO ASSESSMENT METHOD (ARAS−F). CASE STUDY: THE ANALYSIS OF FUZZY MULTIPLE CRITERIA IN ORDER TO SELECT THE LOGISTIC CENTERS LOCATION

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Abstract. The main approaches which are applied to select the logistic center are the methods of gravity center, analytic hierarchy process, similarity to ideal solution, fuzzy ranking, assessment, etc. Multiple Criteria Decision-Making (MCDM) combines analytical and inductive knowledge, describing a domain problem, which can be fuzzy and/or incomplete. The fuzzy MCDM (FMCDM) approach can explain the problem more appropriately. The purpose of the paper is to select the most suitable site for logistic centre among a set of alternatives, to help the stakeholders with the performance evaluation in an uncertain environment, where the subjectivity and vagueness of criteria are described by triangular fuzzy numbers. The paper presents a newly-developed ARAS–F method to solve different problems in transport, construction, economics, technology and sustainable development.

Keywords: fuzzy, multiple criteria decision making, ARAS method, ARAS–F method, operation research, logistic centres, location.

1. Introduction

Most decision problems that human beings face in the real world are gambles. Alternatives are the raw material of decision making. The aim of this article is simple: to enable the decision makers to construct successful real-world, fuzzy multiple criteria problem and solve them. There are three forms of applied thinking that all humans need in key situations: creative thinking, problem solving, and making the right choices based on several options. Humans need arguments for explaining choices which are already made, or for selecting among reasonable alternatives. Each alternative usually has pros and cons of various strengths, which are the measurements of rationality of the feasible.

According to (Rasmussen 1983), the solution process of decision making problem could be divided into three broad categories. At the lowest level is skill-based sensorimotor smooth behaviour. It represents a type of actions that requires very little or no conscious control to perform or execute an action once as intention is formed. At the second level is rule-based behaviour. It is characterised by the use of rules and procedures to select a course of action in a familiar work situation.

Knowledge-based behaviour represents the most advanced level of reasoning. A knowledge-based behaviour is those tasks that are new, unfamiliar, unexpected or unique tasks. It represents a more advanced level of reasoning (Wirstad 1988).

Usually, decision-makers acts and make decisions in groups, but not autonomously.

In proper decision making it is necessary to evaluate the impacts of economic, technological, legislative, environmental, and social changes. According to the decision theorists the decision process follows a systematic, well structured procedure of action and uses well formulated rules, values and beliefs. To take the right decision is typically a very complicated process due to a number of factors:

- goal systems are highly complex combinations of single goals;
- relationships between them are unclear;
- it is difficult to find a relevant set of problem criteria, which should meet requirements of completeness, relevance, non-redundancy, meaningfulness, measurability, and value independence;
- a future development, which deals with large number of reasonable alternatives, is fuzzy;
- the decision-maker acts not as autonomous but in a group of stakeholders.

A number of approaches have been proposed for solving Multiple Criteria Decision-Making (MCDM)

problems such as Analytical Hierarchy Process (AHP) (Morkvėnas *et al.* 2008; Maskeliūnaitė *et al.* 2009), Analytical Network Process (ANP) (Satty 1996), which deals with decisions without making assumptions about the independence of higher level elements from lower level elements and about the independence of the elements within a level, data envelopment analysis (DEA), Technique for Order Performance by similarity to Ideal Solution (TOPSIS) (Hwang and Yoon 1981), VIKOR, Complex Proportional Assessment (COPRAS), Simple Additive Weighting (SAW), etc. These are approaches which measure the alternative ratings and weights of the criteria and are crisp or expressed in precise numbers which depends upon decision maker's judgements or preferences (Wang and Lee 2009).

Authors of this paper applied and developed a lot of MCDM methods to solve the problems in transport, economics, management and construction (Zavadskas *et al.* 2008b; Turskis *et al.* 2009): MEW (Завадскас 1987; Zavadskas *et al.* 2009a), ordering feasible alternatives of making solutions in terms of preferability technique (Завадскас 1987; Turskis 2008), Additive Ratio Assessment Method (ARAS) (Zavadskas and Turskis 2010; Zavadskas *et al.* 2010c), SAW–G (Zavadskas *et al.* 2010b; Medineckienė *et al.* 2010), COPRAS–G (Zavadskas *et al.* 2008a, 2009b), a new logarithmic normalization method in the Game Theory (Zavadskas and Turskis 2008), Multi-Objective Optimization on the basis of the Ratio Analysis (MOORA) (Brauers and Zavadskas 2006; Brauers *et al.* 2008) etc.

2. Fuzzy MCDM

However, under many real cases crisp human judgements are vague and cannot be assessed with exact numeric values. Human's thinking and acting deals with the ill-structured decision problems in an uncertain environment. Human's decision making should take into account subjectivity. Fuzzy set theory allows decision makers to use incomplete or partially obtained information into the problem's solution model.

Fuzzy logic is an extension of the traditional logic to intermediate and approximate values that capture the meaning of words, human reasoning, and decision making. Fuzzy systems and logic has its origins in ancient Greek philosophy (Korner 1967). Zadeh (1965, 1971, 1975a–c) made a crucial step in the design of a problem solving tool. He proposed and developed a concept of fuzzy logic in his seminal papers.

Bellman and Zadeh (1970) were the first who introduced fuzzy sets into the field of MCDM as an important tool to represent the uncertainty and imprecision in human decision making process involving multiple criteria.

'*Fuzzy goals and fuzzy constraints can be defined precisely as fuzzy sets in the space of alternatives. A fuzzy decision, then, may be viewed as an intersection of the given goals and constraints. A maximizing decision is defined as a point in the space of alternatives at which the membership function of a fuzzy decision attains its maximum value*'.

Fuzzy sets have a tremendous potential for multiple criteria problems solution in the fields of operation research, and its applications to various branches of sciences, engineering, economics and management. To resolve the ambiguity which frequently arises from human judgement, fuzzy set theory (Zadeh 1965) has been incorporated in many MCDM methods. The decision matrix is composed by fuzzy values. Fuzzy arithmetical operations should be used to produce the final aggregated scores for each alternative. The concept of fuzzy intervals (Yager and Filev 1994) is applied to simplify these computations and to permit operations with distinct kinds of membership functions.

FMCDM methods evaluate the alternative ratings and the weights of criteria on imprecision and vagueness expressed by fuzzy numbers i.e. fuzzy AHP, fuzzy ANP, fuzzy TOPSIS, fuzzy ELECTRE, fuzzy PROMETHEE, and fuzzy ORESTE. In fuzzy AHP and fuzzy ANP cases the preferences are required to be provided by decision makers. The earliest work in fuzzy AHP appeared in Van Laarhoven and Pedrycz (1983). Pan (2009) applied fuzzy Analytical Hierarchy Process (AHP) to evaluate alternatives of excavation construction related to multiple decision criteria.

Chen *et al.* (1992) first applied fuzzy numbers to establish fuzzy TOPSIS.

These methods can be used for strategic decision making. However, when the numbers of alternatives are more AHP, fuzzy AHP, ANP, and fuzzy ANP does not provide feasible solution.

These methods were modified and applied by various researchers to solve the selection problems in different fields.

2.1. Basic Definitions

In most cases, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. A fuzzy set is a class of objects with a continuum of membership grades. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one (Zadeh 1965). A fuzzy set A defined in space X is a set of pairs:

$$A = \left\{ \left(x, \mu_A \left(x \right) \right), x \in X \right\}, \forall x \in X, \tag{1}$$

where: the fuzzy set A is characterized by its membership function $\mu_A : X \to \left[0;1 \right]$ which associates with each element $x \in X$, a real number $\mu_A \left(x \right) \in \left[0;1 \right]$. The value $\mu_A \left(x \right)$ at x represents the grade of membership of x in A and is interpreted as the membership degree to which x belongs to A. So the closer the value $\mu_A \left(x \right)$ is to 1, the more x belongs to A.

A crisp or ordinary subset A of X can also be viewed as a fuzzy set in X with membership function as its characteristic function, i.e.:

$$\mu_A \left(x \right) = \begin{cases} 1, x \in A; \\ 0, x \notin A. \end{cases} \tag{2}$$

The set X is called a universe of discourse and can be written $\subseteq X$. Sometimes a fuzzy set A in X is denoted by list the ordered pairs $\left(x, \mu_a\left(x\right)\right)$, where the elements with zero degree are usually not listed. Thus a fuzzy set A in X can be represented as $A = \left\{\left(x, \mu_A\left(x\right)\right)\right\}$, where $x \in X$ and $\mu_A : X \rightarrow \left[0;1\right]$.

When the universe of discourse is discrete and finite with cardinality n, that is $X = \left\{x_1, x_1, \ldots, x_n\right\}$, the fuzzy set A can be represented as:

$$A = \sum_{i=1}^{n} \frac{\mu_A\left(x_i\right)}{x_i} = \frac{\mu_A\left(x_1\right)}{x_1} +$$
$$\frac{\mu_A\left(x_2\right)}{x_2} + \cdots + \frac{\mu_A\left(x_n\right)}{x_n}. \tag{3}$$

When the universe of discourse X is an interval of real numbers, the fuzzy set A can be expressed as:

$$A = \int_X \frac{\mu_A\left(x\right)}{x}. \tag{4}$$

A fuzzy number is defined to be a fuzzy triangular number (α, β, γ) if its membership function is fully described by three parameters $(\alpha < \beta < \gamma)$:

$$\mu_A\left(x\right) = \begin{cases} \dfrac{1}{\beta - \alpha} x - \dfrac{\alpha}{\beta - \alpha}, & \text{if } x \in \left[\alpha, \beta\right]; \\ \dfrac{1}{\beta - \gamma} x - \dfrac{\alpha}{\beta - \gamma}, & \text{if } x \in \left[\beta, \gamma\right]; \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

2.2. Defuzzification

In order to obtain a crisp output, a defuzzification process is needed to be applied. Defuzzification is the process of producing a quantifiable result in fuzzy logic, given fuzzy sets and corresponding membership degrees. The output of the defuzzification process is a single number. Many defuzzification techniques have been proposed in the literature.

Various types of membership functions are used. The most commonly used membership functions are the following (Dubois and Prade 1988): triangular, trapezoid, linear, sigmoidal, π-type, and Gaussian.

The most typical fuzzy set membership function is triangular membership function (Fig. 1).

The basic operations of fuzzy triangular numbers $\tilde{n}_1$ and $\tilde{n}_2$ (Van Laarhoven and Pedrycz 1983) are defined as follows:

$$\tilde{n}_1 \oplus \tilde{n}_2 = \left(n_{1\alpha} + n_{2\alpha}, n_{1\beta} + n_{2\beta}, n_{1\gamma} + n_{2\gamma}\right) \text{ addition}; \tag{6}$$

$$\tilde{n}_1 (-) \tilde{n}_2 = \left(n_{1\alpha} - n_{2\gamma}, n_{1\beta} - n_{2\beta}, n_{1\gamma} - n_{2\alpha}\right) \text{ substraction}; \tag{7}$$

$$\tilde{n}_1 \otimes \tilde{n}_2 = \left(n_{1\alpha} \otimes n_{2\alpha}, n_{1\beta} \otimes n_{2\beta}, n_{1\gamma} \otimes n_{2\gamma}\right) \text{ multiplication}; \tag{8}$$
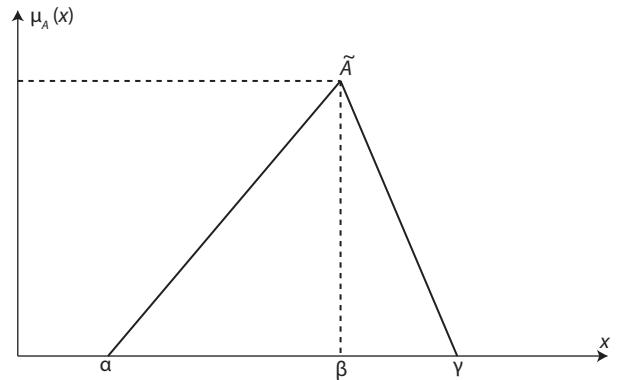

Fig. 1. Triangular membership function

$$\tilde{n}_1 (\div) \tilde{n}_2 = \left(\frac{n_{1\alpha}}{n_{2\gamma}}, \frac{n_{1\beta}}{n_{2\beta}}, \frac{n_{1\gamma}}{n_{2\alpha}}\right) \text{ division}; \tag{9}$$

$$k\tilde{n}_1 = \left(kn_{1\alpha}, kn_{1\beta}, kn_{1\gamma}\right) \text{ multiplication by constant}; \tag{10}$$

$$\left(\tilde{n}_1\right)^{-1} = \left(\frac{1}{n_{1\gamma}}, \frac{1}{n_{1\beta}}, \frac{1}{n_{1\alpha}}\right). \tag{11}$$

3. Criteria Weights Determination Applying Analytic Hierarchy Process (AHP)

There are various approaches for assessing weights (Zavadskas *et al.* 2010a), e.g., the eigenvector method, SWARA (Keršulienė *et al.* 2010), expert method (Zavadskas and Vilutienė 2006), Entropy method, etc.

The decision is made by using the derived weights w of the evaluative criteria (Saaty 1980). According to the Saaty, his experiments have shown that most individuals cannot compare more than seven objects (plus/minus two). Based on this, Saaty established 9 objects as the upper limit of his integer scale for multiple pair wise comparisons (Table 1).

Table 1. The nine-point scale of pair wise comparison (according to Saaty (1980))

Definition	Intensity of importance
Criteria i and j have equal importance	1
Criterion i is weakly more important than criterion j	3
Criterion i is essentially (strongly) more important than criterion j	5
Criterion i is very strongly (demonstrably) more important than criterion j	7
Criterion i is absolutely more important than criterion j	9
Intermediate values between the two adjacent judgements	2, 4, 6, 8
If activity i has one of the nonzero numbers assigned to it when compared with activity j then, then j has reciprocal value when compared with i	Reciprocals nonzero

In AHP the decision matrix is always a square matrix:

$$A = \left(a_{ij} \right) = \begin{bmatrix} 1 & \dfrac{w_1}{w_2} & \cdots & \dfrac{w_1}{w_m} \\ \dfrac{w_2}{w_1} & 1 & \cdots & \dfrac{w_2}{w_m} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{w_m}{w_1} & \dfrac{w_m}{w_2} & \cdots & 1 \end{bmatrix}, \tag{12}$$

$$a_{ii} = 1, a_{ij} = \frac{1}{a_{ji}}, a_{ij} \neq 0.$$

Fuzzy group weight is determined as follows:

After obtaining the criteria weights from AHP the synthesising of ratio judgements is done.

Suppose $\tilde{W} = \left[\overline{\tilde{w}_1, \tilde{w}_n} \right] = \left[\tilde{w}_j \right]$ is fuzzy group weight for n criteria and $\tilde{w}_j$ is fuzzy triangular number:

$$\tilde{w}_j = \left(w_{jl}, w_{jm}, w_{ju} \right), \tag{13}$$

where: $w_{jl} = \min\limits_k y_{jk}, j = \overline{1,n}, k = \overline{1,p}$ is minimum possible value; $w_{j\alpha} = \left(\prod\limits_{k=1}^{p} y_{jk} \right)^{\frac{1}{p}}, j = \overline{1,n}, k = \overline{1,p}$ is the most possible value and $w_{j\beta} = \max\limits_k y_{jk}, j = \overline{1,n}, k = \overline{1,p}$ is the maximal possible value of j-th criterion; $w_{j\gamma} = \max\limits_k y_{jk}, j = \overline{1,n}, k = \overline{1,p}.$

4. A New Fuzzy *Additive Ratio Assessment* (ARAS) method ARAS−F

ARAS method (Zavadskas and Turskis 2010) is based on the argument that the phenomena of complicated world could be understood by using simple relative comparisons. It is argued that the ratio of the sum of normalized and weighted criteria scores, which describe alternative under consideration, to the sum of the values of normalized and weighted criteria, which describes the optimal alternative, is degree of optimality, which is reached by the alternative under comparison.

According to the ARAS method (Zavadskas and Turskis 2010; Tupėnaitė *et al.* 2010), a utility function value determining the complex relative efficiency of a reasonable alternative is directly proportional to the relative effect of values and weights of the main criteria considered in a project.

The first stage is fuzzy decision-making matrix (FDMM) forming. In the FMCDM of the discrete optimization problem any problem which has to be solved is represented by the following DMM of preferences for m reasonable alternatives (rows) rated on n criteria (columns):

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{01} & \cdots & \tilde{x}_{0j} & \cdots & \tilde{x}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{i1} & \cdots & \tilde{x}_{ij} & \cdots & \tilde{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mj} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \tag{14}$$

$$i = \overline{0, m}; j = \overline{1, n},$$

where: m – number of alternatives; n – number of criteria describing each alternative; $\tilde{x}_{ij}$ – fuzzy value representing the performance value of the i alternative in terms of the j criterion; $\tilde{x}_{0j}$ – optimal value of j criterion. A tilde '~' will be placed above a symbol if the symbol represents a fuzzy set.

If optimal value of j criterion is unknown, then:

$$\tilde{x}_{0j} = \max_i \tilde{x}_{ij}, \quad \text{if } \max_i \tilde{x}_{ij} \text{ is preferable, and}$$

$$\tilde{x}_{0j} = \min_i \tilde{x}_{ij}^*, \quad \text{if } \min_i \tilde{x}_{ij}^* \text{ is preferable.} \tag{15}$$

Usually, the performance values $\tilde{x}_{ij}$ and the criteria weights $\tilde{w}_j$ are viewed as the entries of a DMM. The system of criteria as well as the values and initial weights of criteria are determined by experts. The information can be corrected by the interested parties, taking into account their goals and opportunities.

Then the determination of the priorities of alternatives is carried out in several stages.

Usually, the criteria have different dimensions. The purpose of the next stage is to receive dimensionless weighted values from the comparative criteria. In order to avoid the difficulties caused by different dimensions of the criteria, the ratio to the optimal value is used. There are various theories describing the ratio to the optimal value. However, the values are mapped either on the interval [0;1] or the interval [0;∞) by applying the normalization of a DMM.

In the second stage the initial values of all the criteria are normalized – defining values $\tilde{\tilde{x}}_{ij}$ of normalised decision-making matrix $\overline{\tilde{X}}$:

$$\overline{\tilde{X}} = \begin{bmatrix} \tilde{\tilde{x}}_{01} & \cdots & \tilde{\tilde{x}}_{0j} & \cdots & \tilde{\tilde{x}}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\tilde{x}}_{i1} & \cdots & \tilde{\tilde{x}}_{ij} & \cdots & \tilde{\tilde{x}}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\tilde{x}}_{m1} & \cdots & \tilde{\tilde{x}}_{mj} & \cdots & \tilde{\tilde{x}}_{mn} \end{bmatrix}, \tag{16}$$

$$i = \overline{0, m}; j = \overline{1, n}.$$

The criteria, whose preferable values are maxima, are normalized as follows:

$$\tilde{\tilde{x}}_{ij} = \frac{\tilde{x}_{ij}}{\sum\limits_{i=0}^{m} \tilde{x}_{ij}}. \tag{17}$$

The criteria, whose preferable values are minima, are normalized by applying two-stage procedure:

$$\tilde{x}_{ij} = \frac{1}{\tilde{x}_{ij}^*}; \quad \tilde{\tilde{x}}_{ij} = \frac{\tilde{x}_{ij}}{\sum\limits_{i=0}^{m} \tilde{x}_{ij}}. \tag{18}$$

When the dimensionless values of the criteria are known, all the criteria, originally having different dimensions, can be compared.

The third stage is defining normalized-weighted matrix – $\hat{\tilde{X}}$. It is possible to evaluate the criteria with

weights $0 < \tilde{w}_j < 1$. Only well-founded weights should be used because weights are always subjective and influence the solution. The values of weight w_j are usually determined by the expert evaluation method. The sum of weights w_j would be limited as follows:

$$\sum_{j=1}^{n} w_j = 1. \tag{19}$$

$$\tilde{\tilde{X}} = \begin{bmatrix} \tilde{\tilde{x}}_{01} & \cdots & \tilde{\tilde{x}}_{0j} & \cdots & \tilde{\tilde{x}}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\tilde{x}}_{i1} & \cdots & \tilde{\tilde{x}}_{ij} & \cdots & \tilde{\tilde{x}}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{\tilde{x}}_{m1} & \cdots & \tilde{\tilde{x}}_{mj} & \cdots & \tilde{\tilde{x}}_{mn} \end{bmatrix}, \tag{20}$$

$$i = \overline{0, m}; \; j = \overline{1, n}.$$

Normalized-weighted values of all the criteria are calculated as follows:

$$\tilde{\tilde{x}}_{ij} = \tilde{\tilde{x}}_{ij}\tilde{w}_j; \qquad i = \overline{0, m}, \tag{21}$$

where: w_j is the weight (importance) of the j criterion; $\overline{x}_{ij}$ is the normalized rating of the j criterion.

The following task is determining values of optimality function:

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{\tilde{x}}_{ij}; \qquad i = \overline{0, m}, \tag{22}$$

where $\tilde{S}_i$ is the value of optimality function of i-th alternative.

The biggest value is the best, and the least one is the worst. Taking into account the calculation process, the optimality function $\tilde{S}_i$ has a direct and proportional relationship with the values $\tilde{x}_{ij}$ and weights $\tilde{w}_j$ of the investigated criteria and their relative influence on the final result. Therefore, the greater the value of the optimality function $\tilde{S}_i$, the more effective the alternative. The priorities of alternatives can be determined according to the value $\tilde{S}_i$. Consequently, it is convenient to evaluate and rank decision alternatives when this method is used.

The result of fuzzy decision making for each alternative is fuzzy number $\tilde{S}_i$. There are several methods for defuzzification. The centre-of-area is the most practical and simple to apply to:

$$S_i = \frac{1}{3}(S_{i\alpha} + S_{i\beta} + S_{i\gamma}). \tag{23}$$

The degree of the alternative utility is determined by a comparison of the variant, which is analysed, with the most ideal one S_0. The equation used for the calculation of the utility degree of an alternative A_i is given below:

$$K_i = \frac{S_i}{S_0}; \qquad i = \overline{0, m}, \tag{24}$$

where: S_i and S_0 are the optimal criterion values, obtained from Eq. (23).

It is clear, that the calculated values K_i are in the interval [0;1] and can be ordered in an increasing sequence, which is the wanted order of precedence. The complex relative efficiency of the reasonable alternative can be determined according to the utility function values.

5. Case Study: The Analysis of Fuzzy Multiple Criteria in Order to Select the Logistic Centre Location

The transport of freight, ranging from raw materials to finished goods, is essential to economic activity and to the quality of life in the EU. It makes the division of labour possible, allows economies of scale, and mobilises comparative advantages. Freight transport is therefore fundamental for European competitiveness (The EU's Freight Transport... 2007).

Basically an infrastructure project affects the transport system by the way transport of goods will lead to benefits within the logistic system of the company.

Trends towards globalization, integrated logistics and the development of Information and Communication Technologies are the factors reshaping the world's trading patterns and consequently physical trade flows (Vasilis Vasiliauskas and Jakubauskas 2007; Meidutė 2007).

The development of an automatic traffic management helps to improve logistics (Jakimavičius and Burinskienė 2010).

There are five fundamental objectives of the government's transport policy:
- economy;
- environment;
- safety;
- accessibility;
- integration.

A modern society can effectively function only having an effective transport and logistics system. Road infrastructure is complex, and therefore the optimal choice of planned interventions is a delicate task often left to the road managers' subjective judgment (Šelih *et al.* 2008). The transportation system must work effectively to distribute those goods on customer's demand (Mačiulis *et al.* 2009; Vasilis Vasiliauskas and Barysienė 2008).

Strategic level logistical decisions typically have a long-lasting effect. This includes decisions regarding the number, location and capacities of manufacturing plants, and the flow of material through the logistics network (Bramel and Simchi-Levi 1999).The site selection of logistics centre is a key link to the chain of logistics-centre planning.

The financial evaluation of a new Logistics center is mainly performed and based on the viewpoint (and interests) of the private investor.

Logistics center is a space in a defined area which all activities relating to transport, logistics and the distribution of goods – both for national and international transit, are carried out by various operators on a commercial basis. Logistics centres make a significant contribution to the territorial and economic development of the area

they are located. Regional logistics planning and development should be integrated with geography, economy, transportation, and other characteristics. Normally the logistics centres are located in a 100–150 ha territory, however, sometimes the size can reach 500 ha. This creates a problem of security and comfort. At this point in time it is important to separate the terms of optimisation concerning the company and the society. The company is of course trying to arrange the supply chain in a way so it maximises its revenue. In this sense, the transport is a residual of the logistical planning while the minimisation of transport costs gives the same amount of attention as any other costs concerning the company. A welfare maximisation is, however, trying to optimize economic benefits towards the society. In this case an increase in transport can give a potential benefit to the society; however, it also provides a disadvantage by creating more pollution etc. In new EU countries the opposing side is often environmentally-friendly which on the basis of the environmental protection law can even stop the whole investment process (Jaržemskis 2007).

Economic assessment and transport modelling are essential for providing decision-makers with the evidence they need to reach soundly based decisions on transport policies and infrastructure schemes.

Case study describes newly developed solution methodology. The case study under consideration is limited in the optimality terms of logistic company.

The alternative sites are determined according to the planning logistics zone. Four location alternatives of logistics centre are under consideration: A_1, A_2, A_3 and A_4.

The main object is the relationship between the logistic principles and the investments which in this case are translated into four fundamental concepts or criteria (stakeholders' point of view):

- C_1 – investment cost [10^6 €], (optimal value is minimum);
- C_2 – operation time [years] (optimal value is maximum);
- C_3 – expansion possibility, [%] (optimal value is maximum);
- C_4 – closeness to the demand market [kilometres], (optimal value is minimum).

The evaluation methodology, developed in this article is a specific tool, which can be integrated into a wider methodology for planning and evaluating investments in a new Logistics centre, when a mix of public and private funds exists.

In conventional mathematics data of problems are usually determined by the experts as crisp values in terms of classical mathematical reasoning. But in reality, in an imprecise and uncertain environment, it will be utmost unrealistic to assume that the knowledge and representation of an expert can come in a precise way (Senhupta and Pal 2009).

The analytic hierarchy process (AHP) was applied to calculate the crisp criteria weights by seven decision makers ($E_1, … , E_7$). The decision making process in this case depends on the risk attitude of the expert, which

has a crucial role. Fig. 2 shows the relative weights based on four criteria in logistics centres location's performance.

The β weights of each criterion (as it is depicted in Fig. 3) were C_1 – investment cost (0.137), C_2 – operation time (0.203), C_3 – expansion possibility (0.343), and C_4 – closeness to the demand market (0.210).

The alternatives versus four criteria C_1, C_2, C_3 and C_4 must be evaluated by seven stakeholders (decision makers) E_1, E_2, … , E_7.

Decision makers' point of view about criteria and the performance of alternatives is given as a fuzzy decision making matrix $\tilde{X}$ (Table 2).

Tables 3–5 shows the solution process according to the algorithm presented above. Table 6 presents the solution of the problem results and graphical representation of the obtained results.
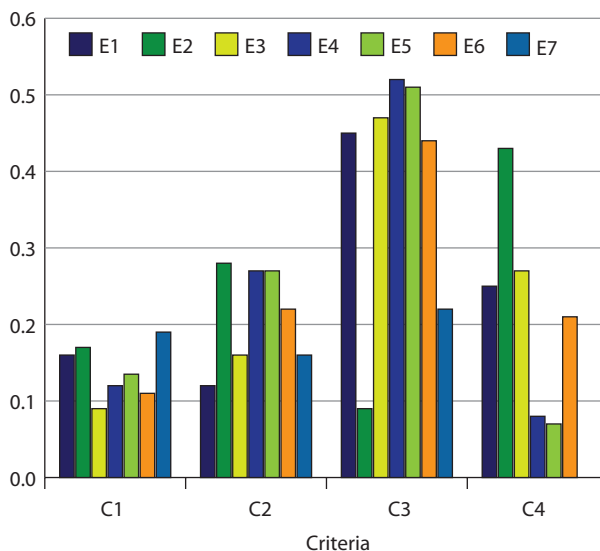

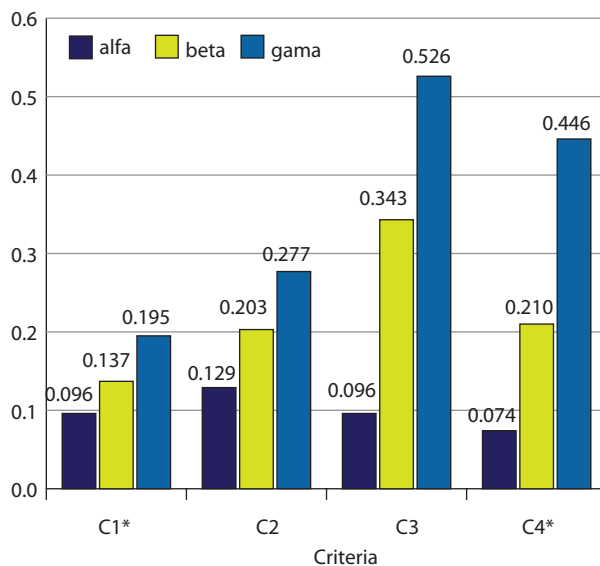**Fig. 2.** Determined weights of criteria (seven stakeholders – experts), which were obtained by applying AHP

Fig. 3. Fuzzy weights of criteria

Table 2. The rating of four criteria by decision makers

Criterion	Decision maker	A_1			A_2			A_3			A_4			Criterion weight (AHP method) w_j
							Ratings							
C_1	D_1	16	17	18	10	11	13	18	22	19	17	20	22	0.156
	D_2	15	16	17	11	12	12	17	18	19	20	23	25	0.161
	D_3	17	17	18	9	10	13	19	20	21	17	20	22	0.096
	D_4	17	18	19	11	12	13	15	17	18	19	22	24	0.124
	D_5	15	16	18	9	11	13	16	19	21	19	22	24	0.138
	D_6	18	18	20	9	11	14	15	17	19	17	20	22	0.111
	D_7	15	17	20	9	10	11	15	18	20	18	21	23	0.195
C_2	D_1	25	27	35	20	25	35	25	33	40	20	28	30	0.129
	D_2	25	27	33	15	28	40	30	33	40	30	33	35	0.277
	D_3	20	22	33	25	33	40	25	28	40	25	33	35	0.161
	D_4	25	27	33	20	30	45	30	33	40	30	33	38	0.273
	D_5	20	27	33	25	30	45	30	33	45	30	38	40	0.275
	D_6	20	22	30	15	25	40	30	38	45	25	28	33	0.222
	D_7	25	32	35	20	30	35	25	28	35	30	33	35	0.149
C_3	D_1	15	20	35	20	25	35	27	30	33	15	22	35	0.448
	D_2	20	25	35	22	25	30	27	30	33	20	22	30	0.096
	D_3	15	25	30	15	20	25	25	30	33	17	27	35	0.466
	D_4	25	30	35	25	30	35	22	25	33	20	27	30	0.526
	D_5	15	20	30	20	25	30	20	30	33	30	32	35	0.512
	D_6	25	30	35	25	28	30	30	35	33	25	27	40	0.444
	D_7	15	25	30	15	22	25	22	25	30	30	32	40	0.231
C_4	D_1	14	16	18	8	9.5	13	8.5	10	12.5	7.5	9.5	12.5	0.269
	D_2	12	14	15	7	10	12	6	8	9	5	7	9	0.446
	D_3	11	13	15	7	9.5	15	7	8.5	10	5	7.5	13	0.277
	D_4	12	14	17	7	9	11	6	8.5	10	6	7.5	11	0.077
	D_5	11	14	16	9.5	11.5	14.5	8	9	10	6	8	9	0.074
	D_6	11	14	19	8	10	11	9	10.5	12	8	10.5	13	0.222
	D_7	14	16	19	10	12.5	15	8	9	11	5	8	10	0.425

Table 3. The fuzzy decision matrix and fuzzy weights of four alternatives

Criterion	A_1			A_2			A_3			A_4			Fuzzy group weight $\tilde{w}_j$		
						Ratings							$w_{j\alpha}$	$w_{j\beta}$	$w_{j\gamma}$
C_1^*	15.0	17.0	20.0	9.0	11.0	14.0	15.0	18.6	21.0	17.0	21.1	25.0	0.096	0.137	0.195
C_2	20.0	26.1	35.0	15.0	28.6	45.0	25.0	32.1	45.0	20.0	32.1	40.0	0.129	0.203	0.277
C_3	15.0	24.7	35.0	15.0	24.8	35.0	20.0	29.1	33.0	15.0	26.7	40.0	0.096	0.343	0.526
C_4^*	11.0	14.4	19.0	7.0	10.2	15.0	6.0	9.0	12.5	5.0	8.2	13.0	0.074	0.210	0.446

Remark: * optimum value of the criterion is minimum

**Table 4.** The changed fuzzy decision making matrix

Criterion	A_0	A_1			A_2			A_3			A_4			Total			Fuzzy group weight $\tilde{w}_j$		
							Ratings										$w_{j\alpha}$	$w_{j\beta}$	$w_{j\gamma}$
C_1	0.111	0.067	0.059	0.050	0.111	0.089	0.071	0.067	0.053	0.045	0.059	0.047	0.040	0.415	0.359	0.317	0.096	0.137	0.195
C_2	45.000	20	25.3	35	20	29.2	40	25	33.0	45	20	32.2	40	130	164.7	205	0.129	0.203	0.277
C_3	40.000	15	25	35	15	25.5	35	20	30.0	35	15	26.2	40	105	146.7	185	0.096	0.343	0.526
C_4	0.200	0.091	0.070	0.053	0.143	0.101	0.067	0.167	0.110	0.080	0.200	0.120	0.077	0.801	0.601	0.477	0.074	0.210	0.446

Table 5. The normalised fuzzy decision making matrix

Criterion	Alternatives															Fuzzy group weight $\tilde{w}_j$		
	A_0			A_1			A_2			A_3			A_4					
	Ratings															$w_{j\alpha}$	$w_{j\beta}$	$w_{j\gamma}$
C_1	0.350	0.309	0.267	0.211	0.164	0.120	0.350	0.248	0.171	0.211	0.148	0.108	0.186	0.131	0.096	0.096	0.137	0.195
C_2	0.220	0.121	0.346	0.098	0.154	0.269	0.098	0.177	0.308	0.122	0.200	0.346	0.098	0.196	0.308	0.129	0.203	0.277
C_3	0.216	0.102	0.381	0.081	0.170	0.333	0.081	0.174	0.333	0.108	0.204	0.333	0.081	0.179	0.381	0.096	0.343	0.526
C_4	0.419	0.151	0.250	0.191	0.116	0.066	0.300	0.168	0.084	0.350	0.183	0.100	0.419	0.200	0.096	0.074	0.210	0.446

Table 6. The normalised-weighted fuzzy decision making matrix and solution results

Criterion	Alternatives														
	A_0			A_1			A_2			A_3			A_4		
	Ratings														
C_1	0.034	0.042	0.052	0.020	0.023	0.023	0.034	0.034	0.033	0.020	0.020	0.021	0.018	0.018	0.019
C_2	0.028	0.025	0.096	0.013	0.031	0.075	0.013	0.036	0.085	0.016	0.041	0.096	0.013	0.040	0.085
C_3	0.021	0.035	0.200	0.008	0.058	0.175	0.008	0.060	0.175	0.010	0.070	0.175	0.008	0.061	0.200
C_4	0.031	0.032	0.111	0.014	0.024	0.030	0.022	0.035	0.037	0.026	0.038	0.045	0.031	0.042	0.043
$\tilde{S}_i$	0.114	0.134	0.460	0.055	0.137	0.303	0.076	0.165	0.331	0.072	0.169	0.337	0.069	0.161	0.347
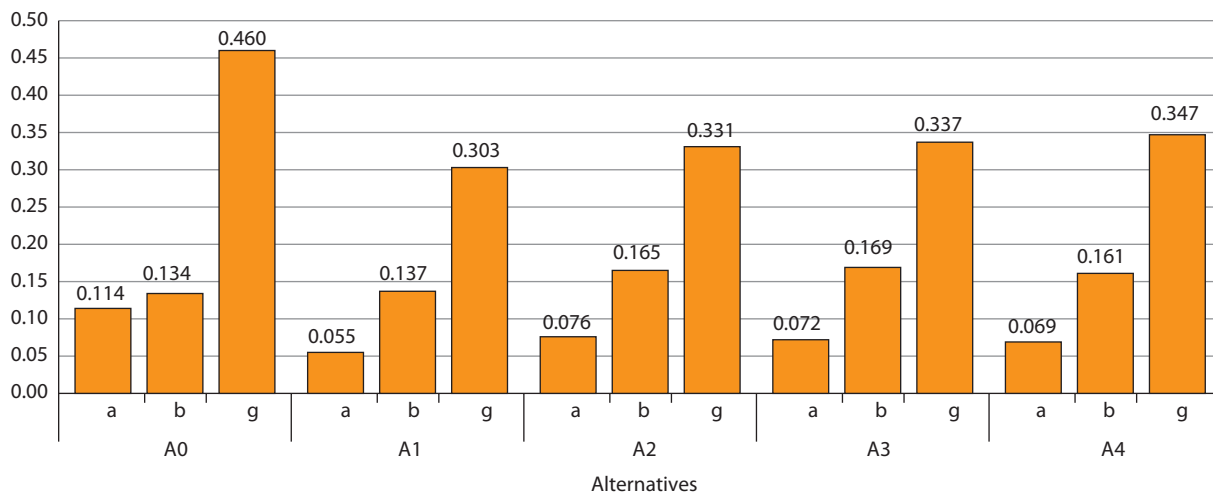
S_i	0.236	0.165	0.191	0.193	0.192
K_i	1.000	0.699	0.809	0.818	0.816
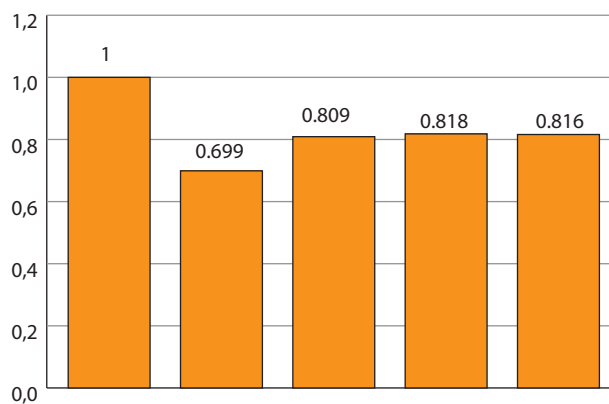
6. Conclusions

The traditional approaches of optimization, statistical and econometric analysis used within the engineering context are often based on the assumption that the considered problem is well formulated and decision-makers usually consider the existence of a single objective, evaluation criterion or point of view that underlies the conducted analysis. In such a case the solution of engineering problems is easy to obtain.

However, the modelling of engineering problems is based on a different kind of logic, taking into consideration the existence of multiple criteria, the conflicting aims of decision maker, the complex, subjective and different nature of the evaluation process, and the participation of several decision makers.

Overall, the main advantages that the MCDM provides in decision making, could be summarized in the following aspects: the possibility to analyze complex problems; the possibility to aggregate both quantitative and qualitative criteria in the evaluation process; the possibility of good evidence of decisions; the possibility for decision-maker to participate actively in the decision-making process and the application of flexible scientific methods in the decision making process.

The applications which may be generated from or adapted to fuzzy logic are wide-ranging.

According to the newly proposed ARAS-F method, the utility function value determining the complex efficiency of a feasible alternative is directly proportional to the relative effect of values and weights of the main criteria considered in a project.

The degree of the alternative utility is determined by a comparison of the variant, which is analysed, with the ideally best one.

It can be stated that the ratio with an optimal alternative may be used in cases when it is seeking to rank alternatives and find ways of improving alternative projects.

In conclusion, ARAS–F method has a promising future in the construction engineering field, because he offers a highly methodological basis for fuzzy decision support.

With an illustrative example: selection of logistics' centre location the proposed methodology is validated. It is found that there are four main criteria which must be considered: investment cost (β weight equals to 0.137), operation time (β weight equals to 0.203), expansion possibility (β weight equals to 0.343), and closeness to the demand market (β weight equals to 0.210).

Weights results show that stakeholders are more concern about the expansion possibility and the closeness to the demand market than the operation time and investment costs.

In this paper it is supposed to deal with fuzzy data to help the model to be very applicable due to lack of certainty and crisp data in real word situations especially about qualitative variables.

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