



THE INVESTIGATION OF DYNAMICS OF MISALIGNED UNITS IN VEHICLE TRANSMISSION

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Abstract. In the article the problems related to the dynamics of a mechanic system on misalignment of shafts in radial direction are presented. The object of the investigation is a two-shaft system connected with an elastic centrifugal ring coupling.

Using equations of static equilibrium it was found that an internal moment of resistance to rotation appears in the coupling connecting the radially misaligned shafts. Using D'Alembert's principle for the rotational movement the differential equation that describes the rotation of the second shaft has been developed. It was shown that after the performance of the corresponding actions and the introduction of a new variable the said equation is transformed in to an equation which character virtually coincides with the equation describing free oscillations of a mathematical pendulum.

Because the value of misalignment of shafts in the radial direction is small in comparison with other parameters, a small parameter method was used for the solution of this equation. The found solutions show that rotational vibrations with the double frequency of rotational movement are excited in the misaligned mechanical system with an elastic centrifugal ring coupling.

The restrictions of the application of a small parameters method have been explored and the limits of its application have been found.

Keywords: vehicle transmission, elastic centrifugal ring coupling, misaligned shafts, rotational vibrations, small parameter method.

1. Introduction

In vehicles with rear driving wheels a rotational movement is transferred from the engine to them via a number of mechanical equipments. On driving the body with the engine vibrates in the direction perpendicular to the longitudinal axis of the transmission and this causes radial misalignments of axes of shafts in units of the transmission. On the rotation of shafts of the units this causes variable loads on their fittings, so the rotational movement becomes uneven. This phenomenon causes sensible vibrations in the vehicle as well as the reduction of the service time of units and assemblies for periodically varying loads. The problem of the reduction of vibrations is discussed in many works, for example [1-3].

2. The Object of the Investigation

One of the methods of the reduction of unevenness of a rotational movement is based on the usage of elastic centrifugal ring couplings in the cardan drive. It was already described in [4]. Herein, we'll discuss the dynamic processes in such drive when shafts being misaligned in radial direction are connected using the above-mentioned couplings.

The construction of the simplest elastic centrifugal ring coupling is presented in Fig 1.

The coupling consists of the driving half-coupling 1 and the driven half-coupling 2. The terminals 3 and 4 of the half-couplings are connected with the elastic steel ring 5, usually made of a wire wound into a circle. When the driving half-coupling is loaded with the moment of rotation a slight angular shifting of the half-couplings in respect of each other takes place on their rotation because

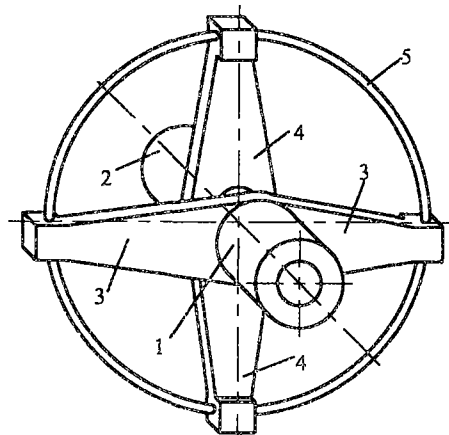


Fig 1. The construction of the simplest elastic centrifugal ring coupling

of elastic deformations of the ring 5 and the shape of the ring becomes similar to an ellipsis. On the rotation of the system the forces of elasticity of the ring and the centrifugal forces of its distributed mass seek for the restoration of the initial round shape of the ring and simultaneously reduce the deformation of the coupling. So links of two types (elastic and centrifugal) participate in the transmission of a rotational movement [5].

3. Connection of Shafts Misaligned in Radial Direction

The above-described situation is characteristic for the connection of ideally aligned shafts. In most cases the axes of the shafts do not coincide, for example, because of vibrations of the body in respect of driving wheels that appear in the transmission of the vehicle on driving.

In the case of radial misalignment of connected shafts the shape of the deformed elastic ring will be another (Fig 2); the reactions P_x and P_y appear in the fitting points of the terminals of the half-couplings.

In this case potential energy of the deformed elastic ring is found from the following expression:

$$\Pi = \sum_n \int_0^{2\pi} \frac{M_1 R d\varphi}{2EI} + \sum_n \int_0^{2\pi} \frac{N_a R d\varphi}{2EF}, \quad (1)$$

where M_1 – the moment of bending in the ring; N_a – the axial force in it; R – the radius of the bend of the ring; EI , EF – the stiffness of the ring for bending and lengthening; φ – an angular coordinate; n – the number of ring segments between the terminals of half-couplings.

Taking into consideration that

$$\begin{aligned} \frac{\partial \Pi}{\partial P_x} &= e \cos \gamma, \\ \frac{\partial \Pi}{\partial P_y} &= e \sin \gamma, \end{aligned} \quad (2)$$

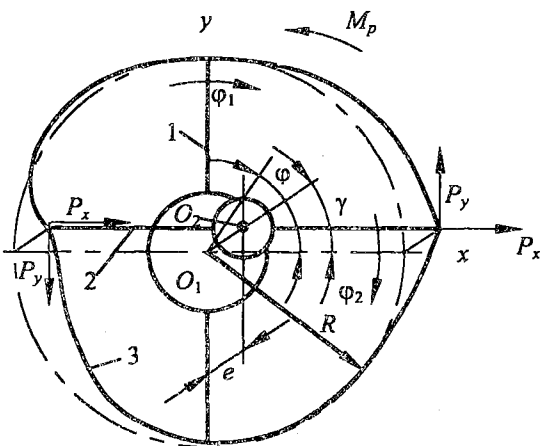


Fig 2. Deformation of the elastic ring of the coupling in case of radial misalignment

we find the values of the reactions P_x and P_y as the functions of radial misalignment e :

$$\begin{aligned} P_x &= \frac{2EI}{R} (a \cos \gamma + b \sin \gamma), \\ P_y &= \frac{2EI}{R} (a \sin \gamma + b \cos \gamma), \end{aligned} \quad (3)$$

where

$$a = \frac{\pi^2 - 8}{4\pi}, \quad b = \frac{\pi - 4}{2\pi}.$$

On the rotation of the system, an internal moment of resistance to rotation M_p appears in the coupling that is found from the equation of moments around the point O_1 :

$$M_p = 2e(P_y \cos \gamma - P_x \sin \gamma). \quad (4)$$

Using the expressions of the reactions P_x and P_y from (3), we obtain:

$$M_p = 157 \frac{EI}{R^3} e^2 \cos 2\gamma. \quad (5)$$

In the case under discussion we may examine the rotating system as a two-mass system with two degrees of freedom considering that its generalized coordinates are the angle of rotation of the first shaft with the half-coupling 1 φ_1 and the angle of rotation of the second shaft with the half-coupling 2 φ_2 (Fig 2).

Taking into account that the angle of rotation γ of the coupling in respect of the direction of misalignment may be interpreted as

$$\gamma = \frac{\varphi_1 + \varphi_2}{2}, \quad (6)$$

the expression of internal moment of resistance to rotation becomes the following:

$$M_p = 157 \frac{EI}{R^3} e^2 \cos(\varphi_1 + \varphi_2). \quad (7)$$

On the investigation of the impact of the misalignment of shafts upon the dynamics of a rotating system we suppose that rotation of the first shaft is even. In such case the generalized coordinates that describe the movement of the system will be the following:

$$\begin{aligned} \varphi_1 &= \alpha_1 + \omega t, \\ \varphi_2 &= \alpha_2 + \omega t + x_e, \end{aligned} \quad (8)$$

where α_1, α_2 – the initial angles of rotation, ω – the angular speed, t – time, x_e – the value (angle) that characterizes the unevenness of rotation of the second shaft.

According to D'alambert's principle rotation of the second shaft is described by the following equation:

$$I_2 \ddot{\varphi}_2 - M_p = 0, \quad (9)$$

where: I_2 – the moment of inertia of the second shaft.

From the equations (7), (8) and (9), we find:

$$\ddot{x}_e - 157 \frac{e^2 EI}{I_2 R^3} \cos(\alpha_1 + \alpha_2 + 2\omega t + x_e) = 0 \quad (10)$$

We introduce a new variable:

$$y_e = \frac{\pi}{2} - (\alpha_1 + \alpha_2 + 2\omega t + x_e) \quad (11)$$

and find its second derivative on time and obtain the equation that virtually describes the free oscillations of mathematical pendulum:

$$\ddot{y}_e + A_e \sin y_e = 0, \quad (12)$$

where

$$A_e = 157 \frac{e^2 EI}{I_2 R^3}.$$

If we suppose that at the moment t_0 $y_e(t_0) = \alpha_y$, $\dot{y}_e(t_0) = \beta_y$, we find:

$$\dot{y}_e = \pm \sqrt{2A_e \cos y_e + \beta_y^2 - 2A_e \cos \alpha_y}. \quad (13)$$

Then after the corresponding transformations:

$$t - t_0 = \int_{\alpha_y}^{y_e} \frac{d y_e}{\sqrt{2A_e (\cos y_e - \cos \alpha_y) + \beta_y^2}}. \quad (14)$$

Then the introduction of the following designations:

$$\sin \frac{1}{2} y_e = k_1 u_1, \quad k_1^2 = \sin^2 \frac{1}{2} \alpha_y + \frac{\beta_y^2}{4A_e}, \quad (15)$$

the expression (14) turns into an elliptic integral of the first degree:

$$\sqrt{A_e} (t - t_0) = \int_{\frac{1}{k_1} \sin \frac{\alpha_y}{2}}^{u_1} \frac{d u_1}{\sqrt{(1 - u_1^2)(1 - k_1^2 u_1^2)}}, \quad (16)$$

that may be solved in respect of x_e .

4. The Solution Using the Small Parameter Method

Bearing in mind that the value of misalignment e is small in comparison with other sizes and, in addition, it is raised to the second power in the expression, we'll try to find a solution of the expression (12) using the small parameter method:

$$y_e = y_{e_0} + \varepsilon y_{e_1} + \varepsilon^2 y_{e_2} + \dots + \varepsilon^n y_{e_n} + \dots, \quad (17)$$

where ε - a small positive parameter.

If we limit ourselves with two first members, we obtain:

$$\begin{cases} \ddot{y}_{e_0} = 0, \\ \ddot{y}_{e_1} + A_e \sin y_{e_0} = 0. \end{cases} \quad (18)$$

Taking into account the initial conditions from the expression (11), we find the solutions of the system:

$$\begin{cases} y_{e_0} = 2\omega t + \delta_0, \\ y_{e_1} = \frac{1}{4\omega^2} A_e \sin(2\omega t + \delta) + 2\omega t + \delta_1, \end{cases} \quad (19)$$

where δ_0, δ_1 - constants.

After the insertion of the expression (19) into the expression (17) and usage of the expression (11), the generalized coordinate φ_2 , describing rotation of the second shaft will be equal to:

$$\varphi_2 = \alpha_2 + \omega t + \frac{39,25 e^2 EI}{\omega^2 I_2 R^3} \sin(2\omega t + \delta). \quad (20)$$

The last member of the expression (20) indicates that the rotation of the second shaft is uneven, its angle of rotation is supplemented with a periodic component of the double frequency of rotation (Fig 3).

Another very important circumstance: the amplitude of this periodical part decreases increasing the angular rotational speed (ω_2 is included into the denominator). This indicates an increase of the evenness of the movement.

5. The Restrictions of the Applications of the Small Parameter Method

The presence of the angular rotational speed in the denominator of the expression of the amplitude indicates

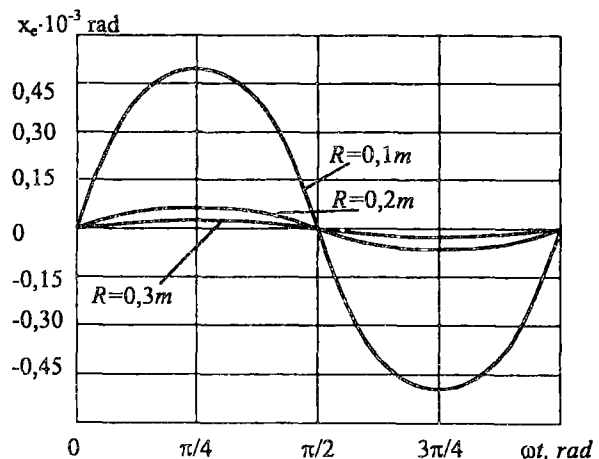


Fig 3. The dependence of the amplitude of rotational vibrations of the second shaft on the angular rotational speed when the diameter of the wire of the ring $d=3$ mm, $I_2=0,05$ kgm², $e=3$ mm, $\omega=100$ s⁻¹, $\alpha_2=0$, $\delta_0=0$

that the small parameter method cannot be applied for a solution of the equation (12) in all cases (when $\omega \rightarrow 0$, the amplitude is growing up to the infinity).

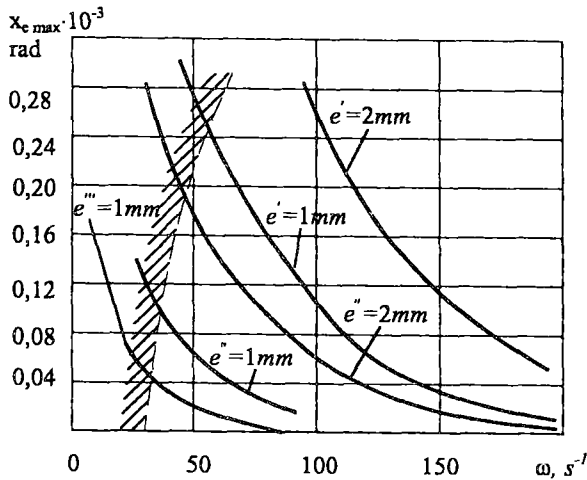


Fig 4. The dependence of the amplitude of rotational vibrations of the second shaft on the angular rotational speed, when the diameter of the wire of the ring $d=3$ mm, ' $-R=0,1$ m, '' $-R=0,2$ m, ''' $-R=0,3$ m

The problem on the application of the expression (20) may be settled on a base of the following considerations. The positive angle of shifting of the half-couplings in respect of each other upon an impact of the moment of resistance to rotation M_p may be found from the following expression:

$$x_e = \frac{M_p}{c_s}, \quad (21)$$

where c_s – the rotational stiffness of the coupling.

Knowing that $c_s = 87 \frac{EI}{R^3}$ [6], we find:

$$x_{e_{\max}} = 1,8 \left(\frac{e}{R} \right)^2. \quad (22)$$

So, on the comparison of the expressions (22) and (20) according to the condition of non-exceeding $x_{e_{\max}}$, we find the lower limit value of the angular speed applicable to the expression (20) (the stroked zone in Fig 4).

6. Conclusions

1. Elastic centrifugal ring couplings may be successfully used for the connection of misligned shafts.
2. The rotation of rotating system misligned in radial direction may be described with the equation that virtually coincides with the equation describing free oscillations of a mathematical pendulum.
3. The frequency of the additional periodical component of the coordinate of unevenness of rotation is equal to the double frequency of the rotation of the system.
4. The application of the small parameter system is not fit in all cases. It is necessary to take into consideration the specific features of the coupling construction.
5. If two misligned shafts are connected with an elastic centrifugal ring coupling, the evenness of the rotation increases with the increase of the rotational speed.

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