



## COMBINED MODAL SPLIT AND ASSIGNMENT MODEL FOR THE MULTIMODAL TRANSPORTATION NETWORK OF THE ECONOMIC CIRCLE IN CHINA

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**Abstract.** Economic circles have been formed and developing in China. An economic circle consists of more than one closely adjoining central cities and their influence zones. It is always the major engine for the development of one country's economy and even for the world economy. A combined modal split and assignment model with deterministic travel demand is proposed for modelling passengers' choices of intercity bus and train which are two main competing modes in the multimodal transportation network of the economic circle. The generalized travel cost model of highway and railway are used incorporating travel time, ticket fare and passenger's discomfort. On the highway network, the interactions of private vehicles and intercity buses are asymmetric. Thus, a variational inequality formulation is proposed to describe the combined model. The streamlined diagonalization algorithm is presented to solve the combined model. The multimodal transportation network based on Yangtze River Delta economic circle is presented to illustrate the proposed method. The results show the efficiency of the proposed model.

**Keywords:** combined model, multimodal transportation network, economic circle, mode choice, network equilibrium, variational inequality.

### 1. Introduction

With the rapid development of China's economy, economic circles have been formed and developing in China, such as Yangtze River Delta economic circle, Bohai Sea rim economic circle, Pearl River Delta economic circle, etc. The present economic competition in China is based on industrial and commercial interactions instead of administrative divisions. The economic circle consists of more than one closely adjoining central cities and their influence zones. The economic circle is always the major engine for the development of one country's economy and even for the world economy. For example, Yangtze River Delta economic circle contributed to 22.1 percent of the country's gross domestic product and 28.5 percent of total foreign trade in 2003. In some other countries, the economic circle is called Metropolitan Area. Neighbouring cities in the economic circle are so close not only because of their physical distances but also due to their employment and commercial dependencies. People can commute between cities in the economic circle.

The transportation network services and the development of economy promotes the growth of the trans-

portation network (Vega and Penne 2008; Afandizadeh and Moayedfar 2008; Vasilis Vasiliauskas and Barysienė 2008; Jaržemskis 2008; Kiisler 2008; Kabashkin 2007; Gromule and Yatskiv 2007). The rapid development of the economic circle in China has incurred travel demand between neighbouring cities in the economic circle dramatically. The existing transportation network cannot satisfy travel demand. Therefore, we need to construct new highways, new railways or update the existing transportation network. In order to scientifically and reasonably configure the transportation network in the economic circle, transportation demand forecasting models for the economic circle needs to be developed. This paper presents a network equilibrium model for the simultaneous prediction of mode choice and route choice over the multimodal transportation network of the economic circle.

Researchers have done hard work dealing with the problem of the transportation network. Since the first mathematical formulation of a user-equilibrium assignment was proposed by Beckmann *et al.* (1959), studies on transportation network equilibrium have developed rapidly. However, they mostly focus on the urban transportation network in the context of which important ad-

vances have been realized over the past 30 years when formulating and analyzing multi-modal network equilibrium models (Florian 1977; Florian and Spiess 1983; Nagurney 1984; Wong 1998; Ferrari 1999). These models considered several alternatives to travel from an origin to a destination by using a 'pure' mode of transport such as the private vehicle mode or the public transit mode and used logit type functions to split travel demand for each travel mode. Peric and Boilé (2006) presented the multimodal network equilibrium model with asymmetric link cost interactions. Wu and Lam (2003) proposed a network equilibrium model with motorized and non-motorized transport modes. As far as mode choice for trips between cities is concerned, logit type models were widely used in the previous studies to predict the proportions of trips taken among several competing transport modes (Vovsha 1997; Hensher 1998; Koppelman and Sethi 2005; Monzón and Rodriguez-Dapena 2006). The faults of these 'pure' logit models are not considering the configuration of the network and how the flows are distributed over the network. In order to overcome these problems, mode choice and route choice should be simultaneously predicted over the economic circle transportation network, and one of the ways is to use combined models which are far from new multimodal network setting. The synthesis and review of these models are presented by Boyce (1990 and 1998) that has made significant contributions in this field. The combined models can be formulated by using the equivalent optimization approach (Florian and Nguyen 1978; Safwat and Magnanti 1988; Lam and Huang 1992; Abrahamsson and Lundqvist 1999), variational inequality (VI) approach (Dafermos 1982; Florian et al. 2002) or the fixed-point approach (Bar-Gera and Boyce 2003).

The proposed model can simultaneously predict mode choice and route choice over the multimodal transportation network of the economic circle. The model herein extends previous works in multi-modal transportation networks. It is a combined modal split and assignment model for the multimodal transportation network which considers the traveller's mode choice of intercity bus and train while considering the asymmetric cost interaction of intercity bus and personal cars over the highway network with deterministic demand. We treat travel demand for public transit and travel demand for personal cars respectively. One of the reasons is that only a few people own cars and that their number can be obtained from the toll station of highways. So, we can think that travel demand for personal cars is known and does not need to consider the mode choice of personal cars that can make the problem easy. Another reason is that intercity bus and train modes play a more important part in connecting the cities of the economic circle. Mode choice is a complex decision process (Sheffi 1985). One of the widely used formulae accounting for mode choice is the logit function. In this paper, we use the logit formula as the mode split function based on generalized travel cost that gives more generality and complexity to the problem.

The remainder of this paper is organized as follows. The second section introduces basic considerations and notation. The third section gives the generalized cost by train and intercity bus, respectively. The fourth section defines equilibrium conditions used in this paper. The fifth section presents the VI formulation of the combined model. Then, a numerical example is followed. Finally, the conclusions are presented.

## 2. Basic Considerations and Notation

### 2.1. Basic Considerations

Trips between the cities of China's economic circle are taken by auto, intercity bus and train. Thus, we suppose that in this study, travellers can complete their trips using three modes - auto, train and intercity bus. Travel choices are denoted by a, tr and b for short respectively. Considering that only a minority of people can complete their trips by car because of the minority of private car ownership, so most trips are completed by public transit and we treat travel demand for automobiles and travel demand for public transits separately. Travel demand for cars can be obtained from highway toll stations. Therefore, we do not need to consider the choice of the above introduced three modes and can only pay attention to study the travellers' choices of intercity buses and trains. Intercity buses also run on highways and share some the same road segments with cars over the highway networks. Hence, asymmetric cost interactions between cars and intercity buses should be considered during the network analysis. Another assumption in this paper is that a traveller makes a trip from one city to another using a single mode which means we do not consider travellers' transfer between modes.

### 2.2. Notation

Consider a multimodal transportation network  $G = (N, L)$ , where  $N$  is the set of nodes and  $L$  is the set of links connecting nodes. The multimodal transportation network  $G$  in an economic circle consisting of the auto sub-network  $G_a = (N_a, L_a)$ , the intercity bus sub-network  $G_b = (N_b, L_b)$  and the train sub-network  $G_{tr} = (N_{tr}, L_{tr})$ . Automobiles can change routes freely from the origin to the destination, so every physical link may be used by automobiles and the auto sub-network  $G_a = (N_a, L_a)$  is the same as the physical highway network. In comparison with private vehicles, intercity buses have their own networks with some fixed routes and some nodes and some physical links may not be included. We defined  $N_b \subseteq N_a, L_b \subseteq L_a$ .  $R$  is the set of origins,  $S$  is the set of destinations,  $R \subset N, S \subset N$ .  $q_{rs}^a$  denotes travel demand for automobile between OD pair  $(r, s)$ ,  $r \in R$  and  $s \in S$ . It is computed in vehicular units.  $q_{rs}^b$  denotes travel demand for intercity bus between OD pair  $(r, s)$ , which is computed in passenger or person units. Passenger units can be transformed into vehicular units by the seating capacity  $\gamma$  of an intercity bus.  $q_{rs}^{tr}$  denotes travel demand for train between OD pair  $(r, s)$ , which is also computed in passenger or person units.  $q_{rs}$  denotes public transit demand which is the sum of  $q_{rs}^{tr}$  and  $q_{rs}^b$  between OD pair  $(r, s)$ .

### 3. Generalized Travel Cost

#### 3.1. Generalized Cost of Train Travel

Travel time is often used as the sole measure of travel cost because it is easier to be measured. However, ticket fare is also a very important factor influencing travellers' mode choices for an intercity trip.

The cost of train travel is composed of two parts including travel time and ticket price. In-vehicle travel time on the train can be given and fixed because a train has an exclusive right-of-way without congestion interactions with other transportation modes and we can consider it as a constant. However, as the number of passengers inside the train increase, passengers will feel uncomfortable because of the crowded trains in which passengers have no seats or small spaces. In order to capture crowding effect, modified in-vehicle travel time is used herein and is expressed as a BPR-type function with regard to constant travel time, the number of passengers and the carrying capacity of the train line which is like metro travel time (Li *et al.* 2007):

$$t_{rs}^{tr} = \omega_{rs}^{tr} \left[ 1.0 + 0.1 \left( \frac{q_{rs}^{tr}}{pC^{tr}} \right)^{2.0} \right], \quad (1)$$

where:  $t_{rs}^{tr}$  is the modified in-vehicle travel time of a train between OD pair  $(r, s)$ ;  $\omega_{rs}^{tr}$  is in-vehicle travel time between OD pair  $(r, s)$  and is a constant;  $C^{tr}$  is the carrying capacity of one train;  $p$  is train frequency during unit time.

Train ticket price is directly proportional to the distance.

Hence, generalized travel cost by train  $G_{rs}^{tr}$  between OD pair  $(r, s)$  measured in terms of equivalent monetary units can be given as:

$$G_{rs}^{tr}(s_{rs}^{tr}) = \tau t_{rs}^{tr} + \lambda_{rs}^{tr} s_{rs}^{tr}, \quad (2)$$

where:  $\pi$  is the travellers' value of time;  $\lambda_{rs}^{tr}$  is the price per kilometre for train between OD pair  $(r, s)$ ;  $s_{rs}^{tr}$  is the length of the railway line between OD pair  $(r, s)$ .

#### 3.2. Generalized Cost of Travelling by Intercity Bus

We also incorporate travel time and bus fare into the generalized cost of travelling by intercity buses. Travel time on the highway is not like that of train as it is associated with traffic conditions rather with a constant. Intercity buses move with automobiles on the highway and experience congestion and delays. Intercity buses and private vehicles share some the same physical road segments. Thus, the travel times of private vehicles are influenced by both automobile flows and intercity bus flows, and the travel times of intercity buses depend on the flows of both intercity buses and private vehicles. In comparison with private vehicles, bus routes and schedules can be assumed fixed in short-run equilibrium analysis, so travel times and flow interactions between intercity buses and automobiles are asymmetric.

Let  $t^{al}$  represents travel times on automobile link  $l$  and  $t^{bl}$  represents travel times on intercity bus link  $l$ , re-

spectively. Link performance functions are given in the following way:

$$t^{al} = t^{al}(x^{al}, v^{bl}) = t^{al(0)} \left[ 1 + \alpha_1 \left( \frac{x^{al}}{c^{al}} \right)^{\beta_1} + \alpha_2 \left( \frac{v^{bl}}{c^{bl}} \right)^{\beta_2} \right], \quad (3)$$

$$t^{bl} = t^{bl}(v^{bl}, x^{al}) = t^{bl(0)} \left[ 1 + \alpha_3 \left( \frac{v^{bl}}{c^{bl}} \right)^{\beta_3} + \alpha_4 \left( \frac{x^{al}}{c^{al}} \right)^{\beta_4} \right], \quad (4)$$

where:  $t^{al(0)}$  and  $c^{al}$  are the free-flow travel time and capacity of automobile link  $l$  respectively;  $t^{bl(0)}$  and  $c^{bl}$  are the free-flow travel time and capacity of intercity bus link  $l$  respectively;  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$ , and  $\beta_4$  are coefficients;  $x^{al}$  is automobile flow on automobile link  $l$  and let  $X^a = (\dots, x^{al}, \dots)$ ,  $v^{bl}$  is the intercity bus flow of intercity bus link  $l$ .

Automobile travel time  $c_{rs}^{ap}$  on route  $p$  from origin  $r$  to destination  $s$  can be given by the sum of travel time on the links comprising this route, i.e.,

$$c_{rs}^{ap} = \sum_{l \in L_a} t^{al} \delta_{l,p}^{rs} \forall r, s, p \in P_{rs}^a, r \in R, s \in S, \quad (5)$$

where:  $\delta_{l,p}^{rs}$  equals to 1 if link  $l$  is a part of route  $p$  from origin  $r$  to destination  $s$ , and 0 otherwise;  $L_a$  is the set of automobile links;  $P_{rs}^a$  is the set of all routes used by automobiles between OD pair  $(r, s)$ .

Travel time of using intercity bus on path  $p$  from origin  $r$  to destination  $s$ ,  $c_{rs}^{bp}$  can be given by the sum of travel time on the links comprising this path, i.e.,

$$c_{rs}^{bp} = \sum_{l \in L_b} t^{bl} \delta_{l,p}^{rs} \forall r, s, p \in P_{rs}^b, r \in R, s \in S, \quad (6)$$

where:  $\delta_{l,p}^{rs}$  equals to 1 if link  $l$  is a part of path  $p$  from origin  $r$  to destination  $s$ , and 0 otherwise;  $L_b$  is the set of intercity bus links and  $P_{rs}^b$  is the set of all paths used by intercity bus between OD pair  $(r, s)$ .

The generalized cost of travelling by intercity bus  $G_{rs}^{bp}$  on path  $p$  between OD pair  $(r, s)$  is expressed as:

$$G_{rs}^{bp}(s_{rs}^{bp}) = \tau c_{rs}^{bp} + s_{rs}^{bp} \lambda_{rs}^{bp}, \quad (7)$$

where:  $\tau$  is the travellers' value of time;  $\lambda_{rs}^{bp}$  is the price per kilometre for intercity bus between OD pair  $(r, s)$ ;  $s_{rs}^{bp}$  is the length of path  $p$  between OD pair  $(r, s)$ .

Note that passenger's discomfort is not considered in the travel cost of travelling by intercity buses because bus operators are not allowed to carry more travellers than their seating capacity.

### 4. Equilibrium Conditions

The UE condition can be described as follows: at user equilibrium, for each OD pair, travel times on all used paths are equal and less than or equal to those would be experienced by a single vehicle on any unused path (Sheffi 1985). The definition of multimodal transportation network equilibrium in the economic circle mentioned in this paper is as follows.

#### 4.1. Choosing an Automobile Route

At equilibrium, auto users' travel choice decisions on satisfying the UE condition stating that for each OD pair, only paths with minimum cost are used can be mathematically expressed as:

$$(c_{rs}^{ap} - \mu_{rs}^a) f_{rs}^{ap} = 0, c_{rs}^{ap} - \mu_{rs}^a \geq 0, \forall r, s, p \in P_{rs}^a, \quad (8)$$

where:  $f_{rs}^{ap}$  is automobile flow on path  $p$  between OD pair  $(r, s)$ ;  $\mu_{rs}^a$  is minimum automobile travel time between OD pair  $(r, s)$ .

#### 4.2. Choosing an Intercity Bus Route

Equilibrium flows over the intercity bus sub network is assumed to satisfy UE conditions. Thus, at equilibrium, flows and travel times over the intercity bus sub network are such that:

$$(G_{rs}^{bp} - G_{rs}^b) f_{rs}^{bp} = 0, G_{rs}^{bp} - G_{rs}^b \geq 0, \forall r, s, p \in P_{rs}^b, \quad (9)$$

where:  $f_{rs}^{bp}$  is passenger flow on path  $p$  between OD pair  $(r, s)$ ;  $G_{rs}^b$  is the minimum generalized travel cost between OD pair  $(r, s)$ .

#### 4.3. Choosing Intercity Bus and Train

It is assumed there is only one railway line between cities and all passengers would use a no-transfer path from their origin to their destination, so the route is fixed when passengers choose the train as a travel mode. The traveller's choice of intercity bus and train is governed by the logit-type formula expressed as:

$$q_{rs}^b = q_{rs} \frac{1}{1 + e^{\theta_{rs}(G_{rs}^b - G_{rs}^{tr} - \varphi_{rs})}}, \quad (10)$$

where:  $\varphi_{rs}$  represents the bias parameter of commuters on train for OD pair  $(r, s)$ ;  $\theta_{rs}$  describes the importance of travel disutility perception of choosing the right mode for OD pair  $(r, s)$ . Two parameters can be calibrated using the observed data of choosing the right mode.

Assuming the logit-based modal split function, equivalent travel cost  $W_{rs}(q_{rs}^{tr})$  on the train for each OD pair can be expressed as:

$$W_{rs}(q_{rs}^{tr}) = \frac{1}{\theta_{rs}} \ln \frac{q_{rs}^{tr}}{q_{rs} - q_{rs}^{tr}} + \varphi_{rs} + G_{rs}^{tr}(q_{rs}^{tr}). \quad (11)$$

### 5. Variational Inequality Model Formulation

Since travel time and flow interactions between the inter-city bus and automobile modes are asymmetric, the problem under consideration cannot be formulated and solved as an equivalent minimization program. The problem is thus formulated as variational inequality. The equivalent VI formulation for the network equilibrium conditions presented in the previous section is given below.

$$\sum_{rs} \sum_{l \in L_a} t^{al}(x^{al*}, v^{bl*})(x^{al} - x^{al*}) + \sum_{rs} \sum_{p \in P_{rs}^b} G_{rs}^{bp}(f_{rs}^{bp} - f_{rs}^{bp*}) +$$

$$\sum_{rs} \left( \frac{1}{\theta_{rs}} \ln \frac{q_{rs}^{tr*}}{q_{rs}} + \varphi_{rs}^{tr} \right) (q_{rs}^{tr} - q_{rs}^{tr*}) + \sum_{rs} \left( \frac{1}{\theta_{rs}} \ln \frac{q_{rs}^{b*}}{q_{rs}} + \varphi_{rs}^b \right) (q_{rs}^b - q_{rs}^{b*}) \geq 0. \quad (12)$$

Subject to:

$$q_{rs}^{tr} + q_{rs}^b = q_{rs}, \forall r, s; \quad (13)$$

$$\sum_{p \in P_{rs}^a} f_{rs}^{ap} = q_{rs}^a, \forall r, s; \quad (14)$$

$$\sum_{p \in P_{rs}^b} f_{rs}^{bp} = q_{rs}^b, \forall r, s; \quad (15)$$

$$f_{rs}^{tr} = q_{rs}^{tr}, \forall r, s; \quad (16)$$

$$x^{al} = \sum_{rs} \sum_{p \in P_{rs}^a} f_{rs}^{ap} \delta_{l,p}^{rs}, \forall r, s; \quad (17)$$

$$x^{bl} = \sum_{rs} \sum_{p \in P_{rs}^b} f_{rs}^{bp} \delta_{l,p}^{rs}, \forall r, s; \quad (18)$$

$$v^{bl} = \frac{x^{bl}}{\gamma}, \forall r, s; \quad (19)$$

$$f_{rs}^{ap}, f_{rs}^{bk} \geq 0, \forall r, s, p \in P_{rs}^a, k \in P_{rs}^b; \quad (20)$$

$$q_{rs}^{tr} \geq 0, q_{rs}^b \geq 0, \forall r, s, \quad (21)$$

where:  $x^{bl}$  is passenger flow on intercity bus link  $l$  and let  $X^b = (\dots, x^{bl}, \dots)$ .

Equation (13) is conservation constraint on mode demand, Equations (14), (15), and (16) represent a set of flow conservation constraints, Equations (17) and (18) are the relationship between link flow and path flow for automobile and intercity bus respectively and Equations (20) and (21) are nonnegative constraints.

We can prove that the proposed VI formulation (12) leads to equilibrium conditions (8) – (10) according to the KKT conditions of VI formulation.

### 6. Solution Algorithm

The streamlined version of the diagonalization algorithm is one of the methods that can solve the above introduced program of variational inequality. A detailed description of the diagonalization algorithm can be found in literature (Sheffi 1985; Nagurney 1998). A description of the solution algorithm is as follows:

**Step 0:** Initialization. Find a feasible link flow pattern vector. Set  $n = 0$ .

Perform all-or-nothing assignment for the automobile sub-network based on  $t^{al} = t^{al}(0, 0), \forall r, s, l$ . This yields  $\{x^{al(0)}\}$ .

Perform all-or-nothing assignment for inter-city bus and train sub-networks based on the initial modal demands determined by the initial modal splits based on zero flow cost. This yields  $\{x^{bl(0)}\}$  and  $\{q_{rs}^{tr(0)}\}$ .

In order to avoid division by zero or a zero value inside the logarithmic function, the shortest paths for inter-city bus and train sub-networks are found and based on their respective empty flow path cost as well

as the initial modal splits resulting from the application of the logit modal split function are determined. Modal demands are then assigned to the shortest paths in each sub-network, thus giving the initial solution for the diagonalization algorithm.

**Step 1:** Update travel times. Set travel times for automobile and inter-city bus based on the new travel pattern  $\{x^{al(n)}\}$  and  $\{x^{bl(n)}\}$ .

$$t^{al(n)} = t^{al}(x^{al(n)}, x^{bl(n)}) = t^{al(0)} \left[ 1 + \alpha_1 \left( \frac{x^{al(n)}}{c^{al}} \right)^{\beta_1} + \alpha_2 \left( \frac{x^{bl(n)}}{\gamma c^{bl}} \right)^{\beta_2} \right]$$

$$t^{bl(n)} = t^{bl}(x^{bl(n)}, x^{al(n)}) = t^{bl(0)} \left[ 1 + \alpha_3 \left( \frac{x^{bl(n)}}{\gamma c^{bl}} \right)^{\beta_3} + \alpha_4 \left( \frac{x^{al(n)}}{c^{al}} \right)^{\beta_4} \right].$$

Compute the generalized travel cost for intercity-bus,

$$G_{rs}^{bl(n)}(s_{rs}^{bp}) = \tau c_{rs}^{bp(n)} + s_{rs}^{bp} \lambda_{rs}^{bp}.$$

Set augment travel cost for the train sub-network based on the new travel pattern  $\{q_{rs}^{tr(n)}\}$  using:

$$W_{rs}(q_{rs}^{tr(n)}) = \frac{1}{\theta} \ln \frac{q_{rs}^{tr(n)}}{q_{rs} - q_{rs}^{tr(n)}} + \varphi_{rs} + G_{rs}^{tr}(q_{rs}^{tr(n)}).$$

**Step 2:** Direction finding. Perform all-or-nothing assignment over three sub-networks. This yields a travel pattern,  $\{y^{al(n)}\}$  over the auto network,  $\{y^{bl(n)}\}$  the intercity bus network and  $\{z_{rs}^{tr(n)}\}$  the train network.

**Step 3:** Move-size determination. Find a scalar,  $\alpha_n$  which solves the following program:

$$\min Z(X^a, q_{rs}^{tr}, X^b) = \sum_{l \in L_a} \int_0^{x^{al(n)} + \alpha(y^{al(n)} - x^{al(n)})} t_{rs}^{al}(\omega, v^{bl(n)}) d\omega + \sum_{p \in L_b} \int_0^{x^{bl(n)} + \alpha(y^{bl(n)} - x^{bl(n)})} t_{rs}^{bl}(x^{al(n)}, v) dv + \sum_{rs} \int_0^{q_{rs}^{tr(n)} + \alpha(z_{rs}^{tr(n)} - q_{rs}^{tr(n)})} \left( \frac{1}{\theta} \ln \frac{\rho}{q_{rs} - \rho} + \varphi_{rs} + G_{rs}^{tr}(\rho) \right) d\rho.$$

**Step 4:** Updating. Set:

$$x^{al(n+1)} = x^{al(n)} + \alpha_n (y^{al(n)} - x^{al(n)}),$$

$$x^{bl(n+1)} = x^{bl(n)} + \alpha_n (y^{bl(n)} - x^{bl(n)}),$$

$$q_{rs}^{tr(n+1)} = q_{rs}^{tr(n)} + \alpha_n (z_{rs}^{tr(n)} - q_{rs}^{tr(n)}).$$

**Step 5:** Convergence test.

If  $(X^{a(n+1)}, q_{rs}^{tr(n+1)}, X^{b(n+1)}) \approx (X^{a(n)}, q_{rs}^{tr(n)}, X^{b(n)})$ , stop.

The solution is  $(X^{a(n+1)}, q_{rs}^{tr(n+1)}, X^{b(n+1)})$ .

Otherwise, set  $n := n + 1$  and go to step 1.

## 7. Numerical Example

### 7.1. Data Input

In this section, a test network has been developed based on the transportation network of Yangtze River Delta economic circle, as shown in Fig. 1. The highway network consists of eleven nodes, fourteen links, three bus routes, three railway lines and three OD pairs (A–B, B–C and A–C). OD pairs (B–A, C–B and C–A) can be solved applying the same method, thus we only calculate OD pairs (A–B, B–C and A–C) as an example. In Fig. 1, Nodes A, B and C denote Nanjing Shanghai and Hangzhou respectively and are the core cities in Yangtze River Delta economic circle; dotted lines represent the railway links; red lines denote highways the capacity of which is 3600 passenger car units (pcu) per hour and 1800 intercity buses per hour respectively; green lines represent highways the capacity of which makes 3200 pcu per hour. Intercity bus lines are supposed to be fixed, as shown in Fig. 1. The bus line from A to B consists of link 1, 2, 3 and 4, the bus line from A to C consists of link 5, 6, 7 and 8 and the bus line from B to C consists of link 9 and 10.

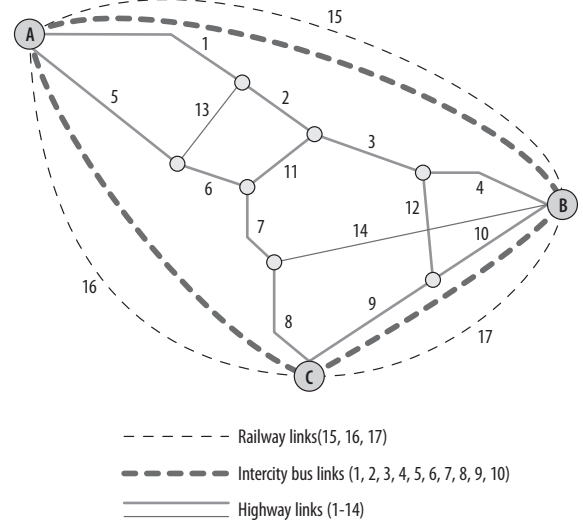


Fig. 1. Test network based on Yangtze River Delta Economic Circle

The length of highway links and the free-flow travel time of automobiles and intercity buses are shown in Table 1. The price per kilometre for train is assumed to be 0.31 Yuan based on the survey. Table 2 shows train fare, bus fare, the seating capacity of a train and an intercity bus. The vehicle occupancy of a train is 615 passengers. The seating capacity of an intercity bus is supposed to be 60 passengers. The value of time is assumed to be 25 Yuan per hour. Other model parameters are:

$$\alpha_1 = 0.15, \alpha_2 = 0.1, \alpha_3 = 0.2, \alpha_4 = 0.15,$$

$$\beta_1 = \beta_2 = \beta_3 = \beta_4 = 4, \theta_{AB} = 0.3, \theta_{BC} = 0.01,$$

$$\theta_{AC} = 0.1, \varphi = 0.$$

**Table 1.** Parameters of the intercity bus link and private vehicle link travel time

Highway link	Length (km)	$t_l^{a(0)}$ (h)	$t_l^{b(0)}$ (h)	Highway link	Length (km)	$t_l^{a(0)}$ (h)	$t_l^{b(0)}$ (h)	Highway link	Length (km)	$t_l^{a(0)}$ (h)	$t_l^{b(0)}$ (h)
(A,1)	113	0.94	1.03	(A,4)	86	0.72	0.78	(7,B)	93	0.78	0.85
(1,2)	36	0.30	0.33	(4,5)	25	0.21	0.23	(6,B)	157	1.57	—
(2,3)	42	0.35	0.38	(5,6)	35	0.29	0.32	(4,1)	40	0.40	—
(3,B)	83	0.69	0.75	(6,C)	91	0.76	0.83	(2,5)	32	0.27	—
(C,7)	73	0.61	0.66	(3,7)	78	0.65	—				

Note: — means no data for that cell.

**Table 2.** Parameters of trains and intercity buses

OD	Railway length (km)	$\omega_{rs}^{tr}$ (h)	$C^{tr}$ (passengers/veh)	Train fare (Yuan/km)	Railway Frequency (veh/hr)	Highway length (km)	Intercity bus Fare cost (Yuan/km)
A-B	301	2.16	615	0.31	4	274	0.33
B-C	173	1.38	615	0.31	1	166	0.33
A-C	504	4	615	0.31	6	237	0.49

## 7.2. Testing Results

Travel demand and public transit demand in 2008 are illustrated in Table 3. Other input data is described in the above section. Now, the model is used to predict how passengers will choose their travel modes. The obtained results are shown in Table 4.

Table 4 shows that the percentage of passengers from B to C by train is less than that of passengers by intercity bus. In order to attract more passengers to make their trips by train, the railway operator can cut train fare. In this study, we have cut train fare between A and C from 156 Yuan to 126 Yuan and found out that 6% of total passengers transfer to take a train and the total travel cost of the system reduces by 0.29%.

The following charts show how passengers' choices in picking a mode change with different levels of demand for OD pairs AB, AC and BC. Fig. 2 shows that the number of passengers who travel by intercity bus increase along with the total demand for public transit. Fig. 3 reveals that the number of passengers who travel by train decreases with the increased total demand for public transit. The reason for this result is that people feel more uncomfortable inside the train as passenger volume inside the train increases. Some passengers change their decisions on accepting the right mode and transfer to make their trips by intercity bus.

**Table 3.** Travel demand

OD	Vehicle travel demand (veh/h)	Public transit demand (passengers/h)
A-B	4 000	4 867
A-C	3 000	1 511
B-C	2 000	11 933

**Table 4.** Results of mode split

OD	Passengers by intercity bus	Percentage by intercity bus	Passengers by train	Percentage by train
A-B	1 444	30%	3 423	70%
A-C	1 067	71%	444	29%
B-C	4 101	34%	7 832	66%

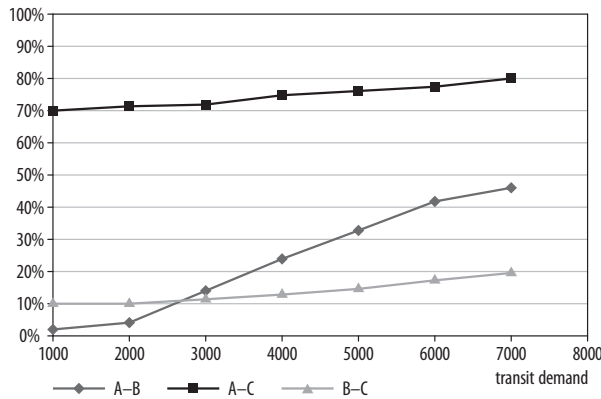


Fig. 2. The percentage of passengers using intercity bus with different levels of transit demand

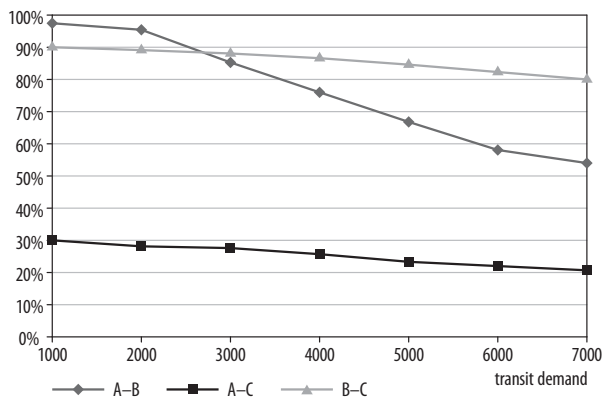


Fig. 3. The percentage of passengers using train with different levels of transit demand

8. Conclusions

- In this paper, a combined modal split and assignment model with determined travel demand is proposed for modelling passengers’ choices of intercity bus and train which are two main competing modes in the multimodal transportation network of the economic circle. It is assumed that the choices of routes for automobiles and passenger flows with asymmetric cost interactions on the highway network satisfy user equilibrium conditions. In addition, the travellers’ choice of intercity bus and train is governed by the logit-type formula. The problem of multimodal network equilibrium has been formulated as a variational inequality formulation. The case study shows that the results of the combined model traffic flow of highway links and the passenger flow of railway can be simultaneously obtained.
- The findings have shown how travellers’ mode choices change with different levels of demand and how much the reduced train fare affects passengers’ choice of travel modes.
- It should be noted that the current model only consider one class of passengers and the value of time is considered as the same. There are many

different classes of passengers in the economic circle because of different income and different travel purpose, so the model should be extended considering the value-of-time of multi-classes for further improvement in the prediction power of the proposed model in the future study. In addition, future studies may also incorporate trip cost in the city into the generalized travel cost.

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